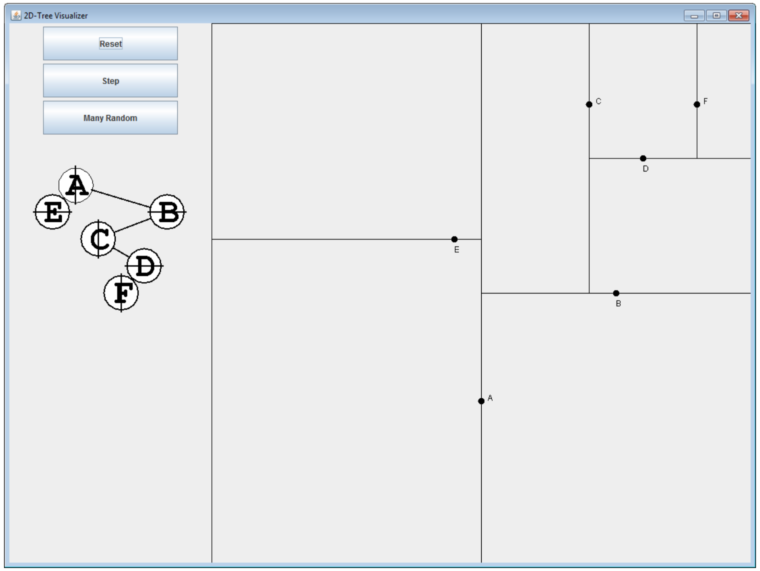
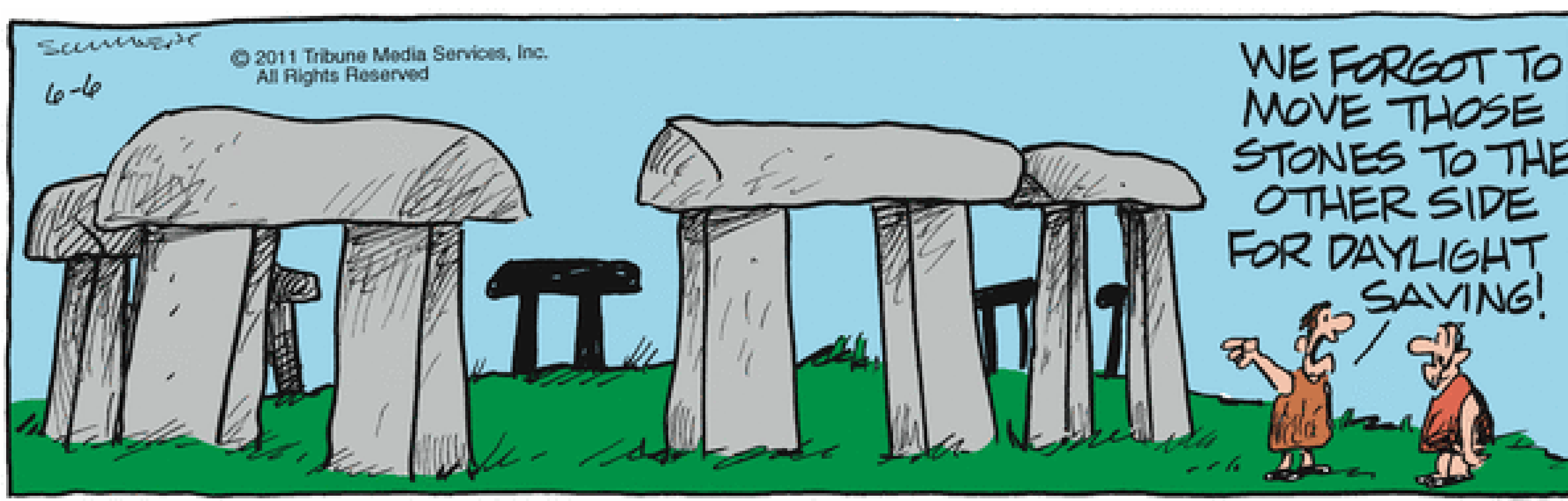


