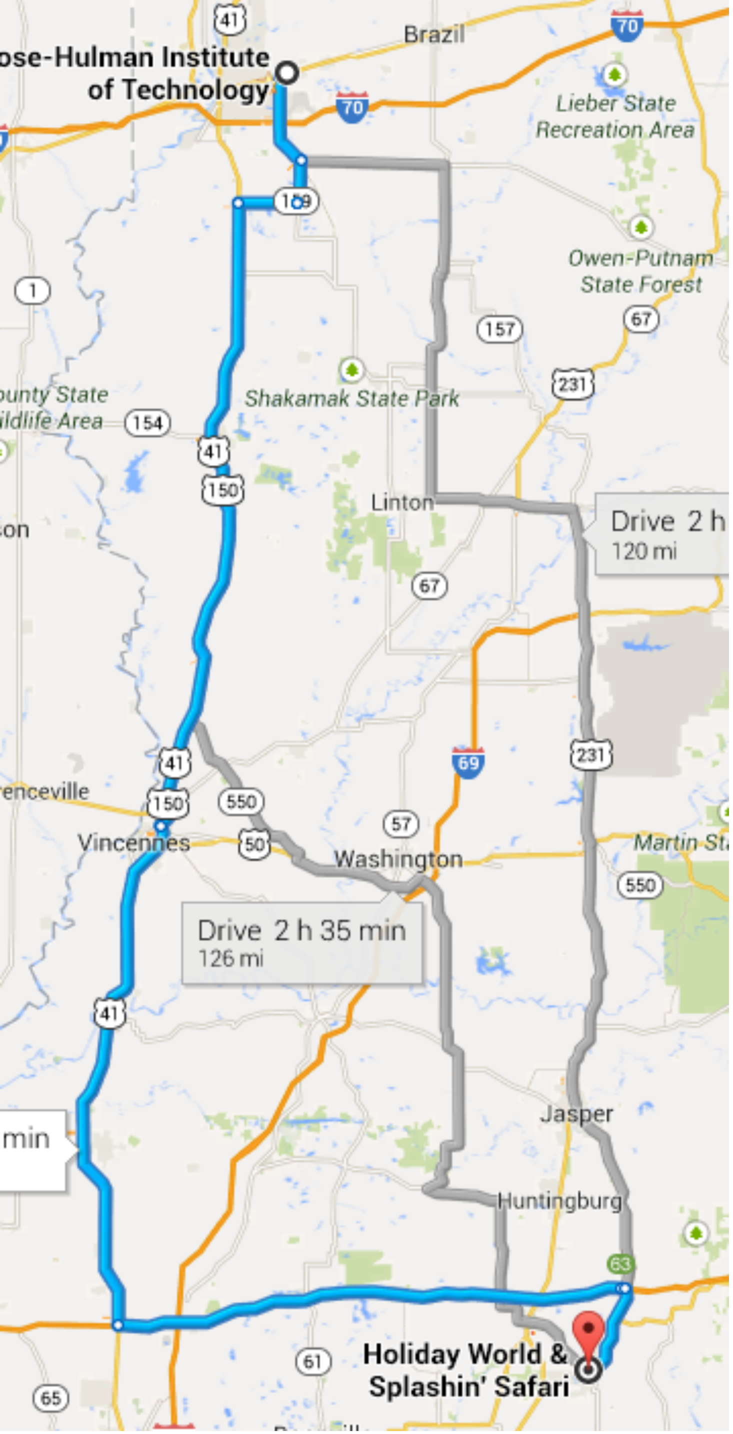
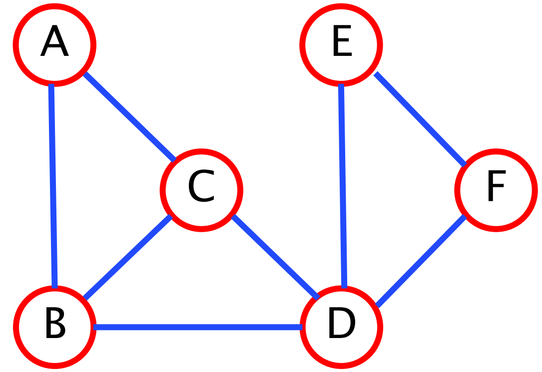




