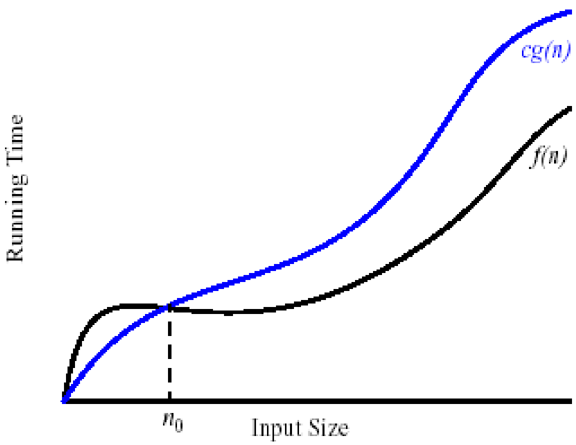
CSSE 230 Day 2

Growable Arrays Continued

Big-Oh notation



Submit Growable Array exercise

Agenda and goals

- Growable Array recap
- Big-Oh definition

- After today, you'll be able to
  - Use the term *amortized* appropriately in analysis
  - State the formal definition of big-Oh notation

Q1-5

Announcements and FAQ

- You will not usually need the textbook in class
- All should do piazza introduction post (a few students left)
- Turn in GrowableArrays now.
- Quiz problems 1-5. Do on your own, then compare with a neighbor.

You must demonstrate programming competence on exams to succeed

- See syllabus for exam weighting and caveats.
- Evening exams (Tuesdays of weeks 3 and 8)
- Think of every program you write as a practice test
  - Especially HW4 and test 2

Review these as needed

- Logarithms and Exponents
  - properties of logarithms:

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x/y) = \log_b x - \log_b y$$

$$\log_b x^\alpha = \alpha \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

  - properties of exponentials:

$$a^{(b+c)} = a^b a^c$$

$$a^{bc} = (a^b)^c$$

$$a^b / a^c = a^{(b-c)}$$

$$b = a^{\log_a b}$$

$$b^c = a^{c*\log_a b}$$

Practice with exponentials and logs

(Do these with a friend after class, not to turn in)

Simplify: Note that log n (without a specified) base means $\log_2 n$. Also, log n is an abbreviation for log(n).

1. $\log(2\, n \log n)$
2. $\log(n/2)$
3. $\log(\mathrm{sqrt}(n))$
4. $\log(\log(\mathrm{sqrt}(n)))$

5. $\log_4 n$
6. $2^{2 \log n}$
7. if $n = 2^{3k} - 1$, solve for k.

Where do logs come from in algorithm analysis?

Solutions

No peeking!

Simplify: Note that log n (without a specified) base means $\log_2 n$.
Also, log n is an abbreviation for log(n).

1. $1+\log n + \log \log n$
2. $\log n - 1$
3. $\frac{1}{2} \log n$
4. $-1 + \log \log n$

5. $(\log n) / 2$
6. n^2
7. $n+1=2^{3k}$
   $\log(n+1)=3k$
   $k= \log(n+1)/3$

A: Any time we cut things in half at each step
(like binary search or mergesort)

Questions?

- About Homework 1?
  - Aim to complete tonight, since it is due after next class
  - It is substantial
  - The last problem (the table) is worth lots of points!
- About the Syllabus?

Homework 1 help

How many times does sum++ run?

```
for (i = 4; i < n; i++)
    for (j = 0; j <= n; j++)
        sum++;
```

Why is this one so easy? (does the inner loop depend on outer loop?)

What if inner were (j = 0; j <= i ; j++) ?

Homework 1 help

How many times does sum++ run?

```
for (i = 1; i <= n; i *= 2)
   sum++;
```

Be precise, using floor/ceiling as needed, to get full credit.