CSSE 230 Day 25

2D Trees

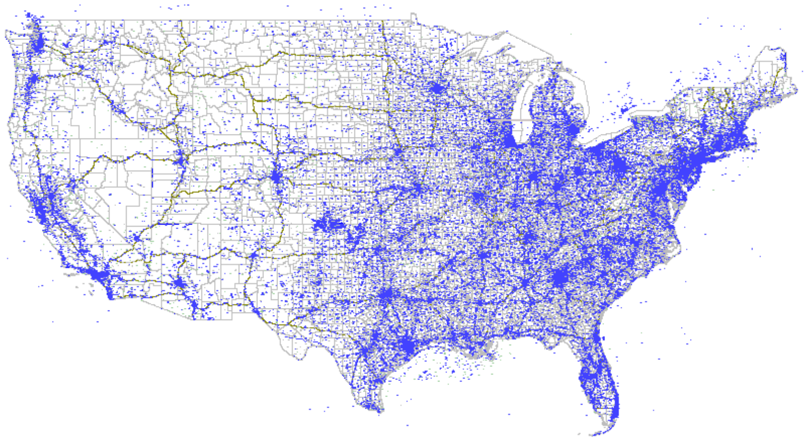
After today, you should be able to …
… explain insert and nearest-neighbor in 2D trees
… implement these algorithms

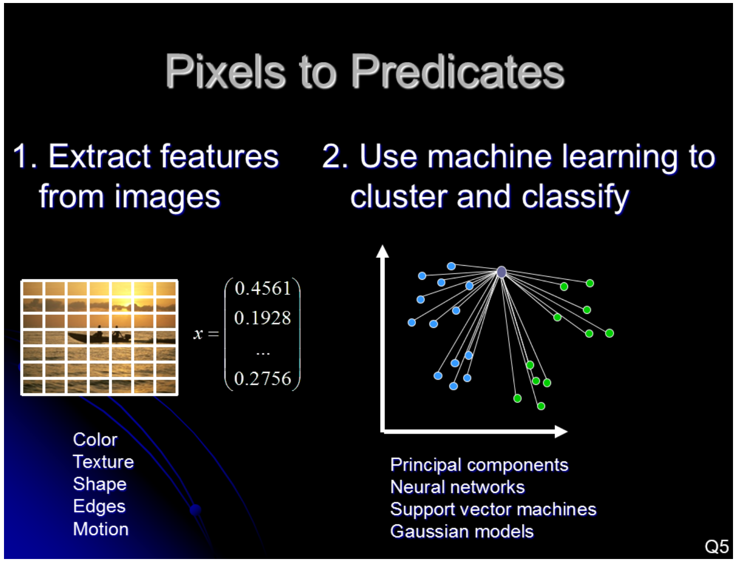
Reminders/Announcements



2D Data

- A large set of (x,y) points
- Which cell phone tower is closest to me?
- Which image is most like this one?
- In general:
  - Find the nearest neighbor of a query point (today).
  - Find or return all points in a certain range.

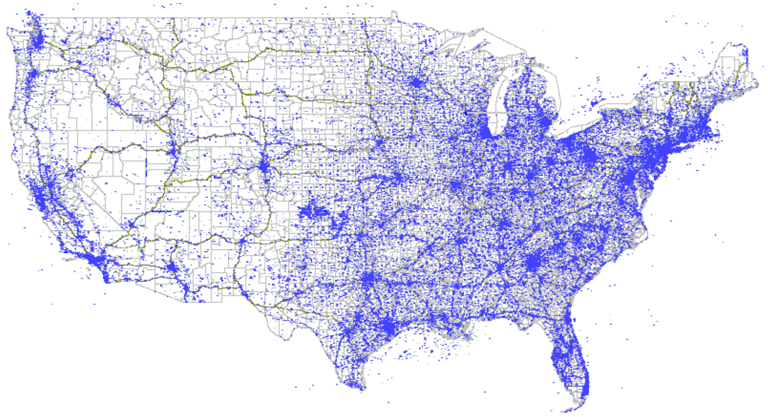




https://personalpages.manchester.ac.uk/staff/m.dodge/cybergeography/atlas/tower_maps_large.gif
https://www.rose-hulman.edu/class/csse/csse463/201520/Slides/01%20Introduction.pdf

How to represent 2D data?

