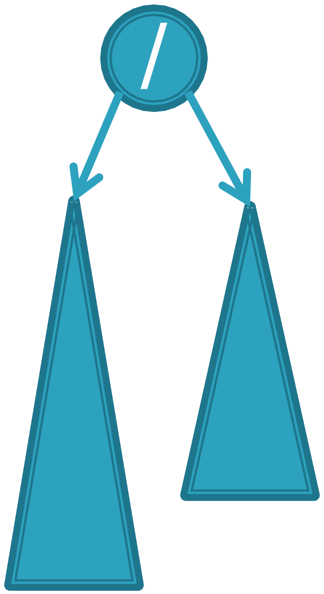


CSSE 230 Day 13

AVL trees and rotations

This week, you should be able to…
…perform rotations on height-balanced trees,
on paper and in code
… write a rotate() method
… search for the kth item in-order using rank

Announcements

- Term project partners posted
  - Sit with partner(s) in the second half of today's class.
  - Read the spec before tomorrow and start planning.

- Exam 2 next class
  - 1^{st} 25 minutes for Day #14 slides
  - Remaining 85 minutes for Exam #2

Exam 2 next class:
Recursive tree traversal methods follow this format

Consider method fooTraverse() defined in BinaryNode class:

fooTraverse()
  If base case:
    Return the appropriate value
  If not at base case:
    1. Compute a value for current node
    2. Call left.fooTraverse() and right.fooTraverse()
    3. Combine all results and return it

- This is O(n) if the computation on the node is constant–time
- Style: pass info through parameters and return values.
  - Do not declare and use extra instance variables (fields) in BinaryTree class

Exam 2 next class:

Recursive tree navigation methods follow this format

Consider method fooNavigate() defined in BinaryNode class

fooNavigate()
  If base case:
    Do required work at target location navigated to
  If not at base case:
    1. Compute which subtree to navigate into
    2. Call either left.fooNavigate() or right.fooNavigate()
    3. Do (optional) work after the recursive call

- This is O(height) and if the BST is height-balanced then O(log(n))
- Style: pass info through parameters and return values.
  - Do not declare and use extra instance variables (fields) in BinaryTree class

Exam 2 next class:
Additional tips

- Sometimes in a traversal, the order nodes are considered matters
  - Preorder, inorder, postorder
- An iterator can be used to manually control a traversal
  - To do lazily, the iterator must have its own stack (or other data structure) replacing the stack of recursive calls
- When editing a tree (inserting/removing a node), we suggest using the “return this” pattern

Summary: for fast tree operations, we must keep tree somewhat balanced in O(log n) time

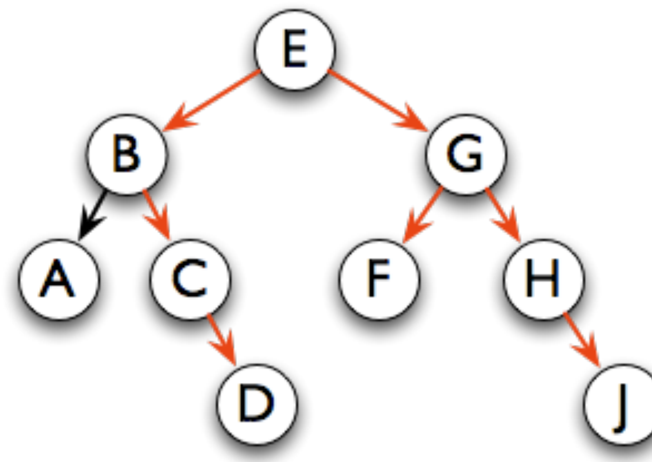
Q1

- Total time to do insert/delete =

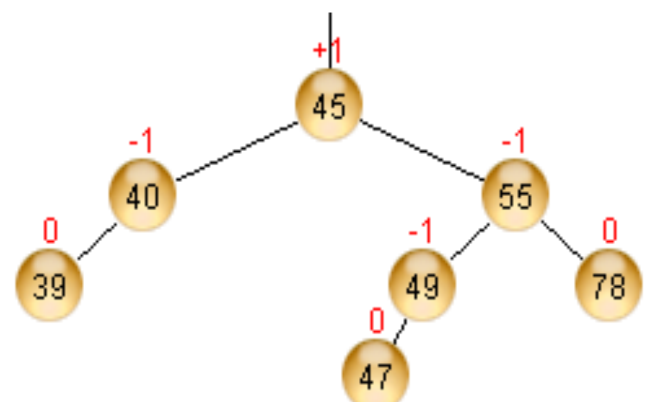
  Time to find the correct place to insert = O(height)