Warm Up and Stretching thoughts

- Short but intense! ~50 lines of code total in our solutions
- Be sure to read the description of how it will be graded. Note how style will be graded.
- **Demo**: Use Subclipse to check out the project
- **Demo**: Running the JUnit tests for test, file, package, and project

Growable Arrays Exercise

Daring to double

Growable Arrays Table

N	E_N	**Answers for problem 2**
4	0	0
5	0	0
6	5	5
7	5	5 + 6 = 11
10	5	5 + 6 + 7 + 8 + 9 = 35
11	5 + 10 = 15	5 + 6 + 7 + 8 + 9 + 10 = 45
20	15	sum(i, i=5..19) = 180 **using Maple**
21	5 + 10 + 20 = 35	sum(i, i=5..20) = 200
40	35	sum(i, i=5..39) = 770
41	5 + 10 + 20 + 40 = 75	sum(i, i=5..40) = 810

Doubling the Size

- Doubling each time:
  - Assume that $N = 5(2^k) + 1$.
- Total # of array elements copied:

k	N	#copies
0	6	5
1	11	5 + 10 = 15
2	21	5 + 10 + 20 = 35
3	41	5 + 10 + 20 + 40 = 75
4	81	5 + 10 + 20 + 40 + 80 = 155
k	$= 5(2^k) + 1$	$5(1 + 2 + 4 + 8 + \ldots + 2^k)$

Express as a closed-form expression in terms of K, then express in terms of N

Doubling the Size (solution)

- Assume that $N = 5(2^k) + 1$.
- Total # of array elements copied
  $= 5(1 + 2 + 4 + 8 + \ldots + 2^k)$
- **Do in terms of k, then in terms of N**

Adding One Each Time

- Total # of array elements copied:

N	#copies
6	5
7	5 + 6
8	5 + 6 + 7
9	5 + 6 + 7 + 8
10	5 + 6 + 7 + 8 + 9
N	???

Express as a closed-form expression in terms of N

- Q6–7

Conclusions

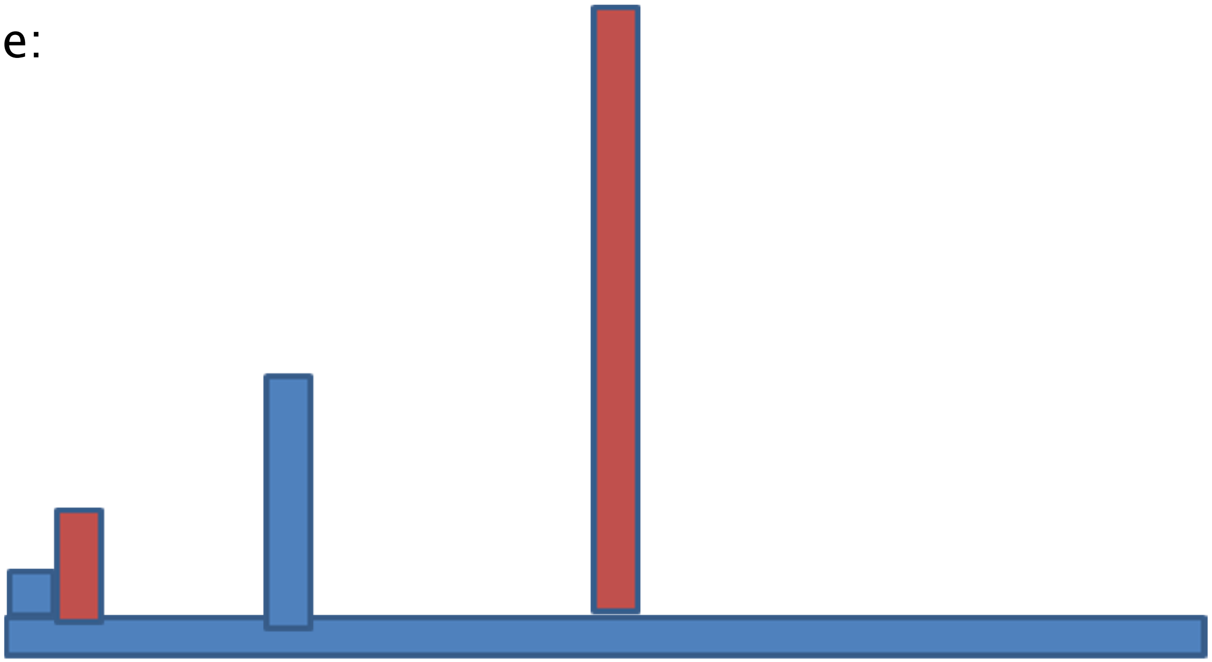
- What's the **amortized** cost of adding an additional string…
  - in the doubling case?
  - in the add-one case?

