

CSSE 230 Day 28

Graphs and their representations

After this lesson, you should be able to ...
... explain what makes a graph different than a tree
... implement simple graph algorithms

<https://www.google.com/maps/preview#!data=!1m4!1m3!1d989355!2d-87.4496039!3d38.8342589!4m2!3m1!7!1m5!1sRose-Hulman+Institute+of+Technology%2C+5500+Wabash+Ave%2C+Terre+Haute%2C+IN+47803!2s0x886d6e421b703737%3A0x96447680305ae1a4!3m2!3d39.482156!4d-87.322345!1m1!1sHoliday+World+%26+Splashin'+Safari%2C+Santa+Claus%2C+IN!3m8!1m3!1d245622!2d-86.923997!3d39.325645!3m2!1i1920!2i955!4f13.1!5m2!13m1!1e1!7m4!1!1m3!1m1!1e1!2b1&fid=0>

Graphs

Terminology

Representations

Algorithms

Example Graph

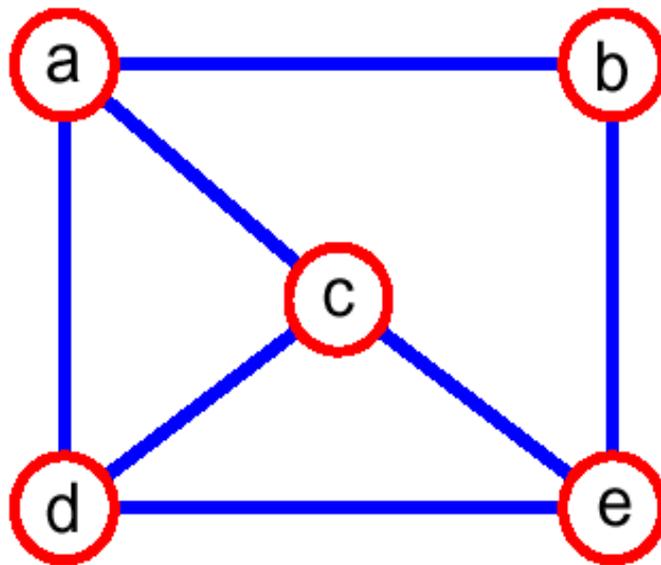
A graph $G = (V, E)$ is composed of:

V : set of *vertices*

E : set of *edges* connecting the *vertices* in V

An **edge** $e = (u, v)$ is a pair of *vertices*

Example:



$V = \{a, b, c, d, e\}$

$E =$

$\{(a, b), (a, c), (a, d),$
 $(b, e), (c, d), (c, e),$
 $(d, e)\}$

- ▶ Size? Edges or vertices?
- ▶ In this class, we use
 - Size: # **V** (number of edges)
- ▶ But the runtime of graph algorithms often depend on the number of edges