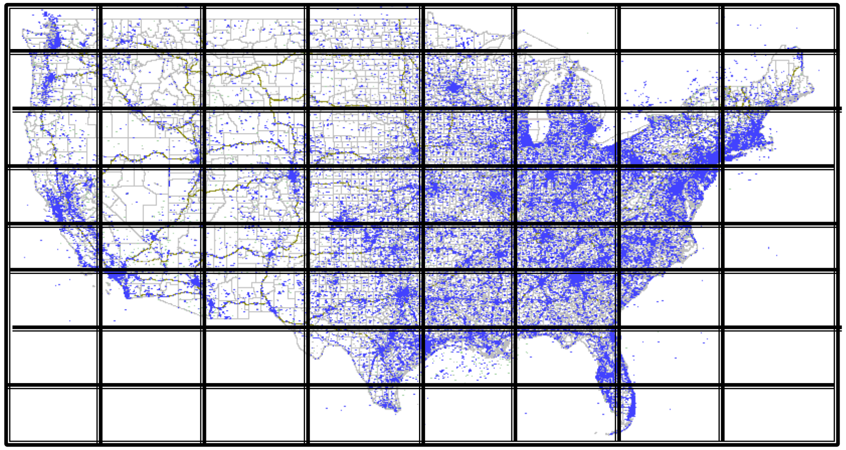
- List of points. Simple but slow
  - [p1, p2, …]
  - Find smallest of dist(q, p1), dist(q, p2), …



Representation	Average nearest-nbr efficiency
List of points	N

How to represent 2D data?

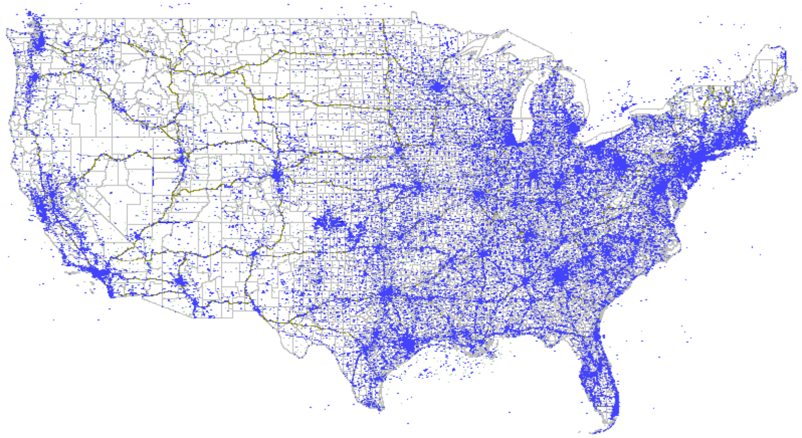
- List of points. Simple but slow
- Use a regular grid.
  - 2D array of lists
  - Faster, but which resolution?
  - Example, M=8



Representation	Average nearest-nbr efficiency
List of points	N
Regular grid	$1 + N/M^2$ but space = $N + M^2$, clustering degrades

How to represent 2D data?

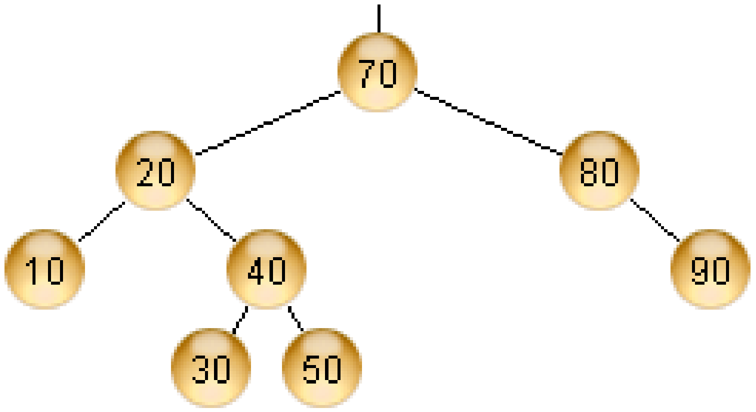
- List of points. Simple but slow
- Use a regular grid.
- ???