Amortized cost means the "average per-operation cost" while adding to a single GrowableArray many times.

- So which should we use?

Worst-case vs amortized cost for adding an element to an array using the doubling scheme

Worst-case:
O(n)



amortized:
O(1)

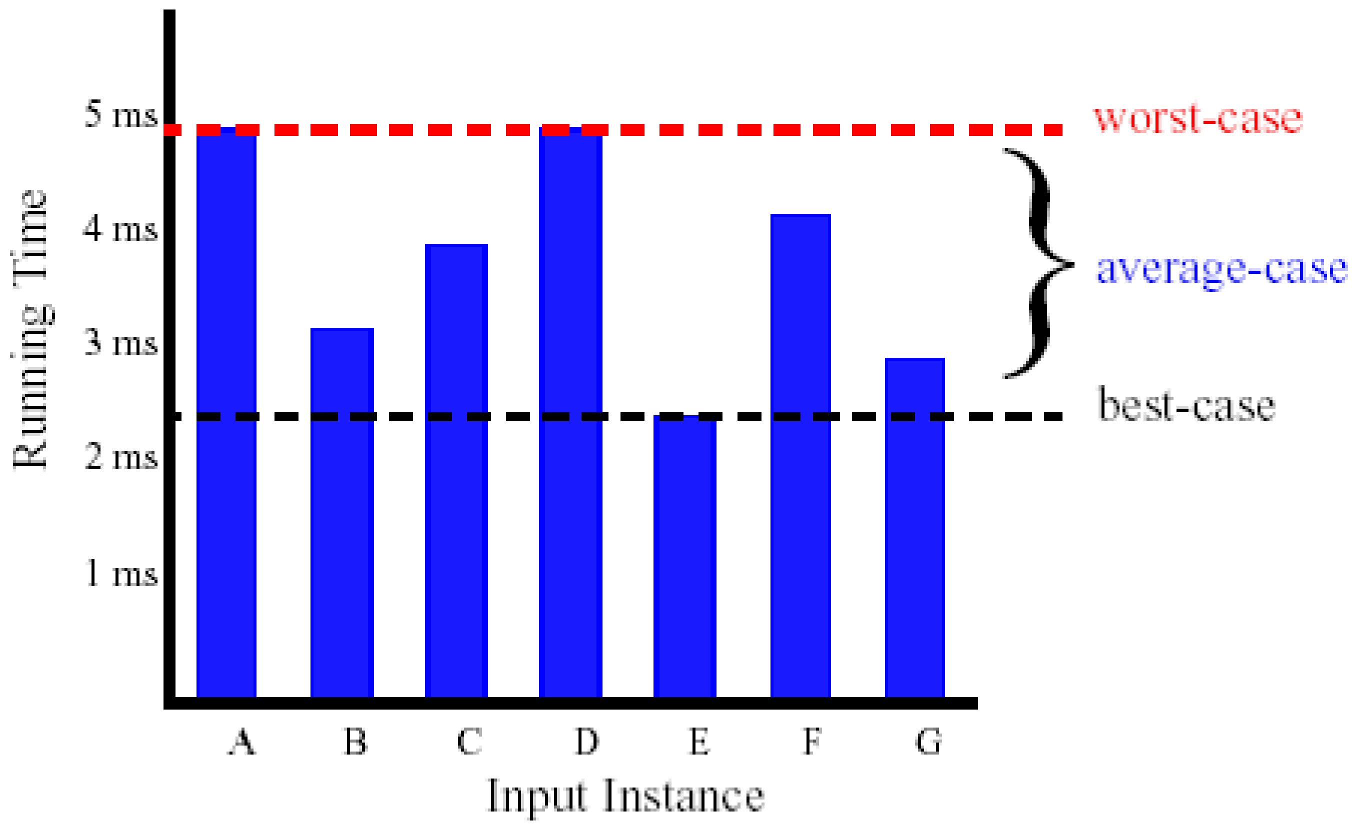
Note: average case means averaged over *input domain*, amortized cost means averaged over *many uses*.