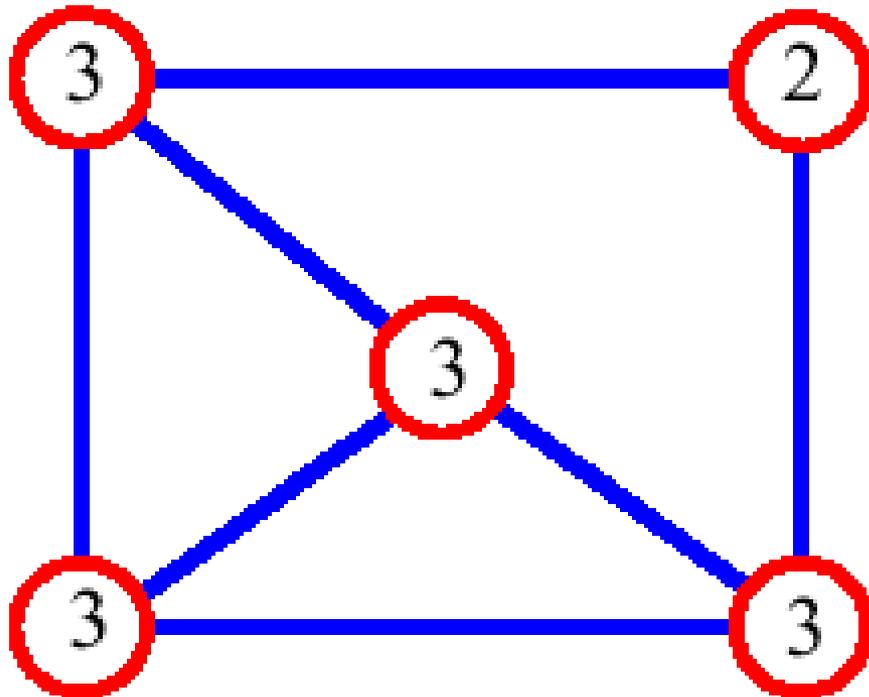
Graph Terminology

also called
“neighbors”

- **adjacent vertices**: connected by an edge
- **degree** (of a **vertex**): # of adjacent vertices

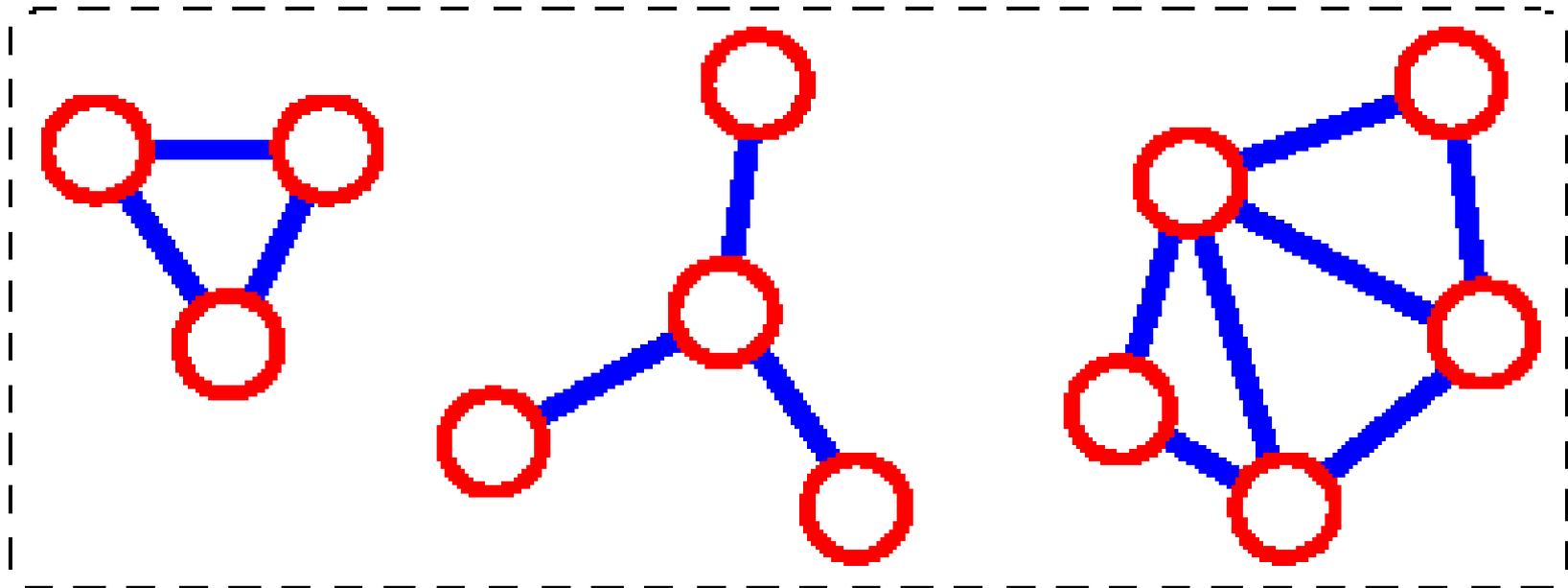
$$\sum_{v \in V} \deg(v) = 2(\# \text{ edges})$$

- Since adjacent vertices each count the adjoining edge, it will be counted twice



Continuing Graph Terminology

connected component: maximal connected subgraph. E.g., the graph below has 3 connected components.



