

$$i = 0..n-1$$

$$j = 0..n-1$$

$$k = i..j$$

| i | j | k | |
|-----|-----|----------|-----|
| 0 | 0 | 0..0 | → 1 |
| | 1 | 0..1 | → 2 |
| | 2 | 0..2 | → 3 |
| | ⋮ | | |
| | n-1 | 0..n-1 | → n |
| 1 | 1 | 1..1 | → 1 |
| | 2 | | 2 |
| | ⋮ | | |
| | n-1 | | n-1 |
| 2 | 1 | | 1 |
| | 2 | | 2 |
| | ⋮ | | |
| | n-2 | | n-2 |
| n-2 | n-2 | n-2..n-1 | 2 |
| | n-1 | n-1 | 1 |
| n-1 | n-1 | | 1 |
| | | | 1 |

$$\frac{n(n+1)}{2}$$

$$\frac{(n-1)(n-1+1)}{2}$$

$$\frac{(n-2)(n-2+1)}{2}$$

$$\frac{2(2+1)}{2} = 3$$

$$\frac{1(1+1)}{2} = 1$$

Sum these; since follow

same pattern

$$\sum_{m=1}^n \frac{m(m+1)}{2}$$

$$= \frac{1}{2} \left(\sum_{m=1}^n m^2 + \sum_{m=1}^n m \right)$$

$$= \frac{1}{2} \left(\frac{m(m+1)(2m+1)}{6} + \frac{m(m+1)}{2} \right)$$

$$= \frac{m(m+1)}{2} \left[\frac{2m+1}{6} + \frac{1}{2} \right]$$

$$= \frac{m(m+1)}{2} \left[\frac{2m+1+3}{6} \right]$$

$$= \frac{m(m+1)(2(m+2))}{2 \cdot 6}$$

$$= \frac{m(m+1)(m+2)}{6}$$

$$O(n^3)$$