CSSE 230 Day 22

Graphs and their representations

After this lesson, you should be able to …
… define the major terminology relating to graphs
… implement a graph in code, using various conventions

https://www.google.com/maps/preview#!data=!1m4!1m3!1d989355!2d-87.4496039!3d38.8342589!4m26!3m17!1m5!1sRose-Hulman+Institute+of+Technology%2C+5500+Wabash+Ave%2C+Terre+Haute%2C+IN+47803!2s0x886d6e421b703737%3A0x96447680305ae1a4!3m2!3d39.482156!4d-87.322345!1m1!1sHoliday+World+%26+Splashin'+Safari%2C+Santa+Claus%2C+IN!3m8!1m3!1d245622!2d-86.923997!3d39.3256455!3m2!1i1920!2i955!4f13.1!5m2!13m1!1e1!7m4!11m3!1m1!1e1!2b1&fid=0

Graphs

Terminology
Representations
Algorithms

Graph Definitions

A graph $G = (V,E)$ is composed of:

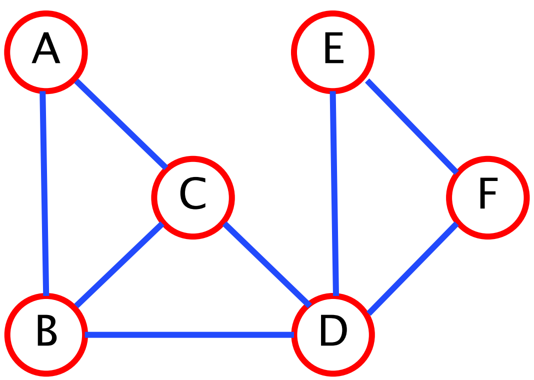
V: set of *vertices* (singular: vertex)

E: set of *edges*

An edge is a pair of vertices. Can be

unordered: $e = \{u,v\}$ (*undirected* graph)

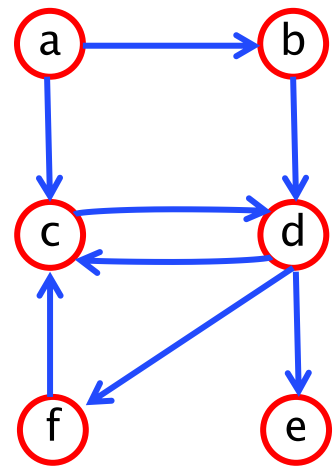
ordered: $e = (u,v)$ (*directed* graph / *digraph*)



Undirected

$V = \{A,B,C,D,E,F\}$

$E = \{\{A,B\},\{A,C\},\{B,C\},\{B,D\},\{C,D\},\{D,E\},\{D,F\},\{E,F\}\}$



Directed

$V = \{a,b,c,d,e,f\}$

$E = \{(a,b),(a,c),(b,d),(c,d),(d,c),(d,e),(d,f),(f,c)\}$

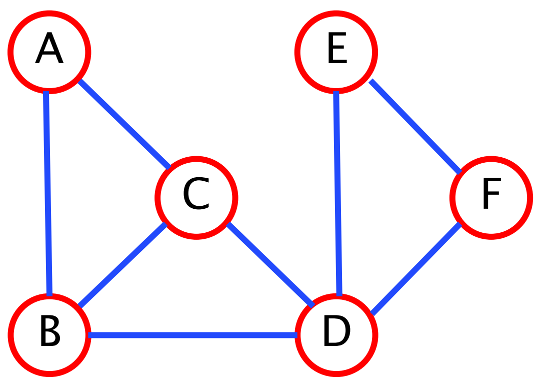
Graph Terminology

- Size? Edges or vertices?
- Usually take size to be n = |V| (# of vertices)
- But the runtime of graph algorithms often depend on the number of edges, |E|
- Relationships between |V| and |E|?

Undirected Graphs: adjacency, degree

- If {u,v} is an edge, then u and v are *neighbors* (also: u is *adjacent* to v)

- *degree* of v = number of neighbors of v



Fact:

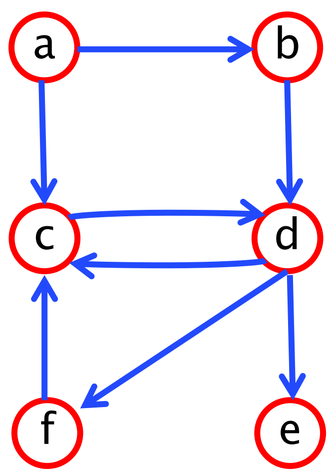
$$\sum_{v \in V} \deg(v) = 2|E|$$

(Why?)

