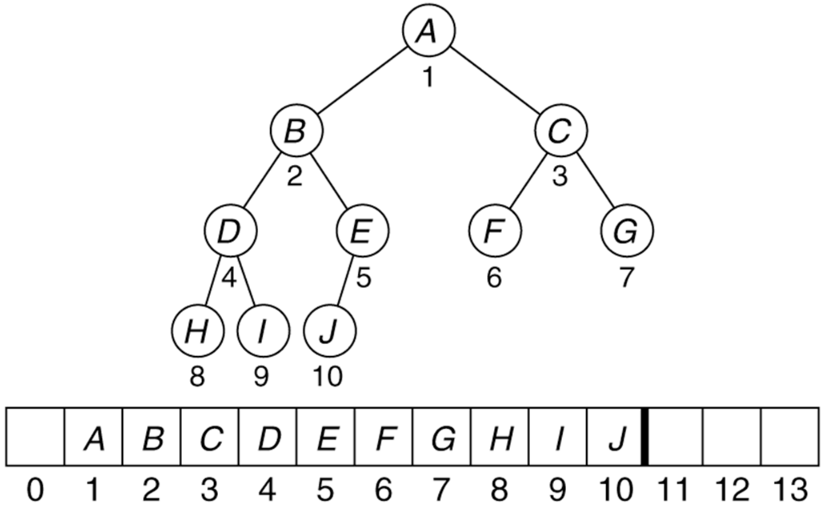


CSSE 230 Day 21

Heapsort

After this lesson, you should be able to …
… explain how and why you can build a heap in O(n) time
… implement heapsort

Sorting Problem

arr

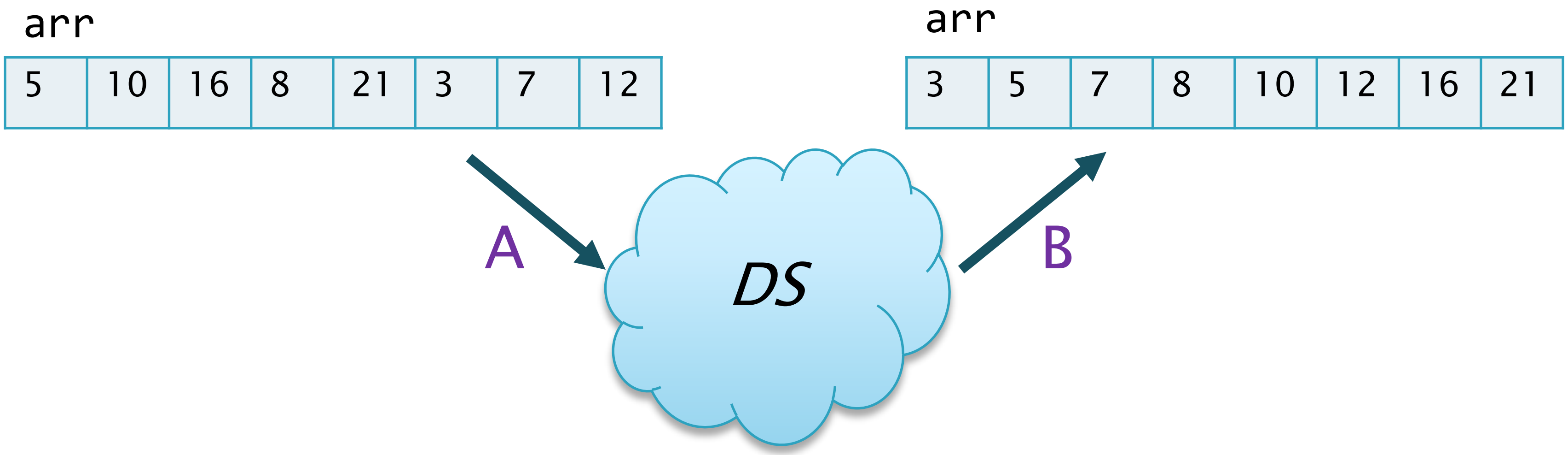
5	10	16	8	21	3	7	12

arr

3	5	7	8	10	12	16	21

- Given array `arr` of Comparables, sort `arr`.