  + time to detect an imbalance

  + time to correct the imbalance



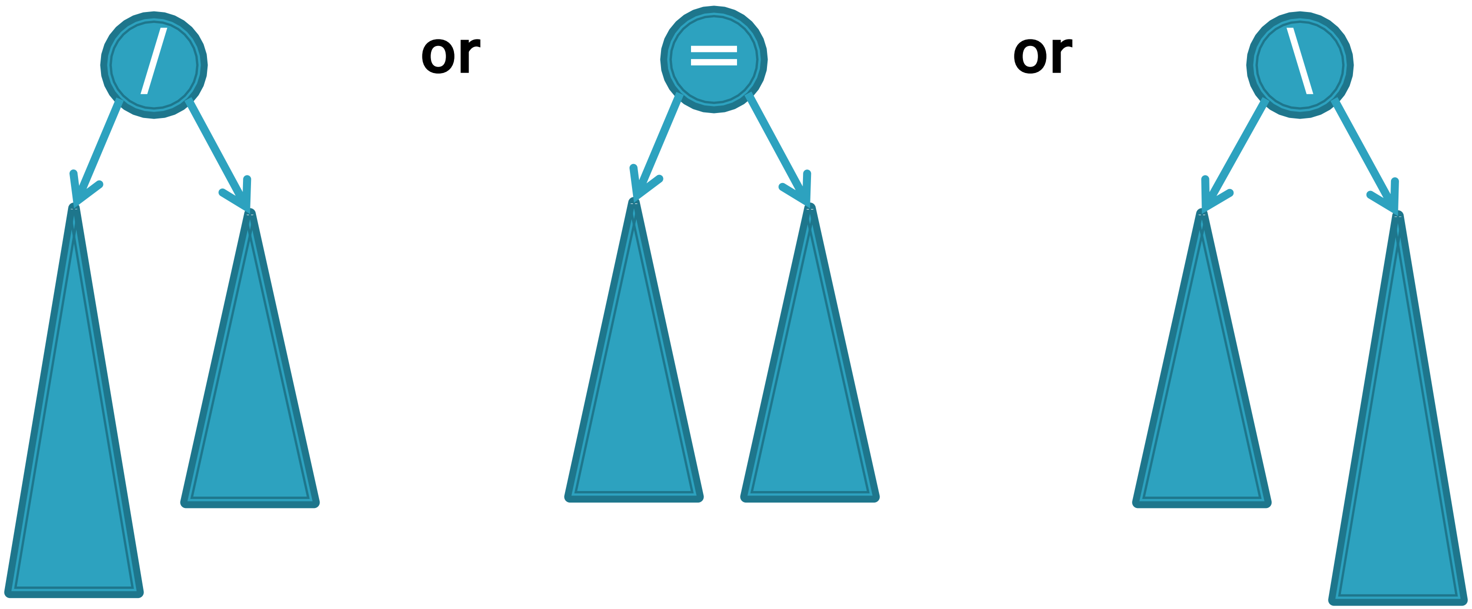
- If we don't bother with balance after insertions and deletions?
- If try to keep perfect balance:
  - Height is O(log n) BUT …
  - But maintaining perfect balance requires O(n) work
- Height-balanced trees are still O(log n)
  - $|\text{Height(left)} - \text{Height(right)}| \leq 1$
  - For T with height h, $N(T) \geq \text{Fib}(h+3) - 1$
  - So $H < 1.44 \log (N+2) - 1.328$ *



- AVL (**A**delson-**V**elskii and **L**andis) trees maintain height-balance using rotations
- Are rotations O(log n)? We'll see…

Q2

AVL tree nodes are just like BinaryNodes, but also have an extra field to store a "balance code"



/ : Current node's left subtree is taller by 1 than its right subtree
= : Current node's subtrees have equal height
\ : Current node's right subtree is taller by 1 than its left subtree