Directed Graphs: adjacency, degree

- If (u,v) is an edge, then v is a *successor* of u and u is a *predecessor* of v

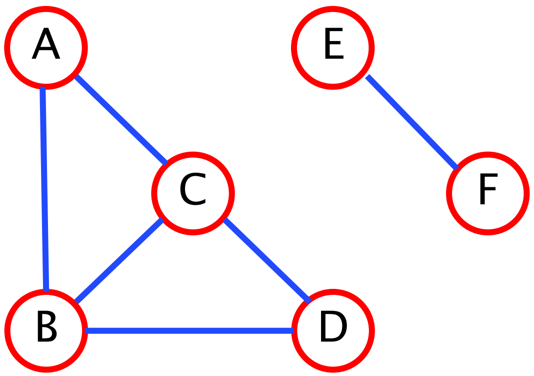
- *Out-degree* of v = number of successors of v
- *In-degree* of v = number of predecessors of v



Undirected Graphs: paths, connectivity

- A *path* is a list of unique vertices joined by edges.
  - For example, [a, c, d, e] is a path from a to e.

- A subgraph is *connected* if every pair of vertices in the subgraph has a path between them.



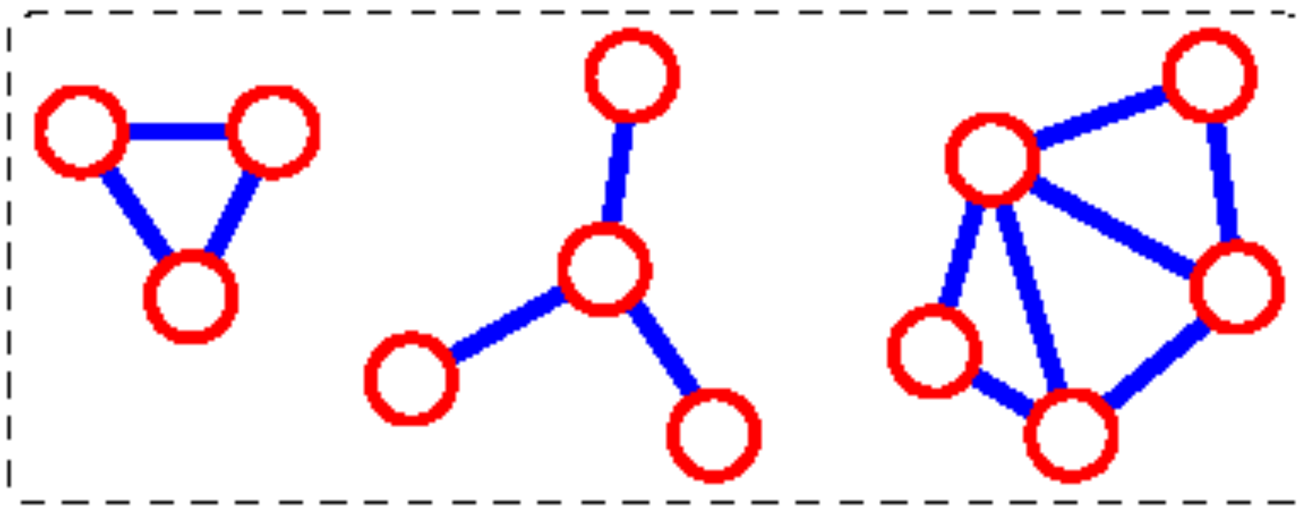
Not a connected graph.

Subgraph	Connected?
{A,B,C,D}	Yes
{E,F}	Yes
{C,D,E}	No
{A,B,C,D,E,F}	No

Undirected Graphs: components

(Connected) *component*: a maximal connected subgraph.