Idea: Using an auxiliary data structure for sorting



`arr`

5	10	16	8	21	3	7	12

`arr`

3	5	7	8	10	12	16	21

- Start with an empty *auxiliary* data structure, *DS*
- Step A. Insert each item from the unsorted array into *DS*
- Step B. Copy the items from *DS* (selecting the most extreme item first, then the next most extreme, etc.) one at a time, back into the original array
- What data structures work for *DS*?
    - BST? Hash set? PQ/heap?

Naïve Heapsort

- Start with empty heap
- Step A. Insert each array element into heap, being sure to maintain the heap property after each insert
- Step B. Repeatedly run deleteMin on the heap, copying elements back into array.

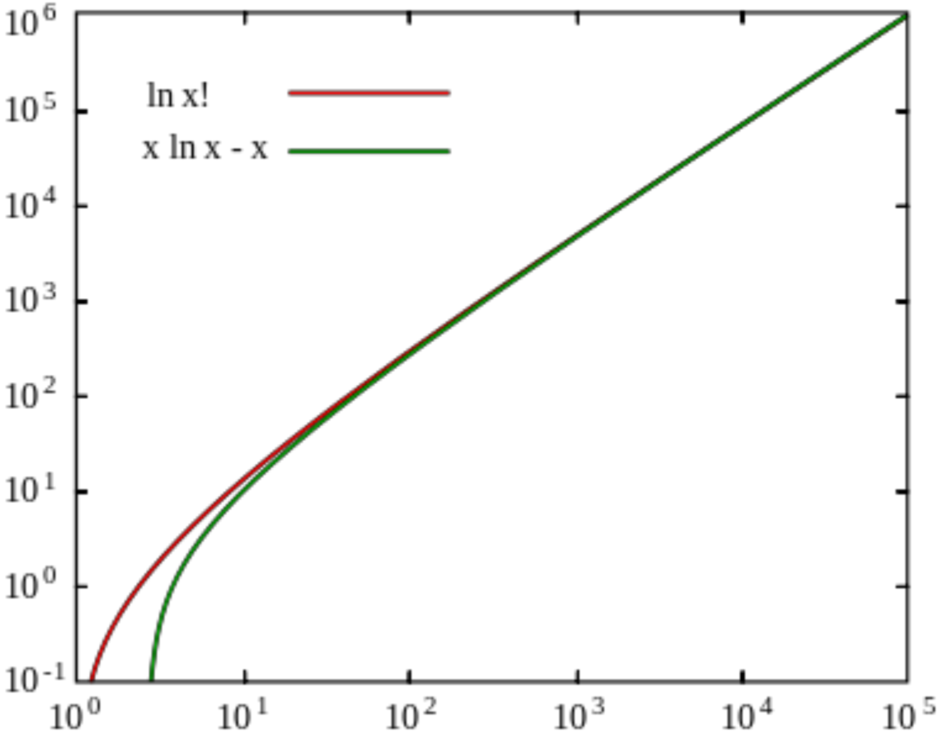
- Analysis?

Analysis of naïve heapsort

- Claim. $\log 1 + \log 2 + \log 3 + \cdots + \log N$ is $\Theta(N \log N)$.