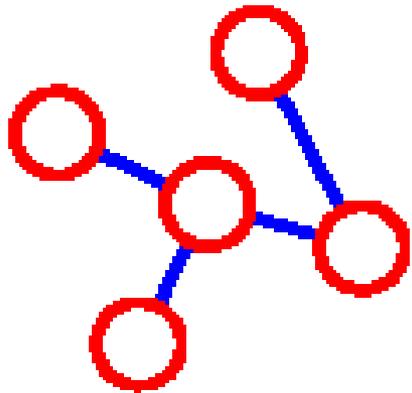
More Connectivity

n = #vertices

m = #edges

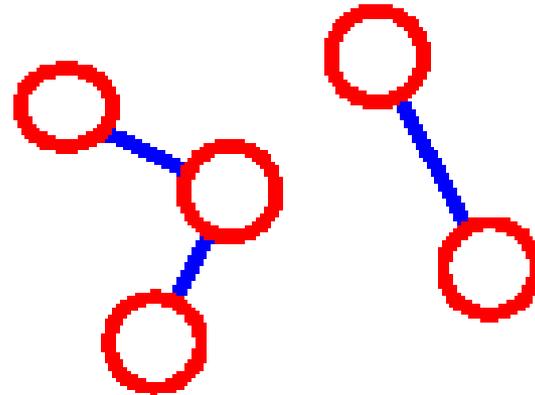
For a tree **m** = **n** - 1

A necessary but not sufficient condition for a graph to be a tree.



n = 5
m = 4

If **m** < **n** - 1, G is not connected



n = 5
m = 3

We represent vertices using a collection of objects

- ▶ Each Vertex object contains information about itself
- ▶ Examples:
 - City name
 - IP address
 - People in a social network
- ▶ Observations about real graphs?

There are many options for representing edges of a graph

- ▶ Adjacency matrix
- ▶ Adjacency list. Each vertex stores...
 - pointers to other vertices?
 - named vertices using a `HashMap<Name,Vertex>`
 - An index into an array of the `Vertex` objects. In each case, we need a way to store the vertex collection
- ▶ Edge list