Algorithm Analysis: Running Time

Running Times

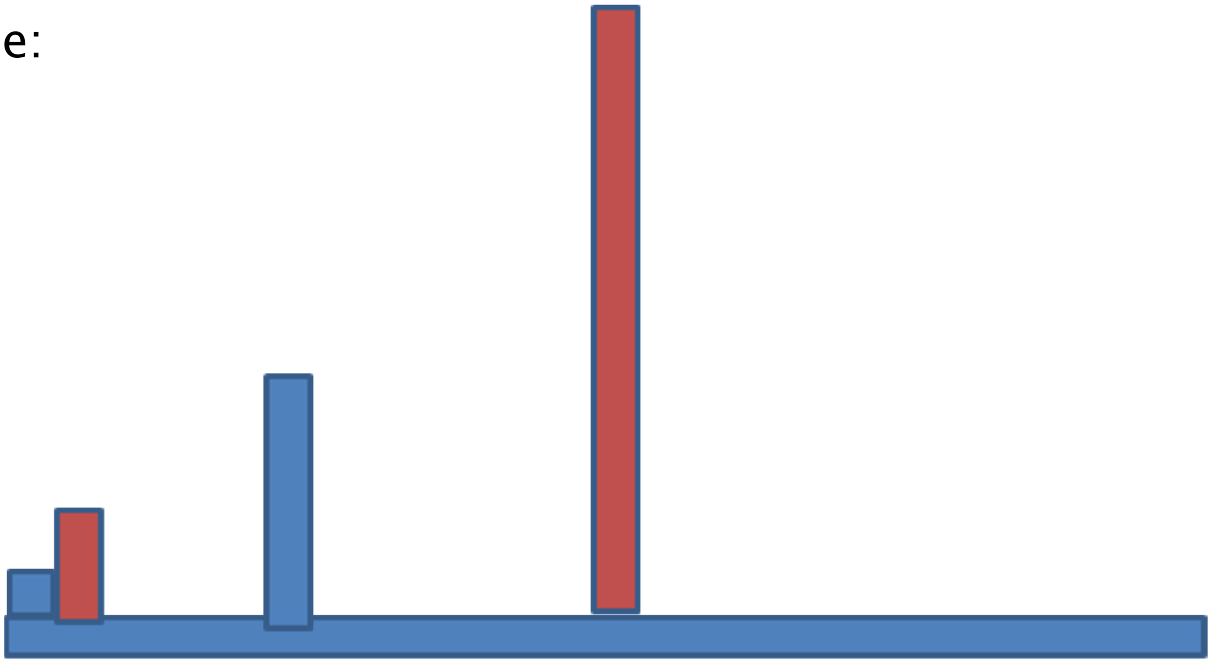
- Algorithms may have different *time complexity* on different data sets
- What do we mean by "Worst Case"?
- What do we mean by "Average Case"?
- What are some application domains where knowing the Worst Case time complexity would be important?
- http://cacm.acm.org/magazines/2013/2/160173-the-tail-at-scale/fulltext

Average Case and Worst Case



Worst-case vs amortized cost for adding an element to an array using the doubling scheme

Worst-case:
O(n)



amortized:
O(1)

Note: average case means averaged over *input domain*, amortized cost means averaged over *many uses*.

Notation for Asymptotic Analysis

Big-Oh

Asymptotic Analysis

- We only care what happens when N gets large
- Is the function linear? quadratic? exponential?