Use **Stirling's approximation**:
Wikipedia link

$$\boxed{\ln n! = n \ln n - n + O(\ln(n))}$$



Analysis of naïve heapsort

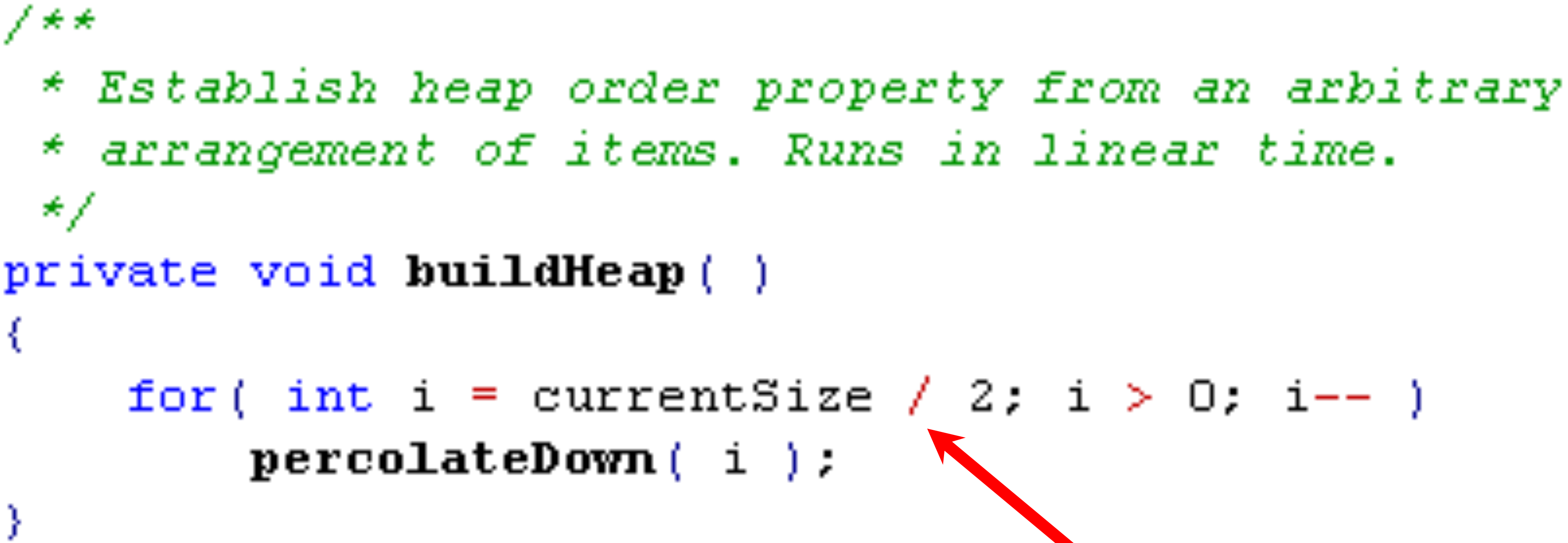
- Add the elements to the heap
  - Repeatedly call insert O(n log n)
- Copy the elements back to the array in order
  - Repeatedly call deleteMin O(n log n)

- Total O(n log n)

- Can we do better for the insertion part?
  - Yes, we don’t need it to be a heap until we are ready to start deleting.
  - insert all the items in arbitrary order into the heap’s internal array and then use **BuildHeap** (next)

