

# CSSE 230 Day 16

Exhaustive search, backtracking, object-oriented Queens

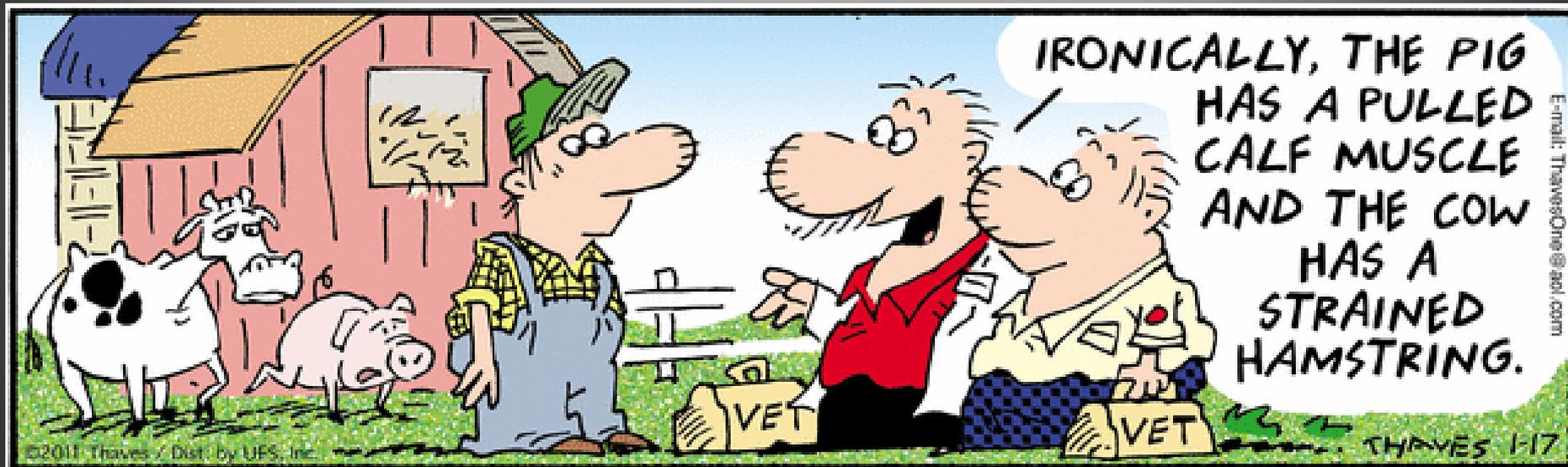
In SVN:  
Queens

After today, you should be able to...

...explain the concept of backtracking

...solve the n-queens problem using backtracking

# Questions



# Exhaustive Search and Backtracking

A taste of artificial intelligence

Check out Queens from SVN

# Exhaustive search

- ▶ Given: a (large) set of possible solutions to a problem
- ▶ Goal: Find all solutions (or an optimal solution) from that set
- ▶ Questions we ask:
  - How do we represent the possible solutions?
  - How do we organize the search?
  - Can we avoid checking some obvious non-solutions?



The “search space”

# In backtracking, we always try to extend a partial solution

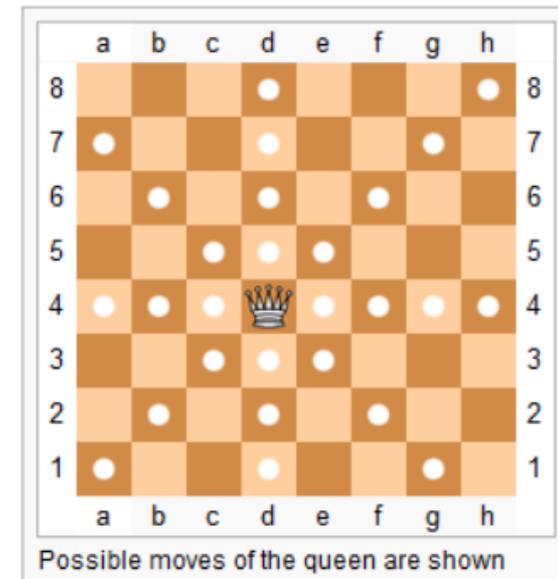
- ▶ Examples: Doublets, solving a maze, the “15” puzzle.
- ▶ Taken from:
  - <http://www.cis.upenn.edu/~matuszek/cit594-2004/Lectures/38-backtracking.ppt>

1	10	2	4
5		3	6
9	11	8	7
13	14	15	12



# The non-attacking chess queens problem is a famous example

- In how many ways can  $N$  chess queens be placed on an  $N \times N$  grid, so that none of the queens can attack any other queen?
  - I.e. there are not two queens on the same row, same column, or same diagonal.
- ▶ There is no "formula" for generating a solution.
- ▶ The famous computer scientist Niklaus Wirth described his approach to the problem in 1971: **Program Development by Stepwise Refinement**  
<http://sunnyday.mit.edu/16.355/wirth-refinement.html#3>



# With a partner, discuss "possible solution" search strategies

- ▶ In how many ways can  $N$  chess queens be placed on an  $N \times N$  grid, so that none of the queens can attack any other queen?
  - I.e. no two queens on the same row, same column, or same diagonal.