Representation	Average nearest-nbr efficiency
List of points	N
Regular grid	$1 + N/M^2$ but space $= N + M^2$, clustering degrades
???	log N

Binary search trees partition the number line

- Split at 70
- Split at 20
- etc



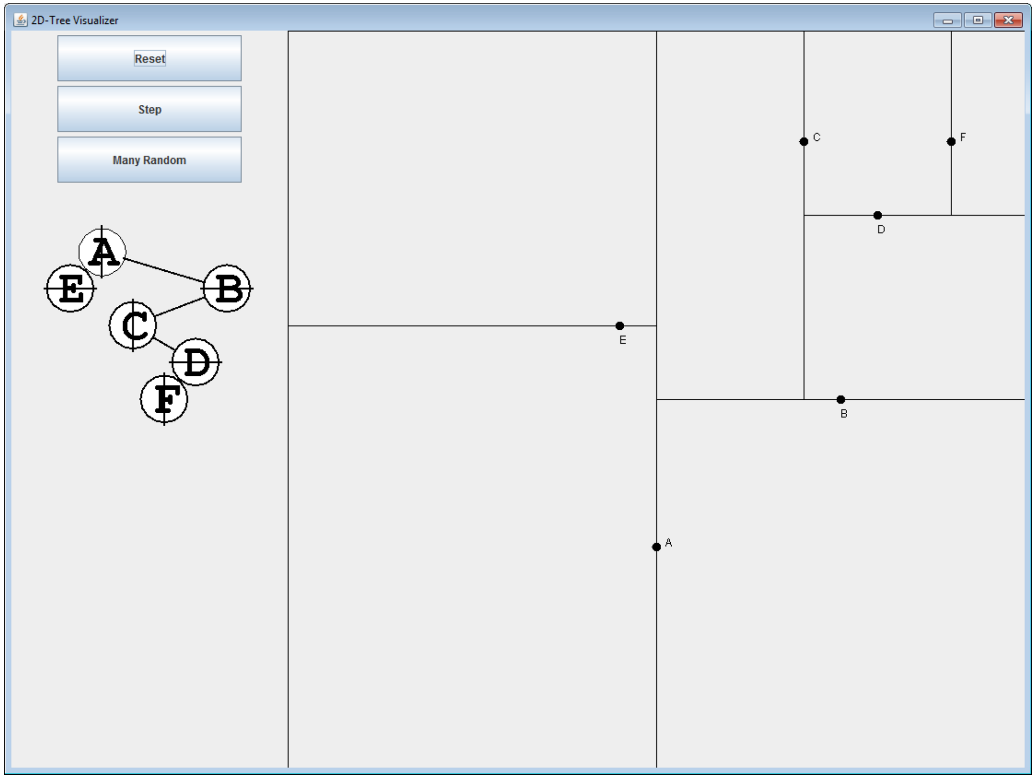
- Any value inserted to the left of 30 must be in what range?

You can partition the coordinate plane with a variation of BSTs

- Each level splits the plane in one direction only

- Use the insert algorithm to build a tree from points:

```
A (0.5, 0.7)
B (0.75, 0.5)
C (0.7, 0.15)
D (0.8, 0.25)
E (0.45, 0.4)
F (0.9, 0.15)
```



Nearest neighbor using a 2D Tree

- Initialize the closest point as the root.
- Recursively go to each side if it could be closer:
  - To left/top and update closest if one found
  - To right/bottom and update closest if one found
  - When hit a null node, just return

- New idea: don't always recurse to left/top first. Instead, first recurse to the **same side** as the query point, and then **only** recurse to the other side **if** it *could* yield a closer point
  - To do this, I suggest that each node also store the bounds of rectangle it is part of