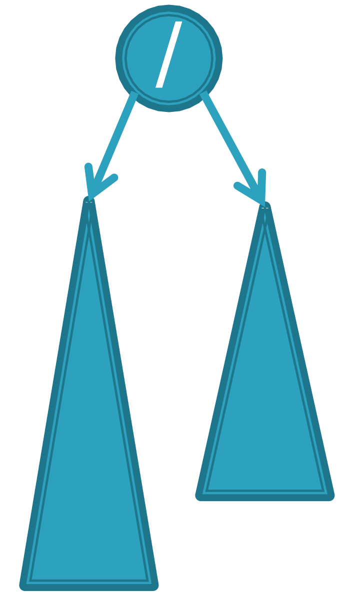
Two possible data representations for: / = \

- Use just two bits, e.g., in a low-level language
- Use `enum` type in a higher-level language like Java

Using balance codes makes AVL Tree rebalancing efficient: O(log n)

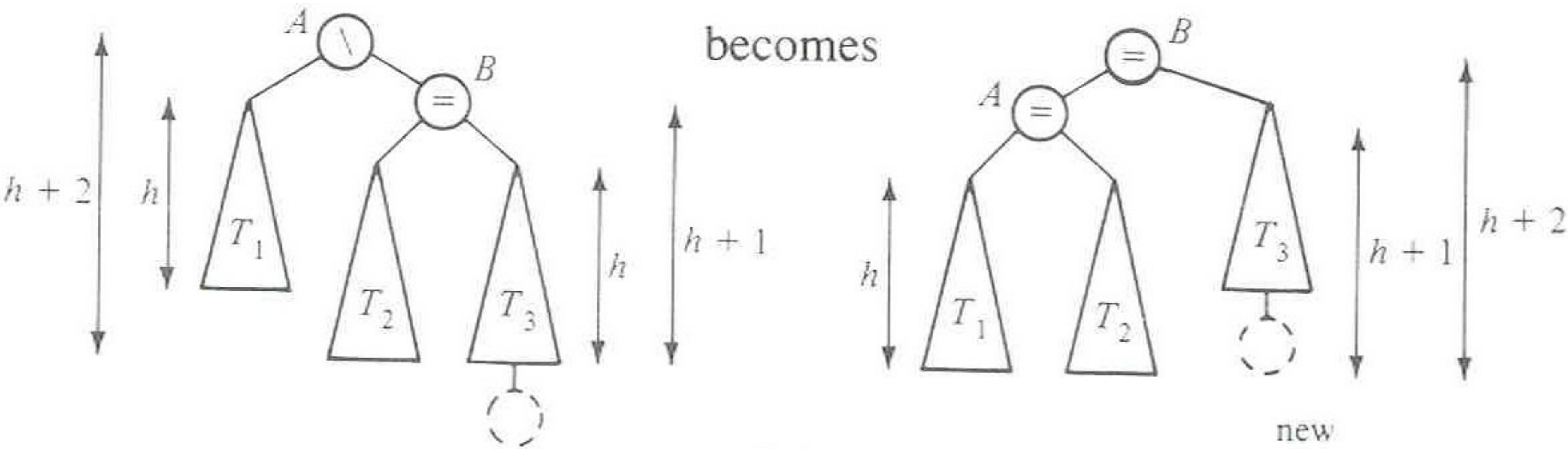
Q3

- Assume tree is height-balanced before insertion
- Insert as usual for a BST
- Move up from the newly inserted node to the **lowest** "unbalanced" node (if any)
  - Use the **balance code** to detect unbalance – how?
  - Why is this O(log n)?
    - We move up the tree to the root in worst case, NOT recursing into subtrees to calculate heights
- Do an appropriate rotation (see next slides) to balance the subtree rooted at this unbalanced node



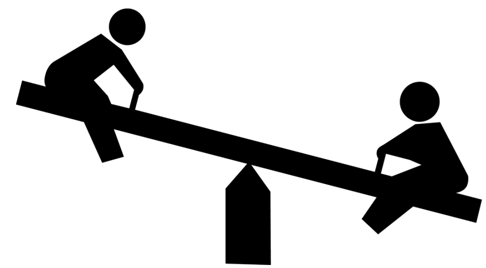
Four types of rotations are required to remove different cases of tree imbalances

- For example, a *single left rotation*:



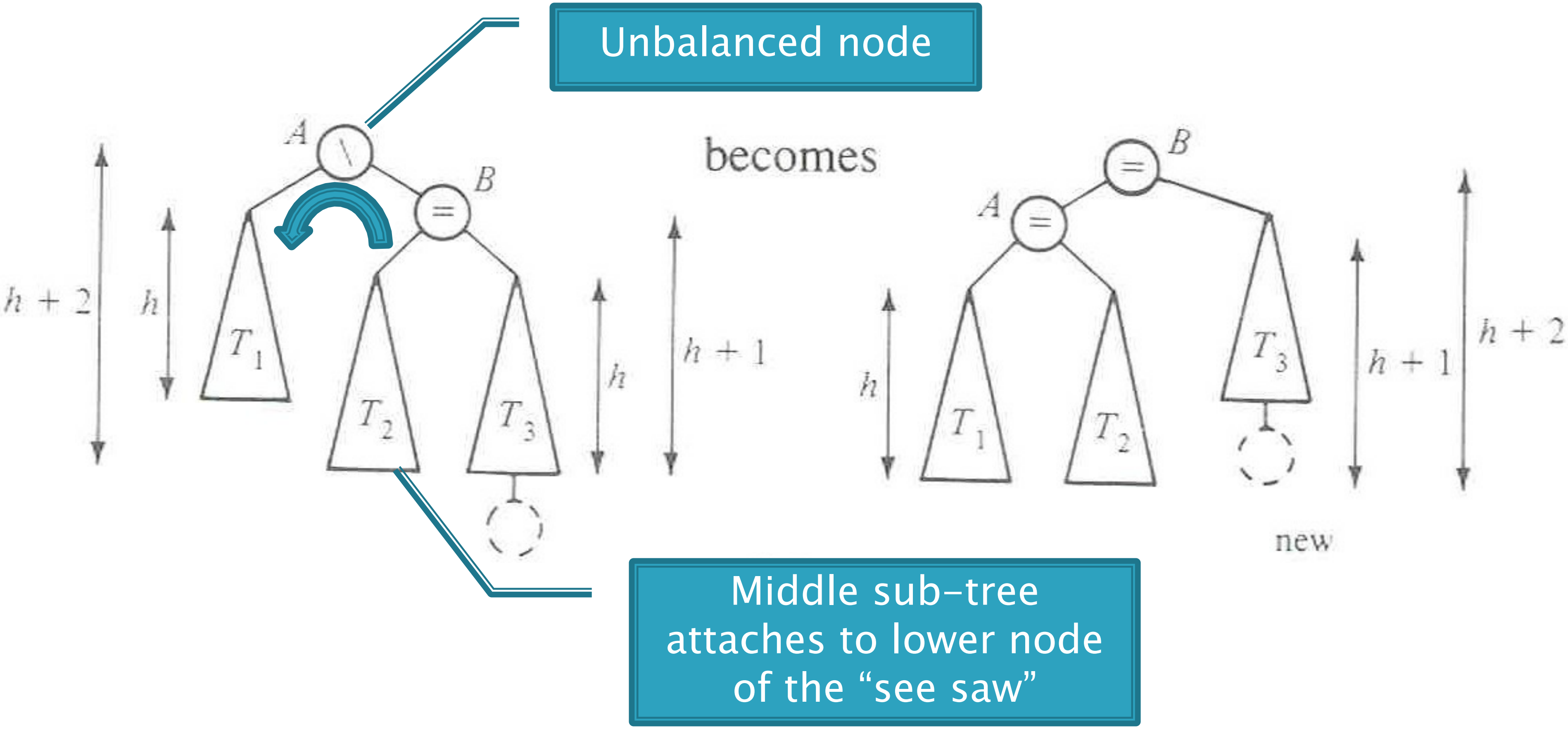
We rotate by pulling the “too tall” sub-tree up and pushing the “too short” sub-tree down

- Two basic cases:
  - “Seesaw” case:
    - Too-tall sub-tree is on the outside
    - So tip the seesaw so it’s level



