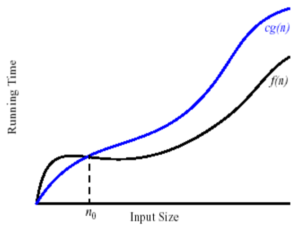
Q1

CSSE 230 Day 2



Growable Arrays Continued
Big-Oh and its cousins

Submit Growable Array exercise
Answer Q1-3 from today's in-class quiz.

Agenda and goals

- Finish course intro
- Growable Array recap
- Big-Oh and cousins

- After today, you'll be able to
  - Use the term *amortized* appropriately in analysis
  - explain the meaning of big-Oh, big-Omega (Ω), and big-Theta (θ)
  - apply the definition of big-Oh to prove runtimes of functions

Announcements and FAQ

- You will not usually need the textbook in class
- All should do piazza introduction post (a few students left)

You must demonstrate programming competence on exams to succeed

- See syllabus for exam weighting and caveats.
- Think of every program you write as a practice test
  - Especially HW4 and test 2a

Warm Up and Stretching thoughts

- Short but intense! ~45 lines of code total in our solutions to all but Adder
- Be sure to read the description of how it will be graded. Note how style will be graded.
- **Demo**: Running the JUnit tests for test, file, package, and project

  Demo: Run the Adder program

Q2–3

Questions?

- About Homework 1?
  - Aim to complete tonight, since it is due after next class
  - It is substantial
  - The last problem (the table) is worth lots of points!
- About the Syllabus?

Homework 1 help

How many times does sum++ run?

```
for (i = 4; i < n; i++)
   for (j = 0; j <= n; j++)
      sum++;
```

Why is this one so easy? (does the inner loop depend on outer loop?)

What if inner were (j = 0; j <= i ; j++) ?

Homework 1 help

How many times does sum++ run?

```
for (i = 1; i <= n; i *= 2)
   sum++;
```

Be precise, using floor/ceiling as needed, to get full credit.

Growable Arrays Exercise

Daring to double

Growable Arrays Table

N	E_N	Answers for problem 2
4	0	0
5	0	0
6	5	5
7	5	5 + 6 = 11
10	5	5 + 6 + 7 + 8 + 9 = 35
11	5 + 10 = 15	5 + 6 + 7 + 8 + 9 + 10 = 45
20	15	sum(i, i=5..19) = 180 **using Maple**
21	5 + 10 + 20 = 35	sum(i, i=5..20) = 200
40	35	sum(i, i=5..39) = 770
41	5 + 10 + 20 + 40 = 75	sum(i, i=5..40) = 810

Doubling the Size

- Doubling each time:
  - Assume that N = 5 (2^k) + 1.
- Total # of array elements copied:

k	N	#copies
0	6	5
1	11	5 + 10 = 15
2	21	5 + 10 + 20 = 35
3	41	5 + 10 + 20 + 40 = 75
4	81	5 + 10 + 20 + 40 + 80 = 155
k	= 5 (2^k) + 1	5(1 + 2 + 4 + 8 + … + 2^k)

Express as a closed-form expression in terms of K, then express in terms of N

Doubling the Size (solution)

- Assume that N = 5 (2^k) + 1.
- Total # of array elements copied = 5(1 + 2 + 4 + 8 + … + 2^k)
- Do in terms of k, then in terms of N

Adding One Each Time

- Total # of array elements copied:

N	#copies
6	5
7	5 + 6
8	5 + 6 + 7
9	5 + 6 + 7 + 8
10	5 + 6 + 7 + 8 + 9
N	???

Express as a closed-form expression in terms of N

Q4-5

Conclusions

- What's the amortized cost of adding an additional string…
  - in the doubling case?
  - in the add-one case?

Amortized cost means the "average per-operation cost" while adding to a single GrowableArray over time.

- So which should we use?

Logarithm review

Q6

Review these as needed

- Logarithms and Exponents
  - properties of logarithms:

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x/y) = \log_b x - \log_b y$$

$$\log_b x^{\alpha} = \alpha \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

  - properties of exponentials:

$$a^{(b+c)} = a^b a^c$$

$$a^{bc} = (a^b)^c$$

$$a^b/a^c = a^{(b-c)}$$

$$b = a^{\log_a b}$$

$$b^c = a^{c*\log_a b}$$

Practice with exponentials and logs

(Do these with a friend after class, not to turn in)

Simplify: Note that log n (without a specified) base means $\log_2 n$.
Also, log n is an abbreviation for log(n).

1. log (2 n log n)
2. log(n/2)
3. log (sqrt (n))
4. log (log (sqrt(n)))

5. $\log_4 n$
6. $2^{2 \log n}$
7. if $n=2^{3k} - 1$, solve for k.

Where do logs come from in algorithm analysis?

Solutions

No peeking!

Simplify: Note that log n (without a specified) base means $\log_2 n$.
Also, log n is an abbreviation for log(n).