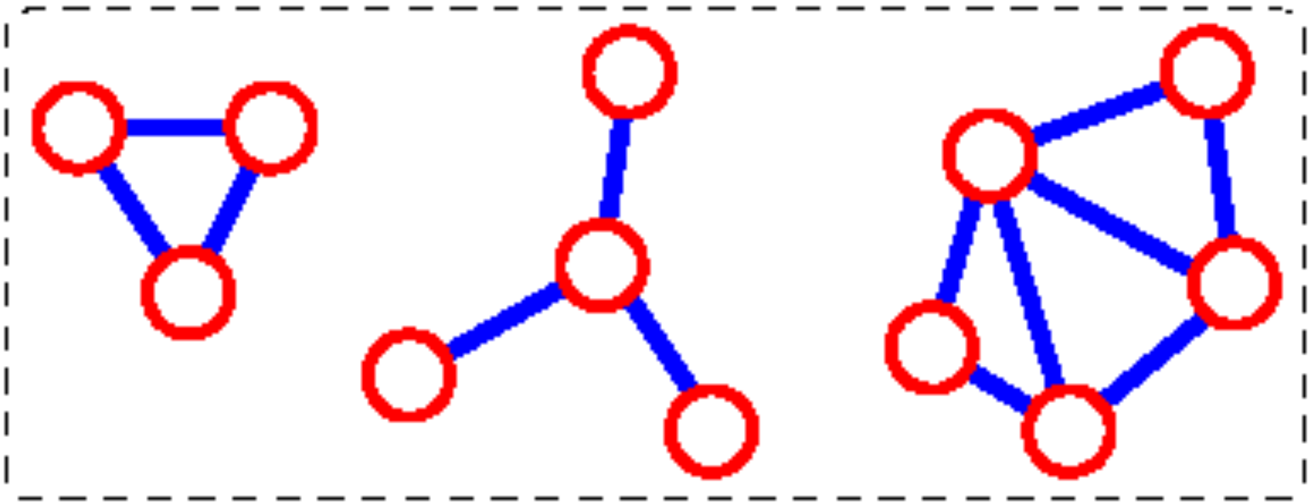
For example, this graph has 3 connected components:



Undirected Graphs: (mathematical) tree

Tree: connected acyclic graph (no cycles)

Example. Which component is a tree?



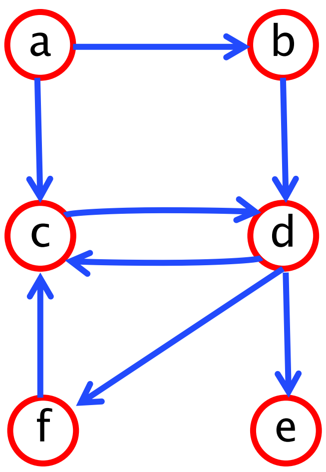
Question: for a tree, what is the relationship between m = #edges and n = #vertices?

$$m = n - 1$$

Directed Graphs: paths, connectivity

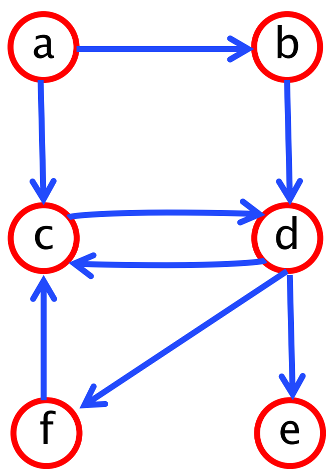
- A *directed path* is a list of unique vertices joined by directed edges.
  - For example, [a, c, d, f] is a directed path from a to f. We say f is *reachable* from a.

- A subgraph is *strongly connected* if for every pair (u,v) of its vertices, v is reachable from u and u is reachable from v.



Directed graphs: components

- *Strongly-connected component*: maximal strongly connected subgraph



Strongly connected components
{a}
{b}
{c,d,f}
{e}

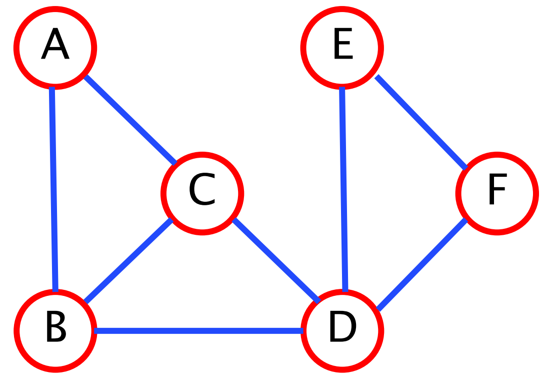
Viewing a graph as a data structure

- Each vertex associated with a name (key)
- Examples:
  - City name
  - IP address
  - People in a social network