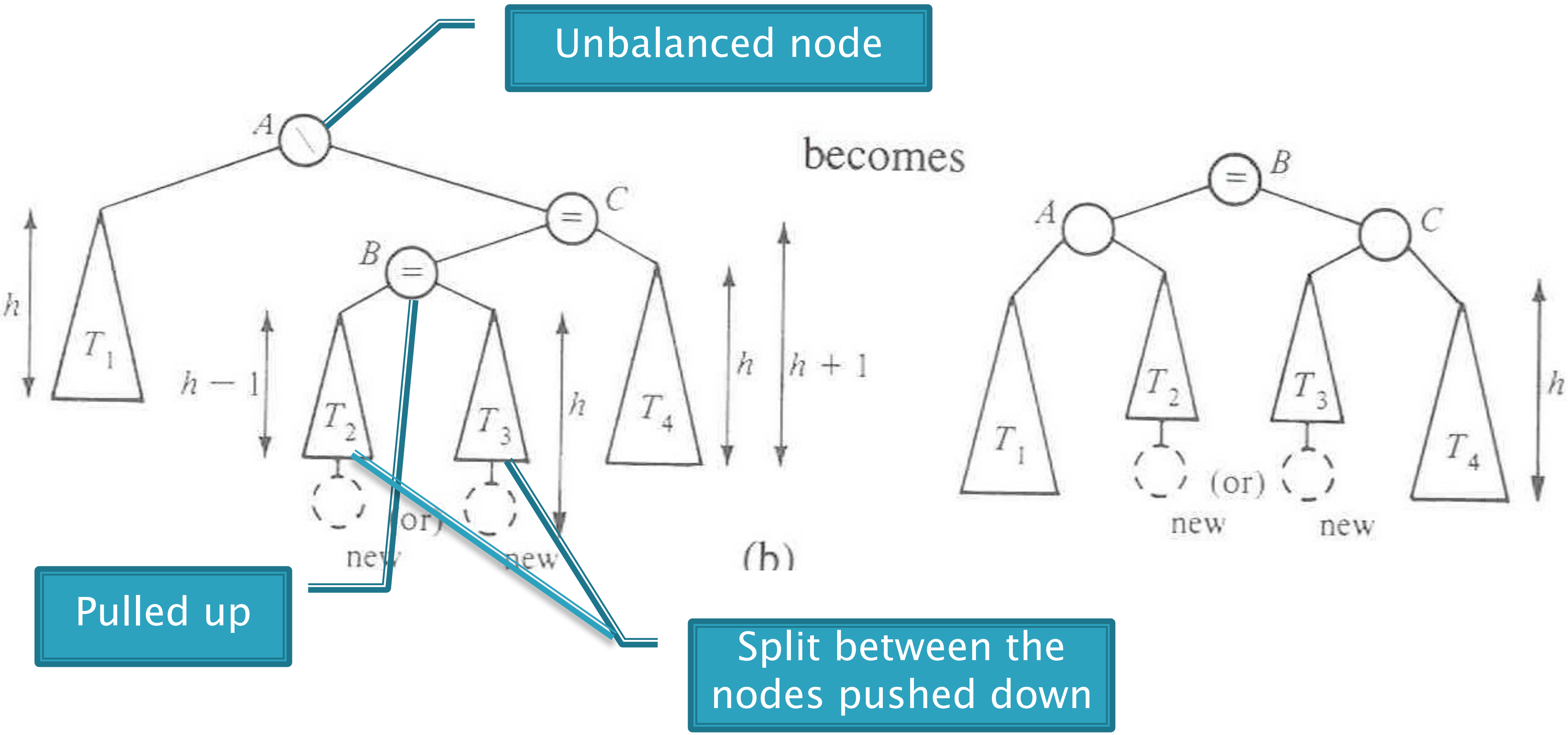
  - “Suck in your gut” case:
    - Too-tall sub-tree is in the middle
    - Pull its root up a level