1. 1+log n + log log n
2. log n - 1
3. ½ log n
4. -1 + log log n

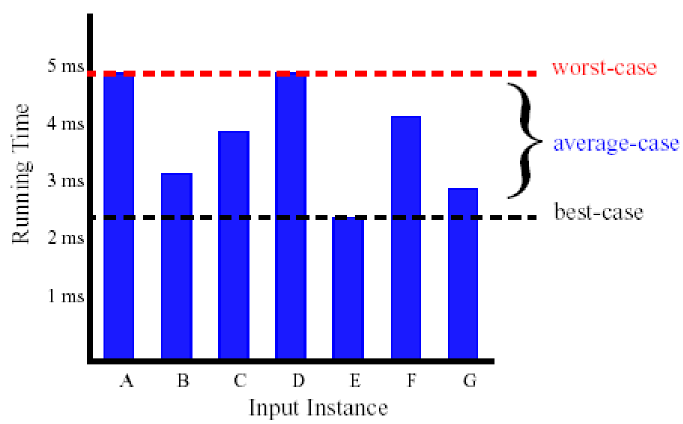
5. (log n) / 2
6. n^2
7. $n+1=2^{3k}$
   log(n+1)=3k
   k= log(n+1)/3

A: Any time we cut things in half at each step
(like binary search or mergesort)

Running Times

- Algorithms may have different *time complexity* on different data sets
- What do we mean by "Worst Case"?
- What do we mean by "Average Case"?
- What are some application domains where knowing the Worst Case time complexity would be important?
- http://cacm.acm.org/magazines/2013/2/160173-the-tail-at-scale/fulltext

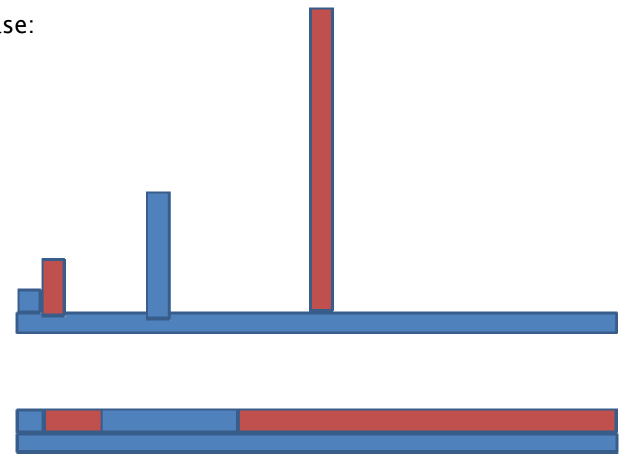
Average Case and Worst Case



Worst-case vs amortized cost for adding an element to an array using the doubling scheme

Worst-case:
O(n)

amortized:
O(1)



Note: average case means averaged over *inputs*, amortized cost means averaged over *time*.

Asymptotics: The "Big" Three

Big-Oh
Big-Omega
Big-Theta

Asymptotic Analysis

- We only care what happens when N gets large
- Is the function linear? quadratic? exponential?