Figure 5.1

Running times for small inputs

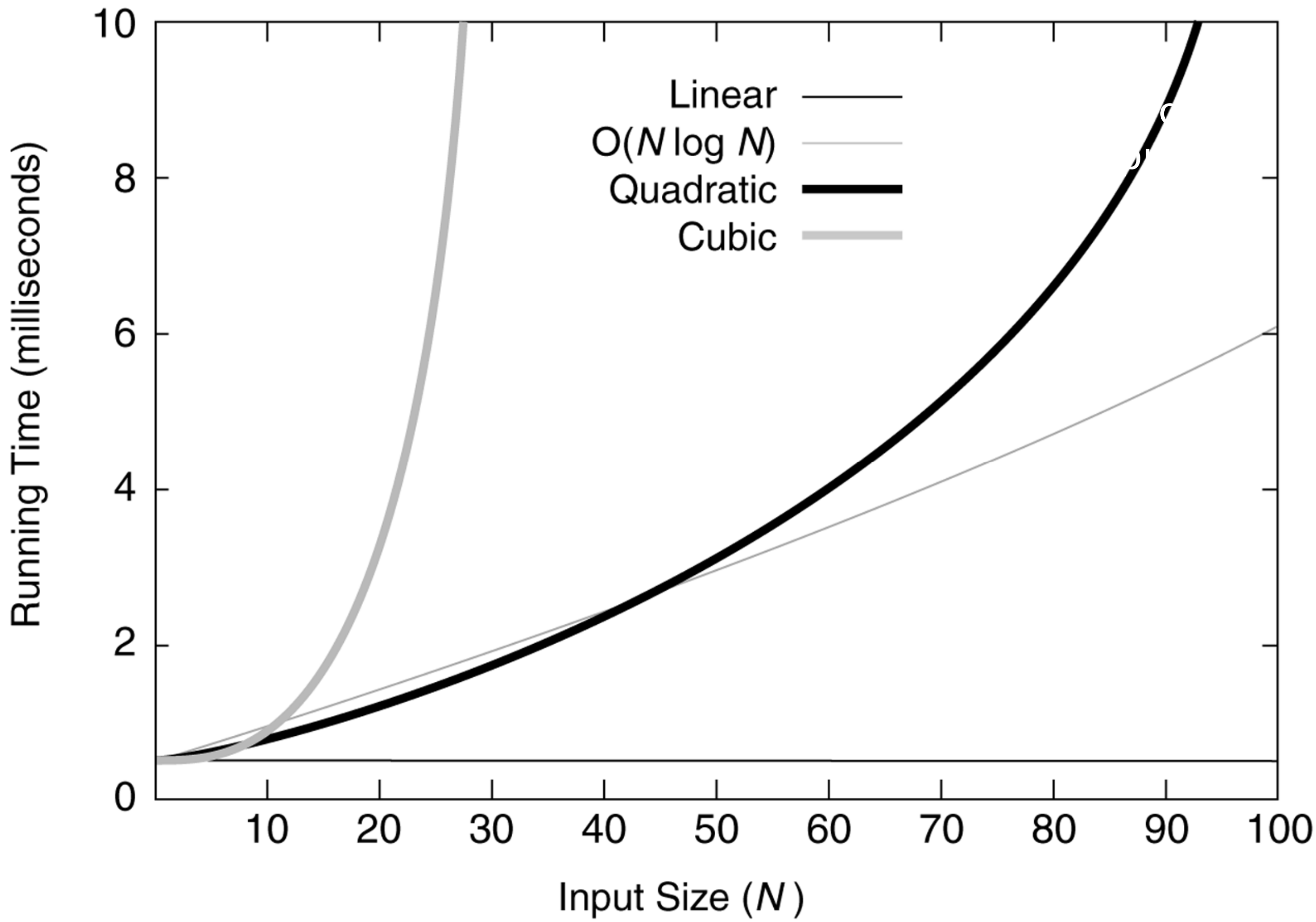


Figure 5.2

Running times for moderate inputs

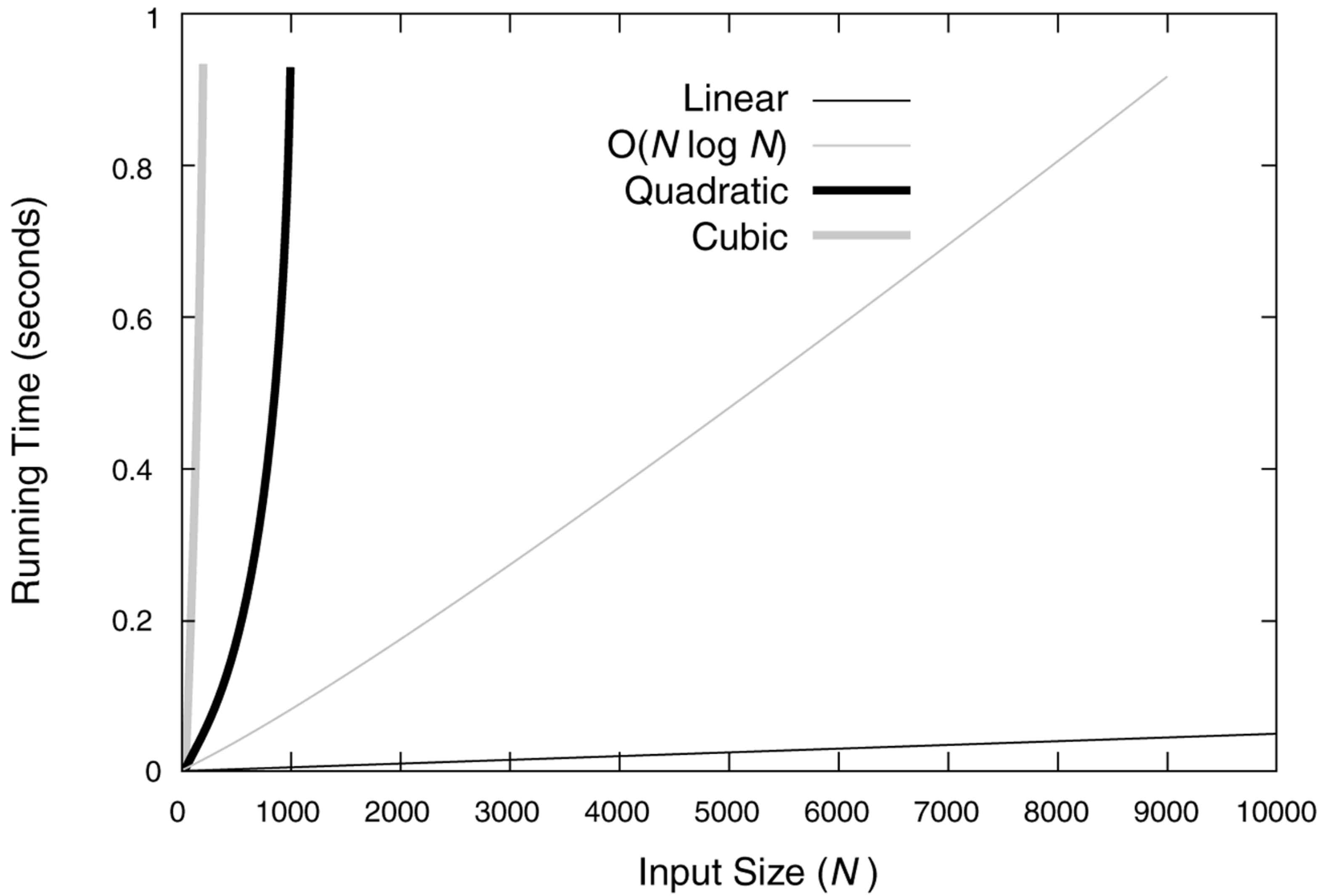


Figure 5.3

Functions in order of increasing growth rate

The answer to most big-Oh questions is one of these functions

Function	Name
c	Constant
$\log N$	Logarithmic
$\log^2 N$	Log-squared
N	Linear
$N \log N$	N log N
N^2	Quadratic
N^3	Cubic
2^N	Exponential

a.k.a "log linear"