BuildHeap takes a complete tree that is not a heap and exchanges elements to get it into heap form

At each stage it takes a root plus two heaps and "percolates down" the root to restore "heapness" to the entire subtree

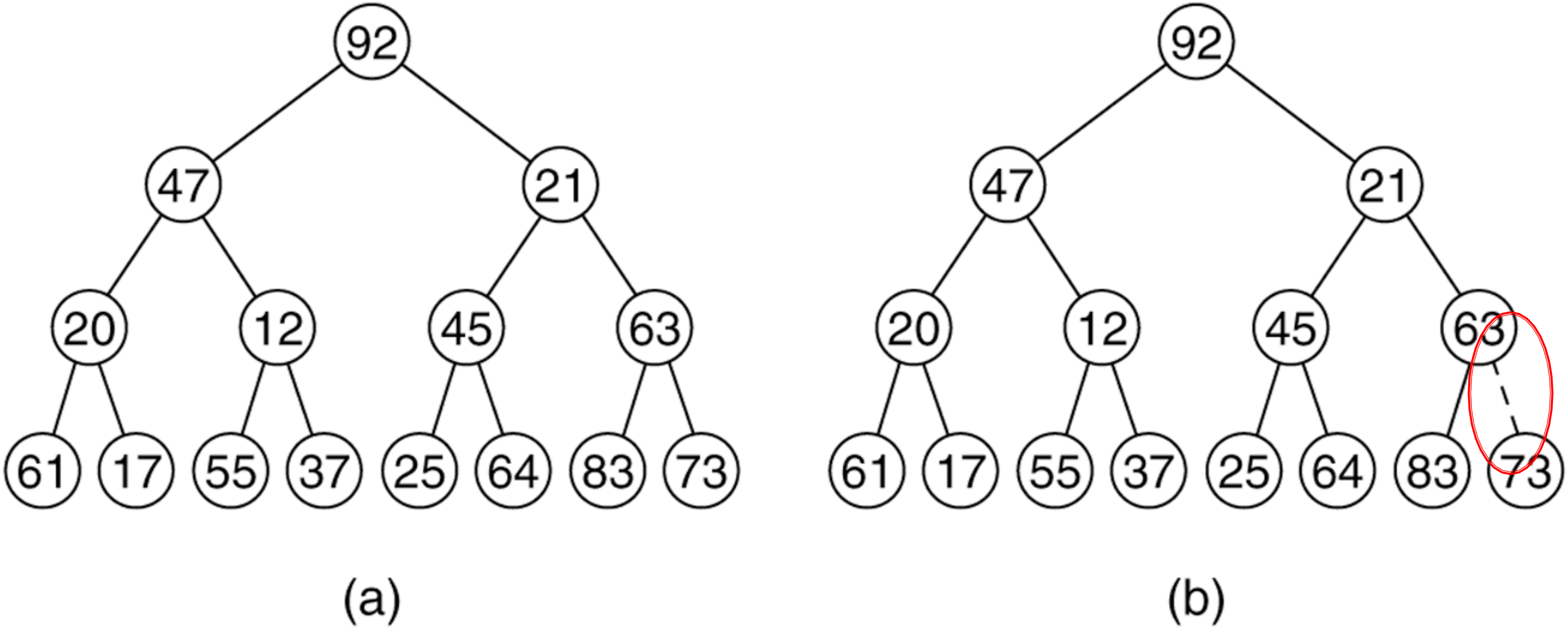


```
/**
 * Establish heap order property from an arbitrary
 * arrangement of items. Runs in linear time.
 */
private void buildHeap( )
{
    for( int i = currentSize / 2; i > 0; i-- )
        percolateDown( i );
}
```