Figure 5.1
Running times for small inputs

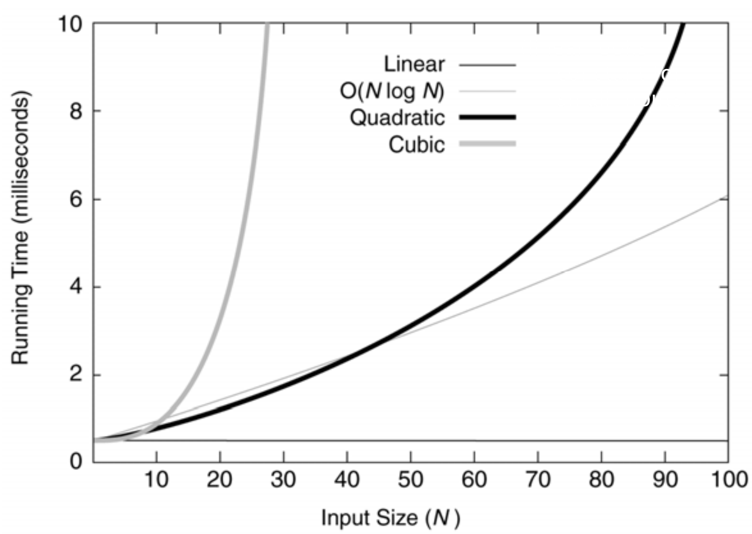


Figure 5.2
Running times for moderate inputs

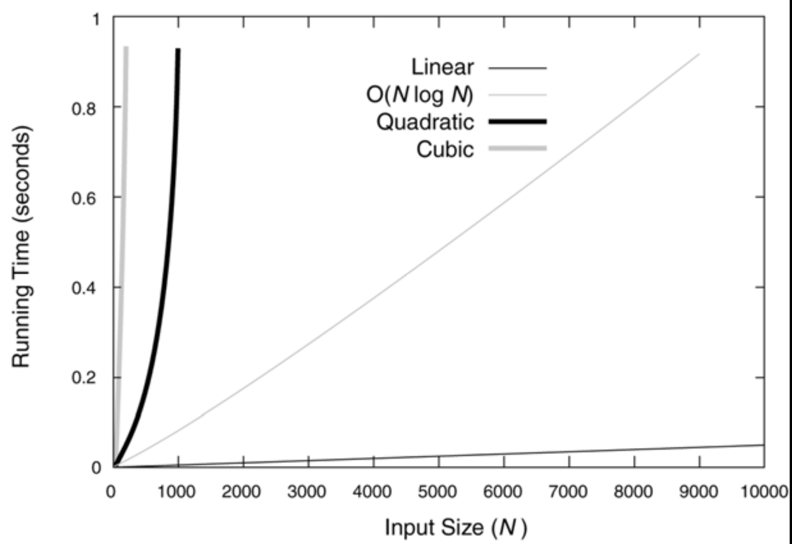


Figure 5.3
Functions in order of increasing growth rate

The answer to most big-Oh questions is one of these functions

Function	Name
c	Constant
$\log N$	Logarithmic
$\log^2 N$	Log-squared
N	Linear
$N \log N$	N log N
N^2	Quadratic
N^3	Cubic
2^N	Exponential

a.k.a "log linear"

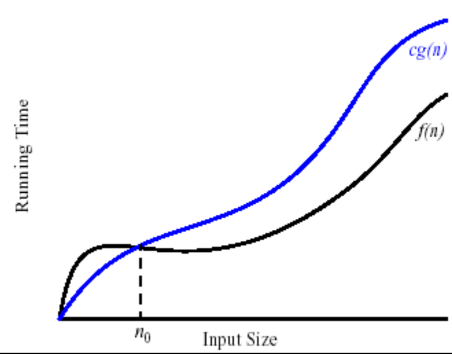
Simple Rule for Big-Oh

- Drop lower order terms and constant factors
- 7n - 3 is O(n)
- $8n^2 \log n + 5n^2 + n$ is $O(n^2 \log n)$

Q7a

Formal Definition of Big-Oh

- Given functions f(n) and g(n), we say that f(n) is O(g(n)) if and only if $f(n) \le c\, g(n)$ for all $n \ge n_0$.
- Two constants: $c > 0$ is a real number and $n_0 \ge 0$ is an integer.
- f(n) and g(n) are functions over non-negative integers.



Q8

To *prove* Big Oh, find 2 constants

- A function f(n) is (in) O(g(n)) if there exist two positive constants **c** and $\mathbf{n_0}$ such that *for all* $n \geq n_0$, $f(n) \leq c\, g(n)$
- **Q: How to prove that f(n) is O(g(n))?**
  A: Give c and $\mathbf{n_0}$

Assume that all functions have non-negative values, and that we only care about $n \geq 0$. For any function g(n), O(g(n)) is a set of functions.

- Ex: f(n) = 4n + 15, g(n) = ???.

Q9

To *prove* Big Oh, find 2 constants

- A function f(n) is (in) O(g(n)) if there exist two positive constants **c** and $\mathbf{n_0}$ such that for all $n \geq n_0$, $f(n) \leq c\, g(n)$
- **Q: How to prove that f(n) is O(g(n))?**
  A: Give c and $\mathbf{n_0}$
- Ex 2: f(n) = n + sin(n), g(n) = ???

Q7bc,10

Big-Oh, Big-Omega, Big-Theta

$O(\,)$ $\Omega(\,)$ $\theta(\,)$

- f(n) is O(g(n)) if $f(n) \leq cg(n)$ for all $n \geq n_0$
  - So big-Oh (O) gives an upper bound
- f(n) is Ω(g(n)) if $f(n) \geq cg(n)$ for all $n \geq n_0$
  - So big-omega (Ω) gives a lower bound
- f(n) is θ(g(n)) if it is both O(g(n)) and Ω(g(n))
  Or equivalently:
- f(n) is θ(g(n)) if $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$
  - So big-theta (θ) gives a tight bound
- True or false: $3n+2$ is $O(n^3)$
- True or false: $3n+2$ is $\Theta(n^3)$

Uses of O, Ω, Θ

- By definition, applied to *functions.*
  "$f(n) = n^2/2 + n/2 - 1$ is $\Theta(n^2)$"
- Can also be applied to an *algorithm*, referencing its **running time**: e.g., when f(n) describes the number of executions of the most-executed line of code.
  "selection sort is $\Theta(n^2)$"
- Finally, can be applied to a *problem*, referencing its **complexity**: the running time of the best algorithm that solves it.
  "The sorting problem is $O(n^2)$"

Big-Oh Style

- Give tightest bound you can
  - Saying $3n+2$ is $O(n^3)$ is true, but not as useful as saying it's $O(n)$
  - On a test, we'll ask for Θ to be clear.
- Simplify:
  - You could also say: $3n+2$ is $O(5n-3\log(n) + 17)$
  - And it would be technically correct…
  - It would also be poor taste … and your grade will reflect that.

Efficiency in context

Q11-13, submit quiz

- There are times when one might choose a higher-order algorithm over a lower-order one.
- Brainstorm some ideas to share with the class

C.A.R. Hoare, inventor of quicksort, wrote:
Premature optimization is the root of all evil.