Single Left Rotation



Diagrams are from *Data Structures* by E.M. Reingold and W.J. Hansen

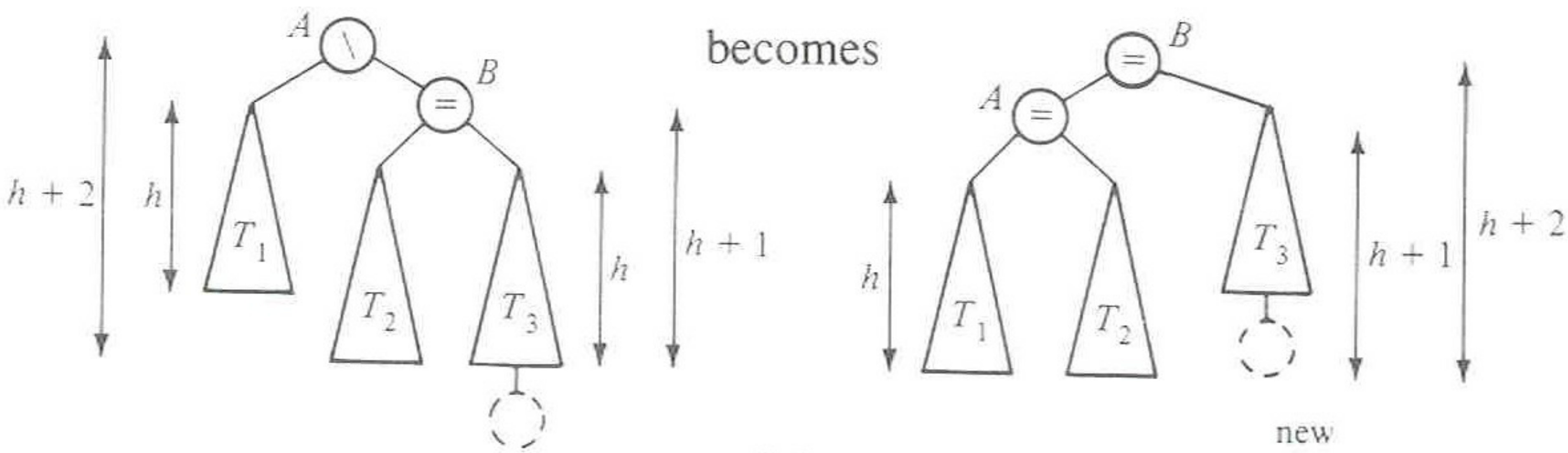
Double Left Rotation



Weiss calls this “right–left double rotation”

Your turn — work with a partner

Q8



- Write the method:

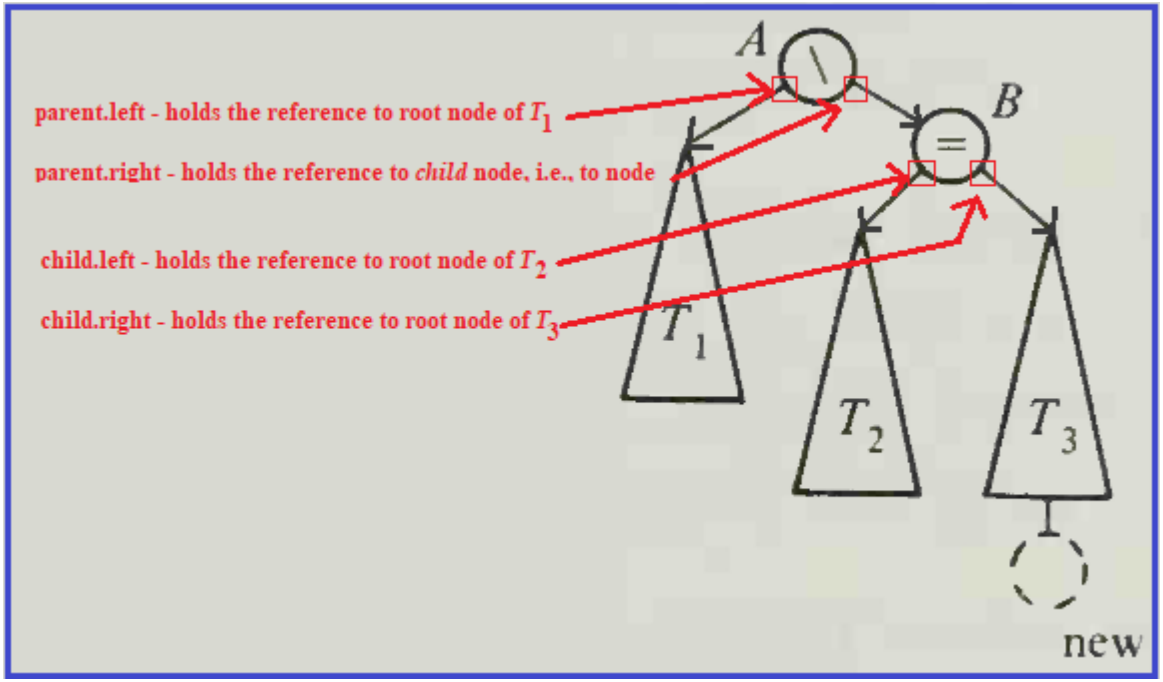
```
static BalancedBinaryNode singleRotateLeft (
    BalancedBinaryNode parent,    /* A */
    BalancedBinaryNode child      /* B */  ) {

}
```

- Returns a reference to the new root of this subtree.
- Don't forget to set the balanceCode fields of the nodes.