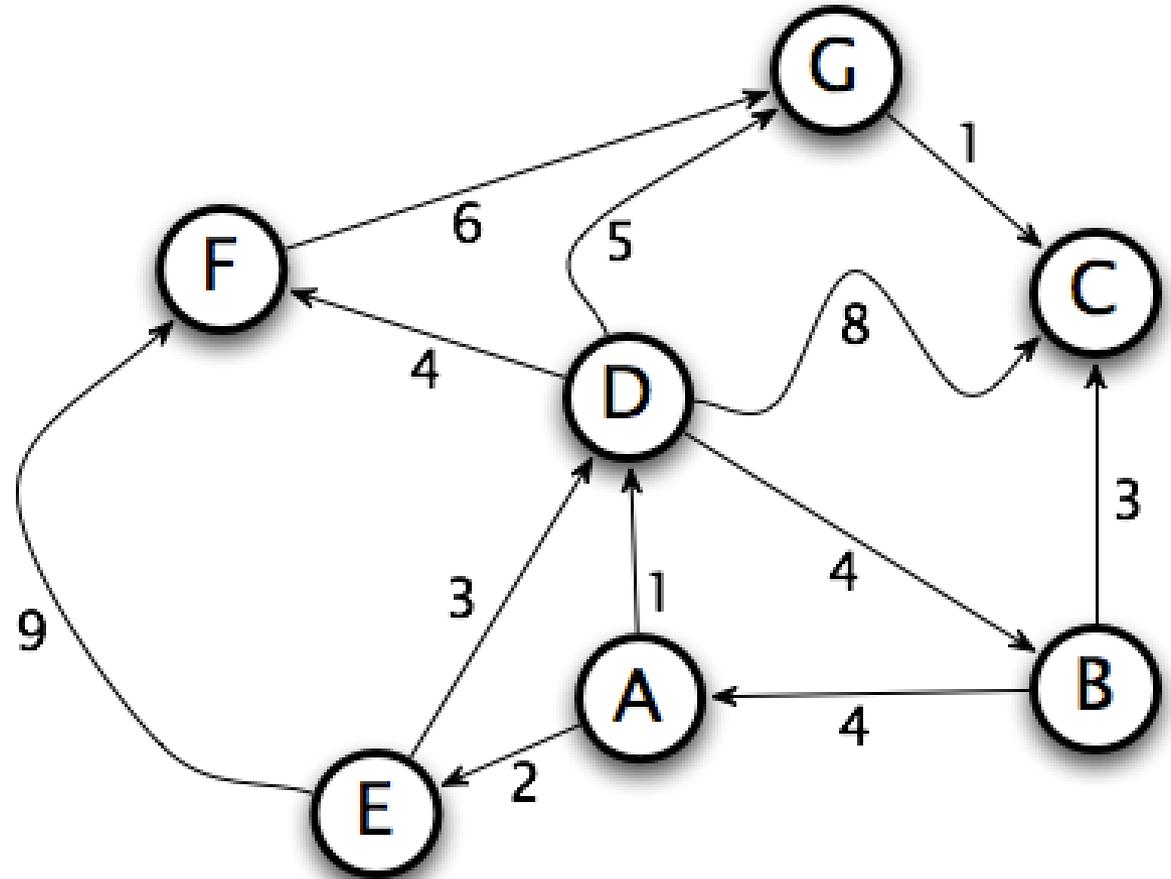
To consider:

Why not just use a triangular “matrix”?

Does a boolean adjacency matrix make sense?

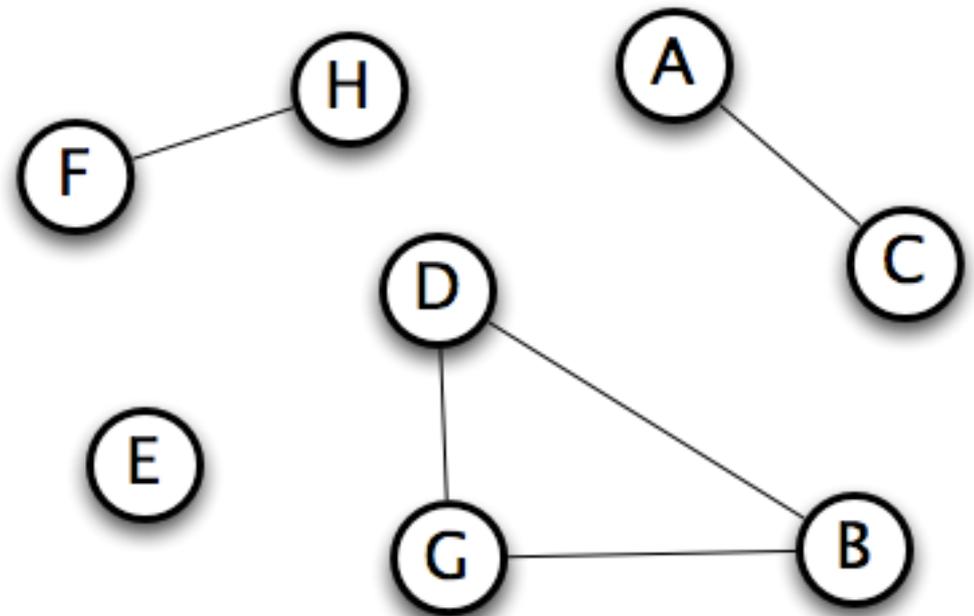
Sample graph problem: Weighted Shortest Path

- ▶ What's the cost of the shortest path from A to each of the other nodes in the graph?



For much more on graphs, take MA/CSSE 473 or MA 477

- ▶ What's the size of the largest connected component?



In SVN: RandomGraphs