- An edge (undirected/directed) represents a link between keys

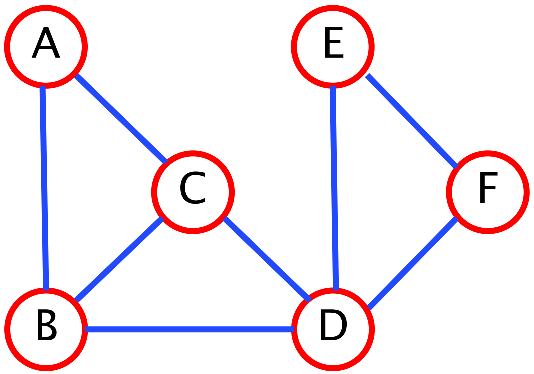
- Graphs are flexible: edges/nodes can have *weights*, *capacities*, or other attributes

There are several alternatives for representing edges of a graph



- Edge list
  - A collection of vertices and a collection of edges
- Adjacency matrix
  - Each key is associated with an index from 0, ..., (n–1)
    - Map from keys to ints?
  - Edges denoted by 2D array (#V x #V) of 0’s and 1’s
- Adjacency list
  - Collection of vertices
    - Map from keys to Vertex objects?
  - Each Vertex stores a List of adjacent vertices

Implementation tradeoffs



Adjacency list

- A → [B,C]
- B → [A,C,D]
- C → [A,B,D]
- D → [B,C,E,F]
- E → [D,F]
- F → [D,E]

Adjacency matrix

A→0
B→1
C→2
D→3
E→4
F→5

	0	1	2	3	4	5
0	0	1	1	0	0	0
1	1	0	1	1	0	0
2	1	1	0	1	0	0
3	0	1	1	0	1	1
4	0	0	0	1	0	1
5	0	0	0	1	1	0

- Running time of degree(v)?
- Running time of deleteEdge(u,v)?
- Space efficiency?

GraphSurfing assignment

- M1: Implement AdjacencyListGraph<T> and AdjacencyMatrixGraph<T>
  - both extend the given ADT, Graph<T>.
- M2: Write methods
  - stronglyConnectedComponent(v)
  - shortestPath(v)

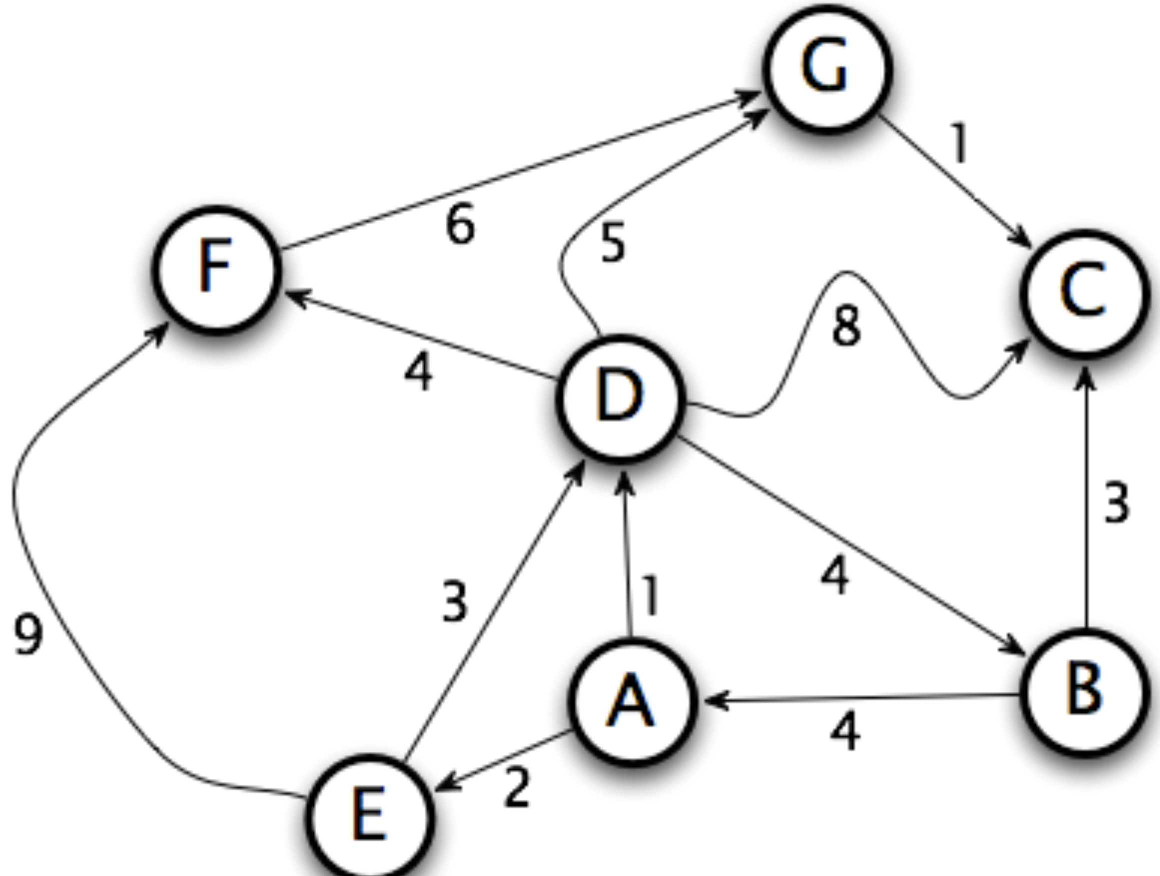
  and use them to go WikiSurfing!