Simple Rule for Big-Oh

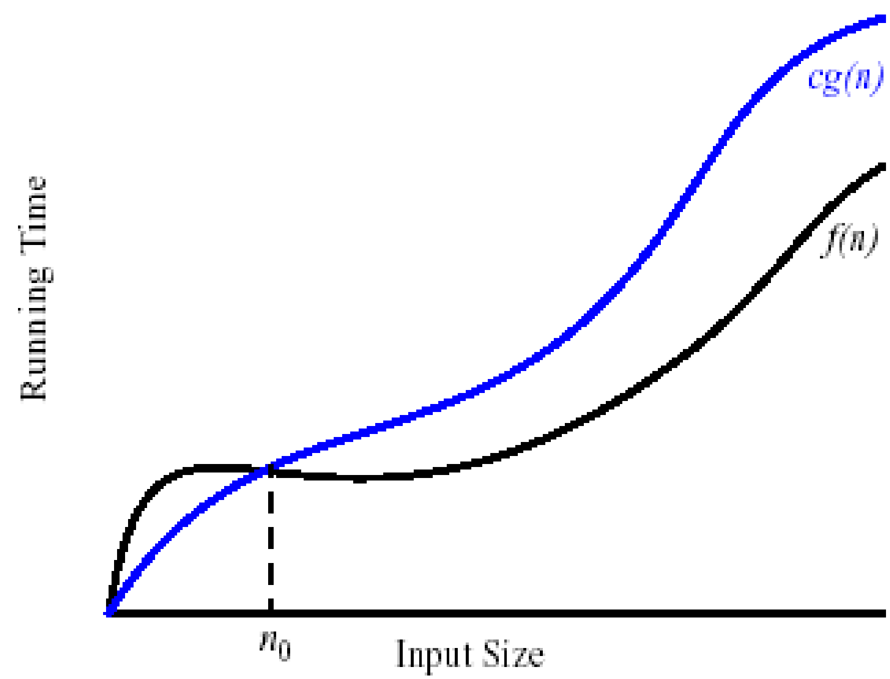
- Drop lower order terms and constant factors
- $7n - 3$ is $O(n)$
- $8n^2 \log n + 5n^2 + n$ is $O(n^2 \log n)$

Q8

Formal Definition of Big-Oh

- Given functions f(n) and g(n), we say that f(n) is O(g(n)) if and only if there exist constants $c > 0$ and $n_0 \geq 0$ such that
$$f(n) \leq c\, g(n) \text{ for all } n \geq n_0.$$

- For this to make sense, f(n) and g(n) should be functions over non-negative integers.



To *prove* Big Oh, find 2 constants and show they work

Q9

- A function f(n) is (in) O(g(n)) if there exist two positive constants **c** and $\mathbf{n_0}$ such that *for all* $n \geq n_0$, $f(n) \leq c\, g(n)$
- **Q: How to prove that f(n) is O(g(n))?**
  A: Give c and $\mathbf{n_0}$

Assume that all functions have non-negative values, and that we only care about $n \geq 0$. For any function g(n), O(g(n)) is a set of functions.

- Ex: $f(n) = 4n + 15$, $g(n) = ???$.

To *prove* Big Oh, find 2 constants and show they work

Q10

- A function f(n) is (in) O(g(n)) if there exist two positive constants **c** and $\mathbf{n_0}$ such that for all $n \geq n_0$, $f(n) \leq c\, g(n)$
- **Q: How to prove that f(n) is O(g(n))?**
  A: Give c and $\mathbf{n_0}$
- Ex 2: $f(n) = n + \sin(n)$, $g(n) = ???$