Why this starting point?

Figure 21.17 Implementation of the linear-time buildHeap method

```
private void buildHeap( )
{
    for( int i = currentSize / 2; i > 0; i-- )
        percolateDown( i );
}
```



(a) (b)

Figure 21.18

(a) After percolateDown(6);
(b) after percolateDown(5)

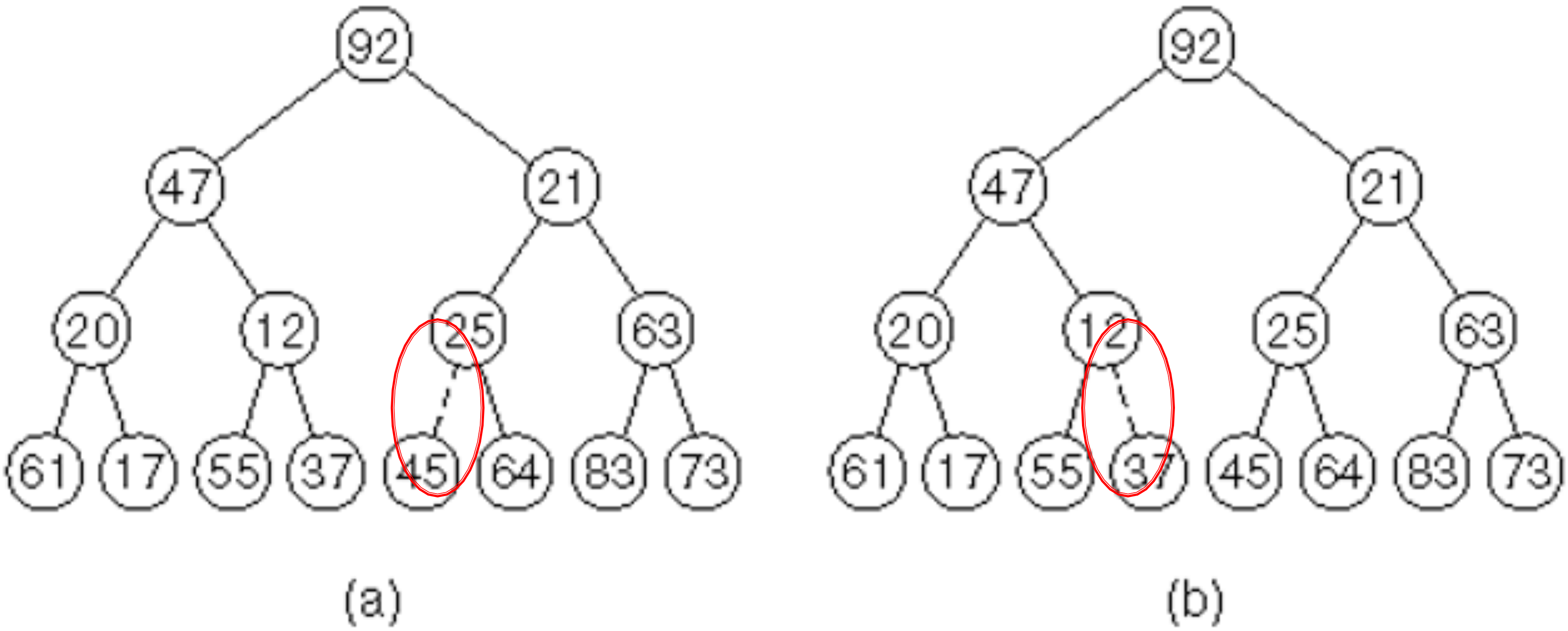


Figure 21.19

(a) After percolateDown(4);
(b) after percolateDown(3)

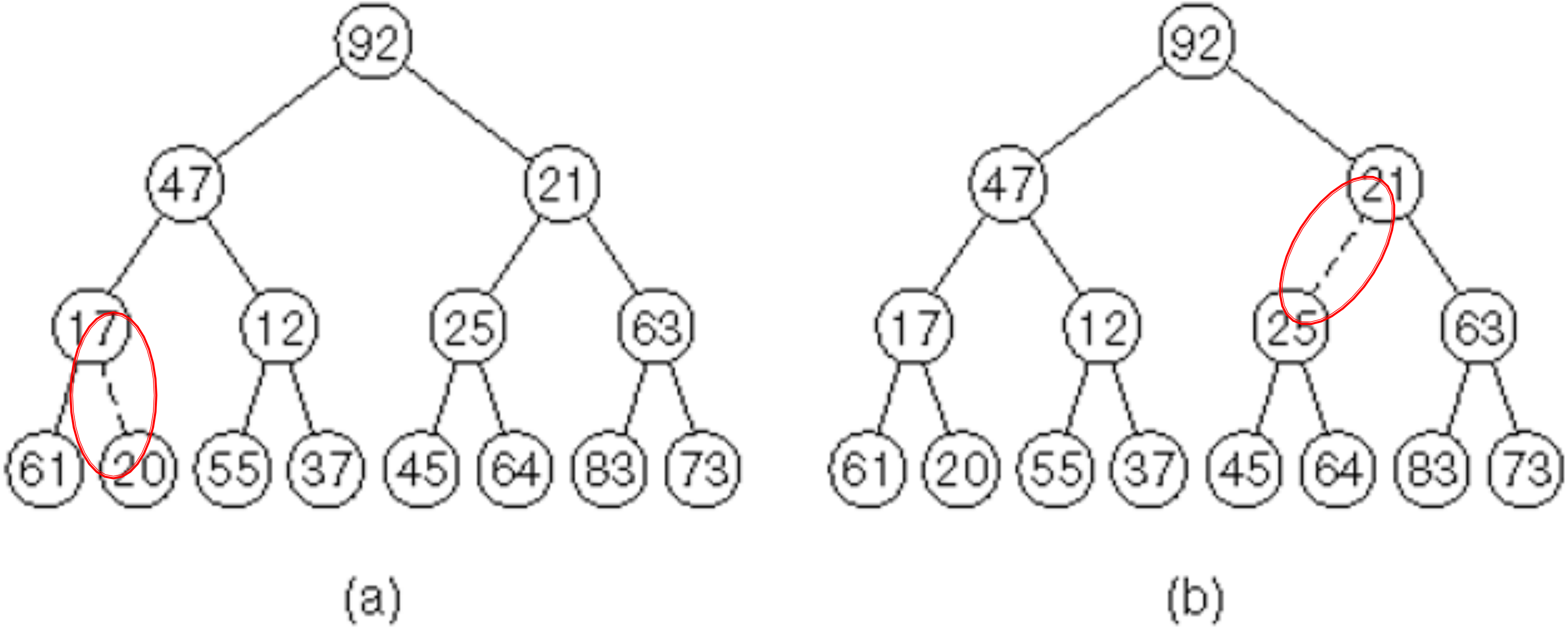
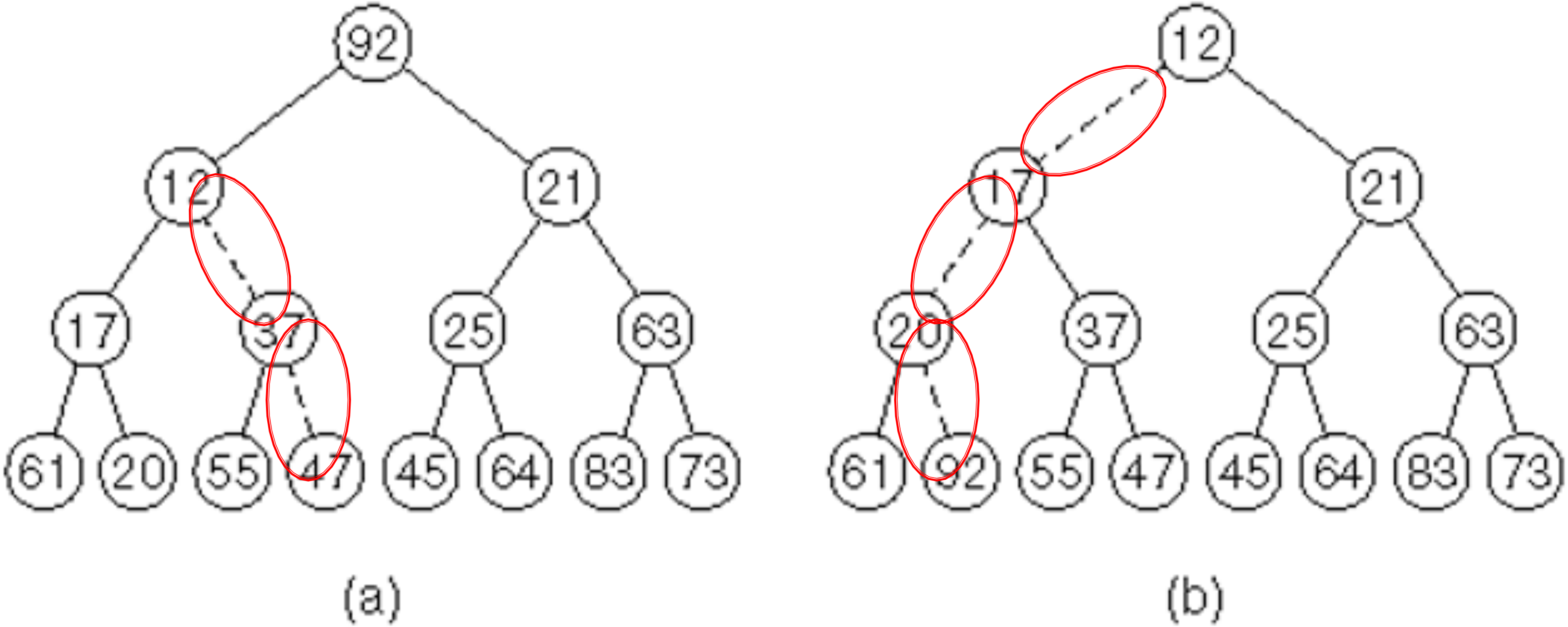


Figure 21.20

(a)After percolateDown(2);
(b) after percolateDown(1) and buildHeap terminates



Analysis of BuildHeap

- Find a summation that represents the maximum number of comparisons required to rearrange an array of $N=2^{H+1}-1$ elements into a heap
  - How many comparisons? The sum of the heights.

- Can you find a summation and its value?

- In HW8, you’ll do this.
- Conclusion: buildHeap is O(N)

Analysis of better heapsort

- Add the elements to the heap
  - ~~Insert n elements into heap~~ (call buildHeap, faster)

- Remove the elements and place into the array
  - Repeatedly call deleteMin

In-place heapsort

- With one final tweak, heapsort only needs O(1) extra space!

- Idea:
  - When we deleteMin, we free up space at the end of the heap’s array.
  - Idea: write deleted item in just-vacated space!
  - Would result in a reverse-sort. Can fix in linear time, but better: use a max-heap. Then, comes out in order!

- http://www.cs.usfca.edu/~galles/visualization/HeapSort.html