Nearest neighbor using a 2D Tree

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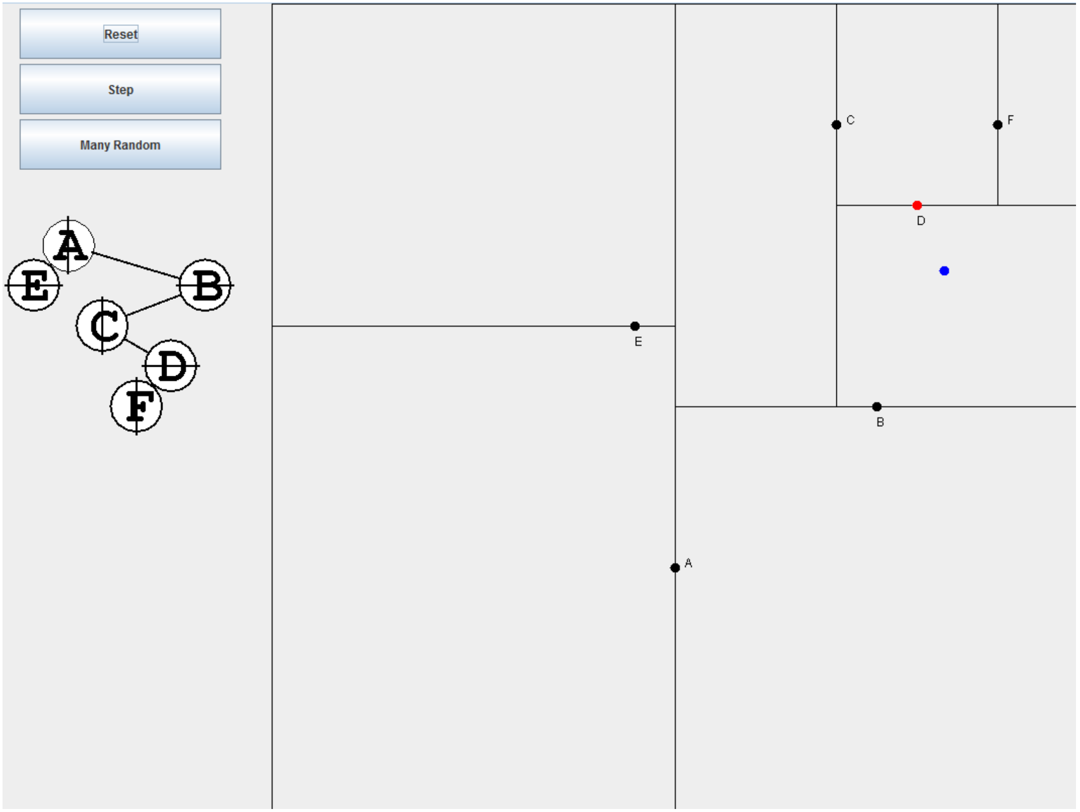
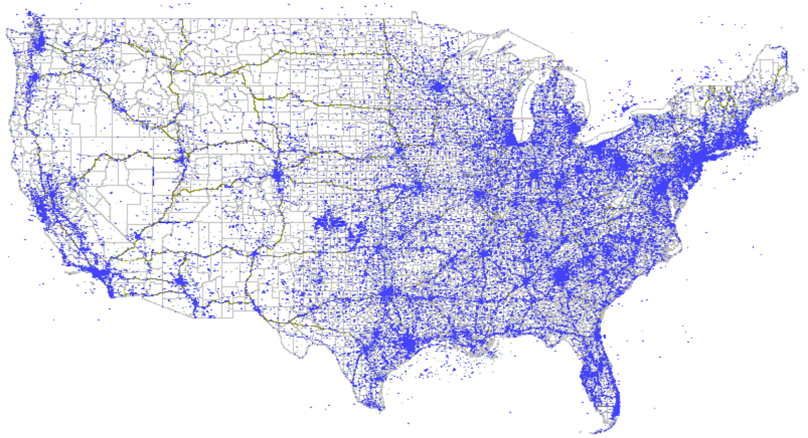


How to represent 2D data?

- List of points. Simple but slow
- Use a regular grid.
- Use a 2D tree
  - You can find the nearest neighbor efficiently



Representation	Average nearest-nbr efficiency
List of points	N
Regular grid	$1 + N/M^2$ but space $= N/M^2 + 1$, clustering degrades
2D tree	log N

2D Trees are useful

- Questions for thought:
  - How would you build a 3D tree?
  - … a k-d tree?
- Summarize now
- Assignment for this week:
  - Implement insert(Point), contains(Point), and nearest(Point) using a 2D tree