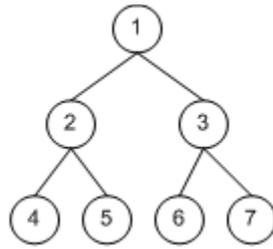


(a)



(b)

# CSSE 230 Day 11

## Size vs height in a Binary Tree

After today, you should be able to...

... use the relationship between the size and height of a tree to find the maximum and minimum number of nodes a binary tree can have

... understand the idea of mathematical induction as a proof technique

# Team project starts end of week

- ▶ Can voice preferences for partners for the term project (groups of 3)
  - EditorTrees partner preference survey on Moodle
    - Preferences balanced with experience level + work ethic
      - If course grades close, I'll honor mutual prefs.
      - If no mutual pref, best to list several potential members.
      - If you don't want to work with someone, I'll honor that. But if your homework or exam average is low, I will put you with others in a similar position. Sorry if that's not your preference, but I can't burden someone who is doing well with someone who isn't.
    - Consider asking potential partners these things:
      - Are you aiming to get an A, or is less OK?
      - Do you like to get it done early or to procrastinate?
      - Do you prefer to work daytime, evening, late night?
      - How many late days do you have left?
      - Do you normally get a lot of help on the homework?
  - Do it tonight, or by Thursday at noon.

# Other announcements

- ▶ Doublets partner evals due tonight
- ▶ Today:
  - Size vs height of trees: patterns and proofs
- ▶ Wrapping up the BST assignment, and worktime.

# Extreme Trees

- ▶ A tree with the maximum number of nodes for its height is a **full tree**.
  - Its height is  **$O(\log N)$**
- ▶ A tree with the minimum number of nodes for its height is essentially a \_\_\_\_\_
  - Its height is  **$O(N)$**
- ▶ Height matters!
  - Recall that the algorithms for search, insertion, and deletion in a binary search tree are  **$O(h(T))$**

To prove recursive properties (on trees), we use a technique called mathematical induction

- ▶ Actually, we use a variant called *strong induction* :



The former  
governor of  
California

# Strong Induction

- ▶ To prove that  $p(n)$  is true for all  $n \geq n_0$ :
  - Prove that  $p(n_0)$  is true (base case), and
  - For all  $k > n_0$ , prove that if we assume  $p(j)$  is true for  $n_0 \leq j < k$ , then  $p(k)$  is also true
- ▶ An analogy for those who took MA275:
  - Regular induction uses the previous domino to knock down the next
  - Strong induction uses all the previous dominos to knock down the next!
- ▶ Warmup: prove the arithmetic series formula
- ▶ Actual: prove the formula for  $N(T)$