Your turn — work with a partner

Q8



- Write the method:

```
static BalancedBinaryNode singleRotateLeft (
    BalancedBinaryNode parent,    /* A */
    BalancedBinaryNode child      /* B */  ) {
}
```

- Returns a reference to the new root of this subtree.
- Don't forget to set the balanceCode fields of the nodes.

More practice—(sometime after class)

- Write the method:

```
BalancedBinaryNode doubleRotateRight (
    BalancedBinaryNode parent,      /* A */
    BalancedBinaryNode child,       /* C */
    BalancedBinaryNode grandChild   /* B */ ) {


}
```

- Returns a reference to the new root of this subtree.
- Rotation is mirror image of double rotation from an earlier slide

O(log N)?

Q9,Q1,Q10-11

- If you have to rotate after insertion, you can stop moving up the tree:
  - Both kinds of rotation leave height the same as before the insertion!

- Is insertion plus rotation cost really O(log N)?

Insertion/deletion in AVL Tree:	O(log n)
Find the imbalance point (if any):	O(log n)
Single or double rotation:	O(1)
Total work:	O(log n)
Foreshadow:	
for deletion # of rotations:	O(log N)

Term Project: EditorTrees

Like BST, except:

1. Keep height-balanced
2. Insertion/deletion by **index**, not by comparing elements. So not sorted

Examples:

```
EditorTree et = new EditorTree()
et.add('a')     // append to end
et.add('b')     // same
et.add('c')     // same. Rebalance!
et.add('d', 2) // where does it go?
et.add('e')
et.add('f', 3)
```

- Notice the tree is height-balanced (so height = O(log n)), but not a BST

To find index quickly, add a **rank** field to BinaryNode

- Gives the in-order position of this node within its own subtree

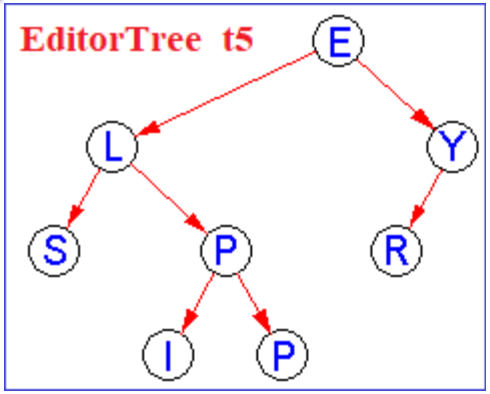
  i.e., rank = the size of its left subtree

  0-based indexing

- How would we do **`get(pos)`**?

