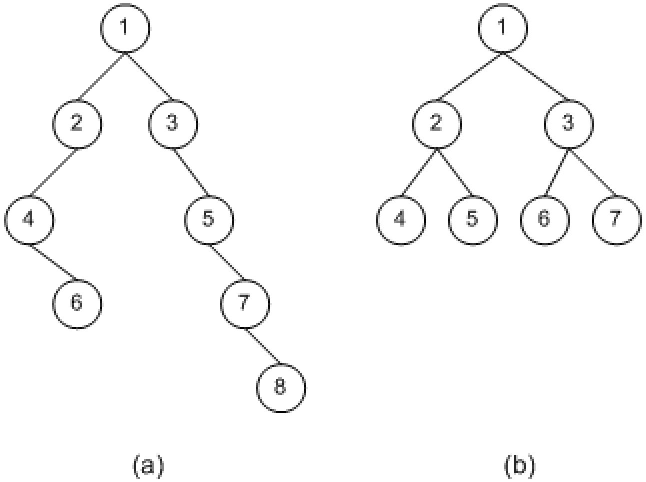

CSSE 230 Day 10

Size vs height in a Binary Tree

After today, you should be able to…
… use the relationship between the size and height of a tree to find the maximum and minimum number of nodes a binary tree can have
…understand the idea of mathematical induction as a proof technique

http://i.msdn.microsoft.com/dynimg/IC71494.gif

Today

- New material:
  - Size vs height of trees: patterns and proofs
- Review for test next class
  - Written (50–70%):
    - big O/θ/Ω: true/false, using definitions, code analysis
    - Choosing an ADT to solve a given problem
    - Maybe a bit with binary trees
  - Programming (30–50%):
    - Implementing one ADT using another ADT
- Due after that:
  - Hardy's Taxi, part two: efficiency boost!
  - Meet partner now

Q1-5

Size and Height of Binary Trees

- Notation:
  - Let T be a tree
  - Write h(T) for the height of the tree, and
  - N(T) for the size (i.e., number of nodes) of the tree
- Given h(T), what are the bounds on N(T)?
  - N(T) <= _______ and N(T) >= _______
- Given N(T), what are the bounds on h(T)?
  - Solve each inequality for h(T) and combine

Extreme Trees

- A tree with the maximum number of nodes for its height is a **full** tree.
  - Its height is O(log N)
- A tree with the minimum number of nodes for its height is essentially a ______ ______
  - Its height is O(N)
- Height matters!
  - Recall that the algorithms for search, insertion, and deletion in a binary search tree are O(h(T))

To prove recursive properties (on trees), we use a technique called mathematical induction

- Actually, we use a variant called *strong induction* :

The former governor of California

Strong Induction

- To prove that p(n) is true for all $n >= n_0$:
  - Prove that $p(n_0)$ is true (base case), and
  - For all $k > n_0$, prove that if we assume p(j) is true for $n_0 \leq j < k$, then p(k) is also true
- An analogy for those who took MA275:
  - Regular induction uses the previous domino to knock down the next
  - Strong induction uses all the previous dominos to knock down the next!
- Warmup: prove the arithmetic series formula
- Actual: prove the formula for N(T)

Exam Review

The Big Picture

- All data structures really boil down to:
  - Continuous memory (**arrays**), or
  - Nodes and pointers (**linked lists, trees, graphs**)
- Let's draw pics of each
- Then you do the questions on the back with a partner as exam review

- Then time for questions