Two minutes  
No Peeking!

- ▶ **Very naive approach. Perhaps stupid is a better word!**

There are  $N$  queens,  $N^2$  squares.

- ▶ For each queen, try every possible square, allowing the possibility of multiple queens in the same square.
  - Represent each potential solution as an  $N$ -item array of pairs of integers (a row and a column for each queen).
  - Generate all such arrays (you should be able to write code that would do this) and check to see which ones are solutions.
  - Number of possibilities to try in the  $N \times N$  case:
  - Specific number for  $N=8$ :

**281,474,976,710,656**

# Search Space Possibilities 2/5

**Slight improvement.** There are  $N$  queens,  $N^2$  squares. For each queen, try every possible square, notice that we can't have multiple queens on the same square.

- Represent each potential solution as an  $N$ -item array of pairs of integers (a row and a column for each queen).
- Generate all such arrays and check to see which ones are solutions.
- Number of possibilities to try in  $N \times N$  case:
- Specific number for  $N=8$ :

**178,462,987,637,760**  
**(vs. 281,474,976,710,656)**

# Search Space Possibilities 3/5

- ▶ **Slightly better approach.** There are  $N$  queens,  $N$  columns. If two queens are in the same column, they will attack each other. Thus there must be exactly one queen per column.
- ▶ Represent a potential solution as an  $N$ -item array of integers.
  - Each array position represents the queen in one column.
  - The number stored in an array position represents the row of that column's queen.
  - **Show array for 4x4 solution.**
    - Generate all such arrays and check to see which ones are solutions.
    - Number of possibilities to try in  $N \times N$  case:
    - Specific number for  $N=8$ :

**16,777,216**

# Search Space Possibilities 4/5

- ▶ **Still better approach** There must also be exactly one queen per row.
- ▶ Represent the data just as before, but notice that the data in the array is a set!
  - Generate each of these and check to see which ones are solutions.
  - **How to generate?** A good thing to think about.
  - Number of possibilities to try in  $N \times N$  case:
  - Specific number for  $N=8$ :

**40,320**

# Search Space Possibilities 5/5

- ▶ **Backtracking solution**
- ▶ Instead of generating all permutations of  $N$  queens and checking to see if each is a solution, we generate "partial placements" by placing one queen at a time on the board
- ▶ Once we have successfully placed  $k < N$  queens, we try to *extend* the partial solution by placing a queen in the next column.
- ▶ When we extend to  $N$  queens, we have a solution.

# Experimenting with 8 x 8 Case

- ▶ Play the game:

- <http://homepage.tinet.ie/~pdpals/8queens.htm>

- ▶ See the solutions:

- <http://www.dcs.ed.ac.uk/home/mlj/demos/queens>

# Program output:

```
>java RealQueen 5
```

```
SOLUTION:  1 3 5 2 4
```

```
SOLUTION:  1 4 2 5 3
```

```
SOLUTION:  2 4 1 3 5
```

```
SOLUTION:  2 5 3 1 4
```

```
SOLUTION:  3 1 4 2 5
```

```
SOLUTION:  3 5 2 4 1
```

```
SOLUTION:  4 1 3 5 2
```