Sample Graph Problems

To discuss algorithms, take MA/CSSE473 or MA477

Weighted Shortest Path

- What's the cost of the shortest path from A to each of the other nodes in the graph?

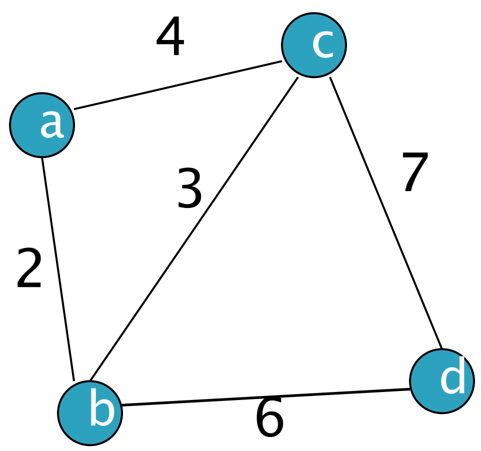


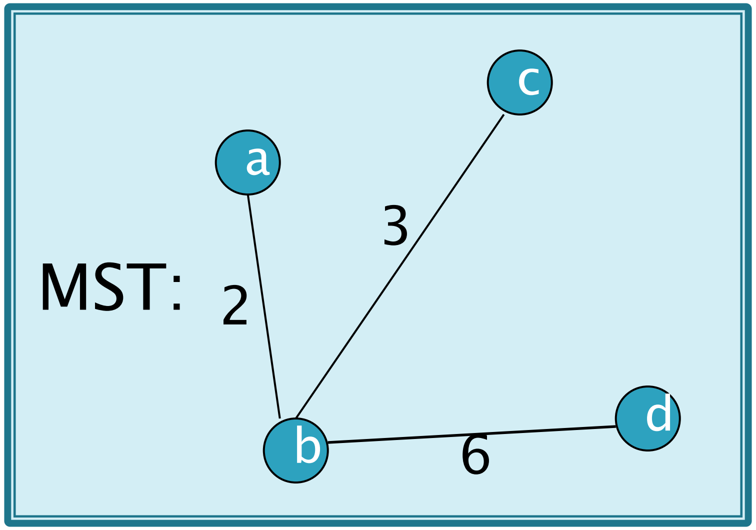
For much more on graphs, take MA/CSSE 473 or MA 477

Minimum Spanning Tree

- *Spanning tree*: a connected acyclic subgraph that includes all of the graph's vertices
- *Minimum spanning tree* of a weighted, connected graph: a spanning tree of minimum total weight

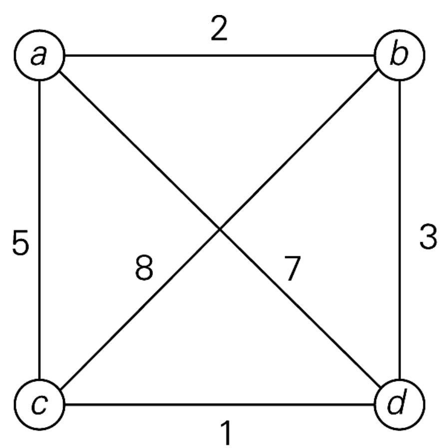
Example:





Traveling Salesman Problem (TSP)

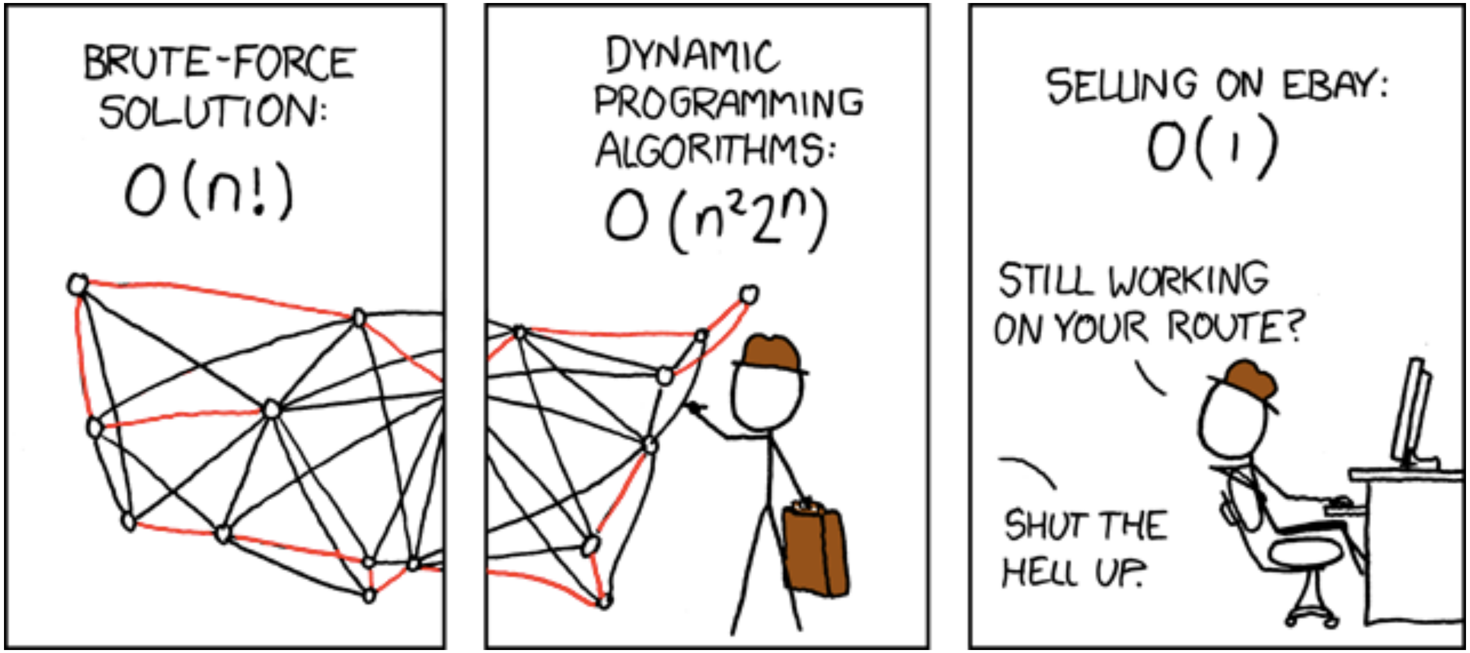
- *n* cities, weights are travel distance
- Must visit all cities (starting & ending at same place) with shortest possible distance



Tour	Length	
a —> *b* —> *c* —> *d* —> *a*	*l* = 2 + 8 + 1 + 7 = 18	
a —> *b* —> *d* —> *c* —> *a*	*l* = 2 + 3 + 1 + 5 = 11	optimal
a —> *c* —> *b* —> *d* —> *a*	*l* = 5 + 8 + 3 + 7 = 23	
a —> *c* —> *d* —> *b* —> *a*	*l* = 5 + 1 + 3 + 2 = 11	optimal
a —> *d* —> *b* —> *c* —> *a*	*l* = 7 + 3 + 8 + 5 = 23	
a —> *d* —> *c* —> *b* —> *a*	*l* = 7 + 1 + 8 + 2 = 18	

- Exhaustive search: how many routes?
- $(n-1)!/2 \in \Theta((n-1)!)$

Traveling Salesman Problem



- Online source for all things TSP:
  - http://www.math.uwaterloo.ca/tsp/

Example graphs for project