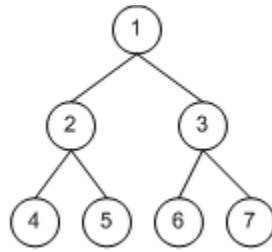


(a)



(b)

CSSE 230 Day 11

Size vs height in a Binary Tree

After today, you should be able to...

... use the relationship between the size and height of a tree to find the maximum and minimum number of nodes a binary tree can have

...understand the idea of mathematical induction as a proof technique

Team project starts Day 13

- Can voice preferences for partners for the term project (groups of 3)
 - EditorTrees partner preference survey on Moodle.
 - Preferences balanced with experience level + work ethic.
 - If course grades are close, I'll honor mutual prefs.
 - If no mutual pref, best to list several potential members.
 - If you don't want to work with someone, I'll honor that. But if your homework or exam average is low, I will put you with others in a similar position. Sorry if that's not your preference, but I can't burden someone who is doing well with someone who isn't.
 - Consider asking potential partners these things:
 - Are you aiming to get an A, or is less OK?
 - Do you like to get it done early or to procrastinate?
 - Do you prefer to work daytime, evening, late night?
 - How many late days do you have left?
 - Do you normally get a lot of help on the homework?
 - Survey is due Day 12; do it as soon as you can.

Some meme humor

If pants wore pants...
would they wear them



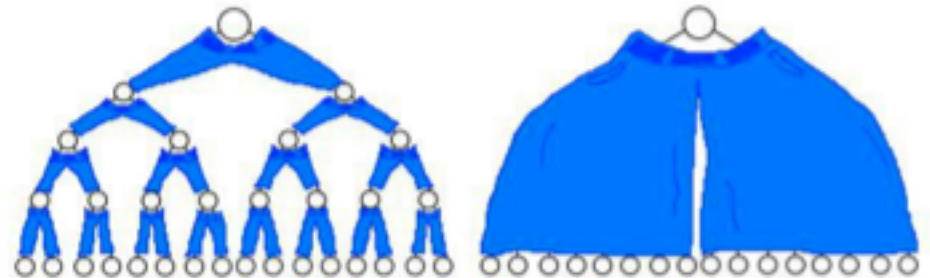
like this? or like this?

If a binary tree wore pants would he
wear them

like this

or

like this?



Other announcements

- Today:
 - Size vs height of trees: patterns and proofs
- Wrapping up the BST assignment, and worktime.

Extreme Trees

- A tree with the maximum number of nodes for its height is a **full** tree.
- A tree with the minimum number of nodes for its height is essentially a _____
- Height matters!
 - Recall that the algorithms for search, insertion, and deletion in a binary search tree are **$O(h(T))$**

Mathematical Induction

- To prove that $p(n)$ is true for all $n \geq n_0$:
 - Prove that $p(n_0)$ is true (base case), and
 - Prove that **if $p(k)$ is true** for any $k \geq n_0$, then $p(k+1)$ is also true.

[This part of the proof must work for all such k !]

To prove recursive properties (on trees), we use a technique called mathematical induction

- Actually, we use a variant called *strong induction* :



The former
governor of
California

Strong Induction

- To prove that $p(n)$ is true for all $n \geq n_0$:
 - Prove that $p(n_0)$ is true (base case), and
 - For all $k > n_0$, prove that if we assume $p(j)$ is true for $n_0 \leq j < k$, then $p(k)$ is also true
- An analogy:
 - Regular induction uses the previous domino to knock down the next
 - Strong induction uses all the previous dominos to knock down the next!
- Warmup: prove the arithmetic series formula
- Actual: prove the formula for $N(T)$