- **`Insert`** and **`delete`** start similarly

Rank and position of element in tree

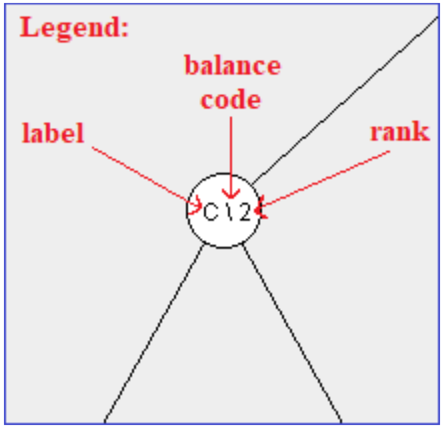


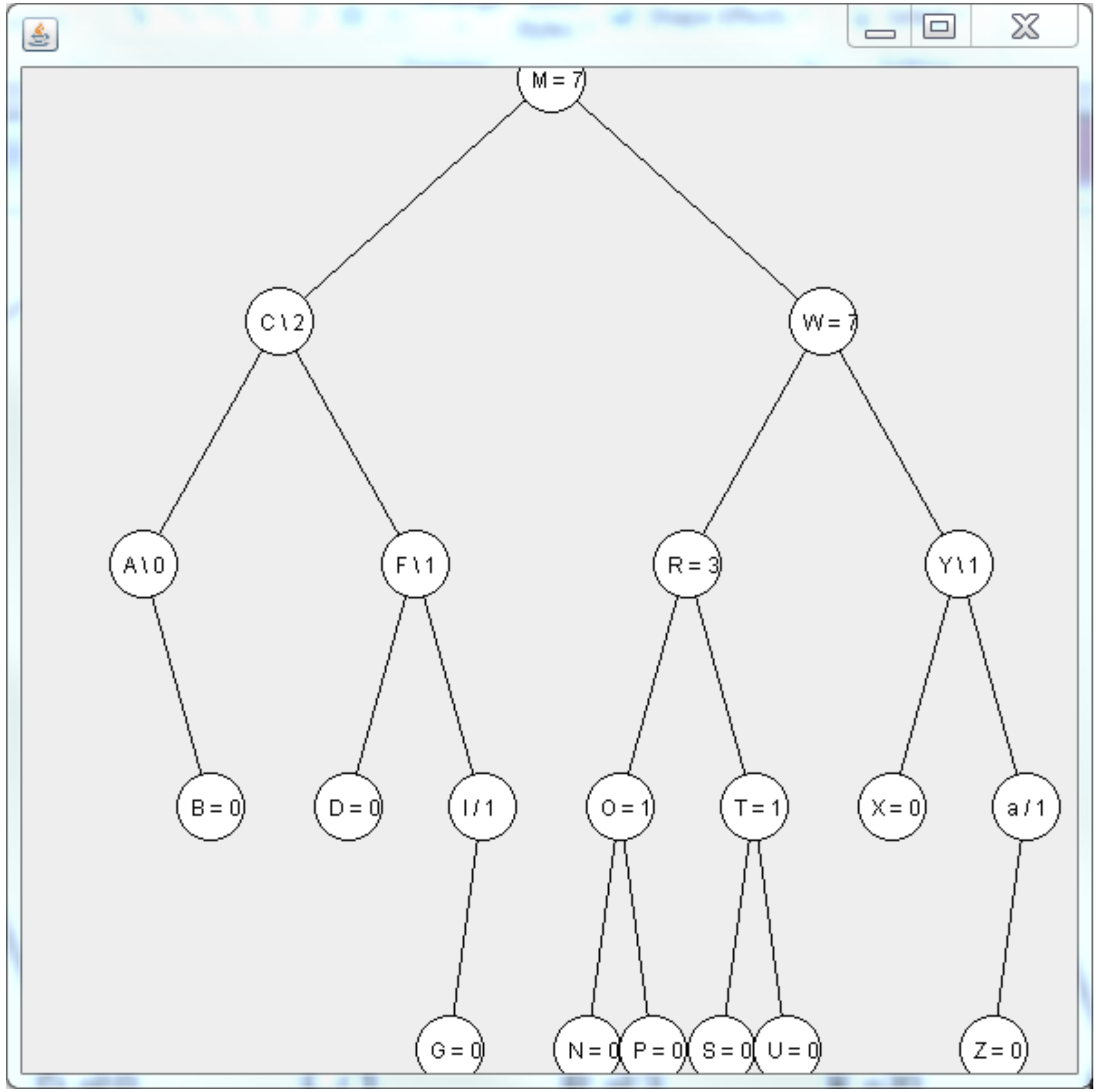
Suppose EditorTree's *toString* method performs an in-order traversal

Then:
String s2 = t5.toString(); // s2 = "SLIPPERY"

- Character 'S' is at position 0, and has rank 0
- Character 'L' is at position 1, and has rank 1
- Character 'I' is at position 2, and has rank 0
- Character 'P' is at position 3, and has rank 1
- Character 'P' is at position 4, and has rank 0
- Character 'E' is at position 5, and has rank 5
- Character 'R' is at position 6, and has rank 0
- Character 'Y' is at position 7, and has rank 1

- |s2| = 8


Legend:
balance
code
label
rank
C \ 2


M = 7
C \ 2
W = 7
A \ 0
F \ 1
R = 3
Y \ 1
B = 0
D = 0
I / 1
O = 1
T = 1
X = 0
a / 1
G = 0
N = 0
P = 0
S = 0
U = 0
Z = 0

With your EditorTrees team

Milestone 1 due in day 17.

Start soon!

Read the specification and check out the starting code