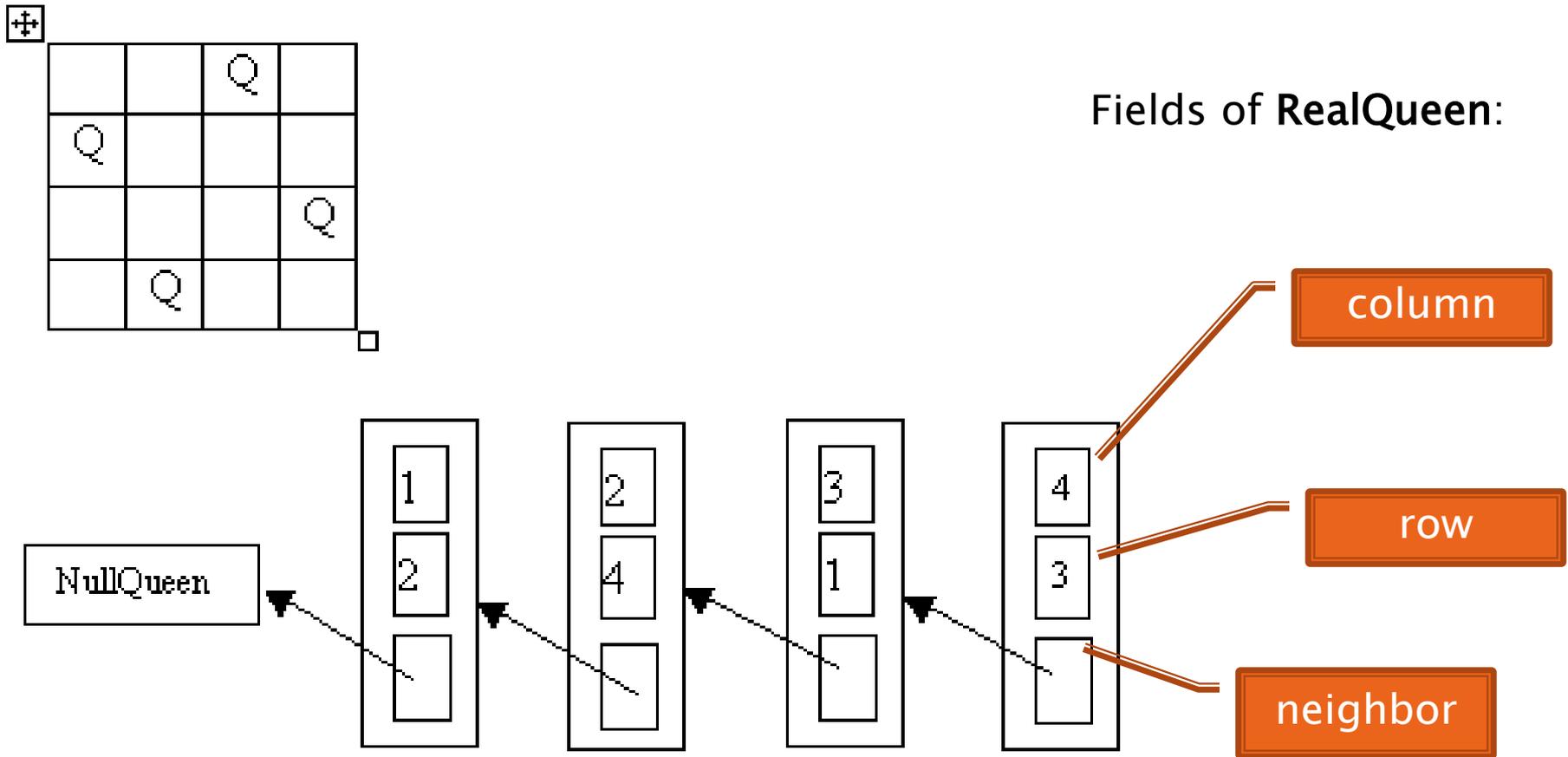
```
SOLUTION:  4 2 5 3 1
```

```
SOLUTION:  5 2 4 1 3
```

```
SOLUTION:  5 3 1 4 2
```

# The Linked List of Queen Objects

- ▶ Board configuration represented by a linked list of **Queen** objects



Designed by Timothy Budd

<http://web.engr.oregonstate.edu/~budd/Books/oopintro3e/info/slides/chap06/java.htm>

# Outline of the algorithm

- ▶ Each queen sends messages directly to its immediate neighbor to the left (and recursively to all of its left neighbors)
- ▶ Return value provides information concerning all of the left neighbors:
- ▶ Example: `neighbor.canAttack(currentRow, col)`
  - Message goes to the immediate neighbor, but the real question to be answered by this call is
  - "Hey, neighbors, can any of you attack me if I place myself on this square of the board?"

- ▶ `findFirst()`
- ▶ `findNext()`
- ▶ `canAttack(int row, int col)`

Your job (part of WA6):

**Understand the job of each of these methods.**

**Javadoc from the Queen interface can help**

Fill in the (recursive) details in the RealQueen class

**Debug**

More details on next slide

# More algorithm outline

1. Queen asks its neighbors to find the first position in which none of them attack each other
  - Found? Then queen tries to position itself so that it cannot be attacked.
2. If the rightmost queen is successful, then a solution has been found! The queens cooperate in recording it.
3. Otherwise, the queen asks its neighbors to find the next position in which they do not attack each other
4. When the queens get to the point where there is no next non-attacking position, all solutions have been found and the algorithm terminates

**And recursion does its magic!**