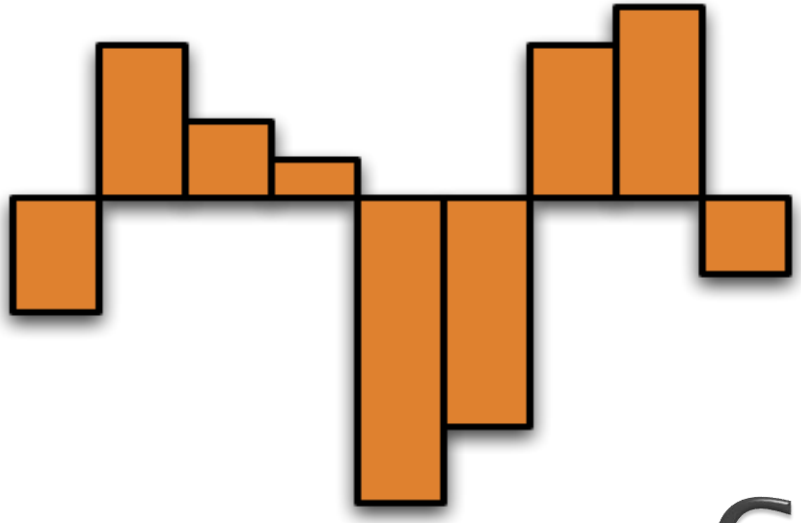


CSSE 230 Day 3

Maximum Contiguous Subsequence Sum

After today's class you will be able to:

- state and solve the MCSS problem on small arrays by observation
- find the exact runtimes of the naive MCSS algorithms

https://openclipart.org/image/2400px/svg_to_png/169467/bow_tie.png

Andrew Hettlinger ▸ Matt Boutell

November 6 at 12:30pm ·

In your class, I never thought I'd actually use big O notation, but now I find myself using it in my complaints to coworkers about how a previous developer would sort a list before doing a binary search to find a single element O(nlogn) + O(logn) instead of just doing a linear search O(n). I feel really nerdy now (as if I didn't before ☺)

Like · Comment

So why would we ever sort first to do binary search?

Limits and Asymptotics

- Consider the limit

$$\lim_{n \to \infty} \frac{f(n)}{g(n)}$$

- What does it say about asymptotic relationship between f and g if this limit is…
  - 0?
  - finite and non-zero?
  - infinite?

Apply this limit property to the following pairs of functions

1. n and n^2
2. log n and n (on these questions and solutions ONLY, let log n mean natural log)
3. n log n and n^2
4. $\log_a n$ and $\log_b n$ $(a < b)$
5. n^a and a^n $(a > 1)$
6. a^n and b^n $(a < b)$

Recall l'Hôpital's rule:
under appropriate conditions,

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

Answers

1. n is $O(n^2)$; n^2 is $\Omega(n)$
2. $\log n$ is $O(n)$; n is $\Omega(\log n)$
3. $n \log n$ is $O(n^2)$; n^2 is $\Omega(n \log n)$. (Use l'Hopital's rule)

4. $\log_a n$ is $\Theta(\log_b n)$ (so $\log_b n$ is $\Theta(\log_a n)$)
   Hint: Rewrite $\log_a n$ as $\log n / \log a$ and $\log_b n$ as $\log n / \log b$. Simplifying, we see that the original limit is a constant: no differentiating is needed here either.

5. n^a is $O(a^n)$ and is $o(a^n)$; a^n is $\Omega(n^a)$ and $\omega(n^a)$
   Hint: use l'Hopital's rule repeatedly until numerator goes to 0.

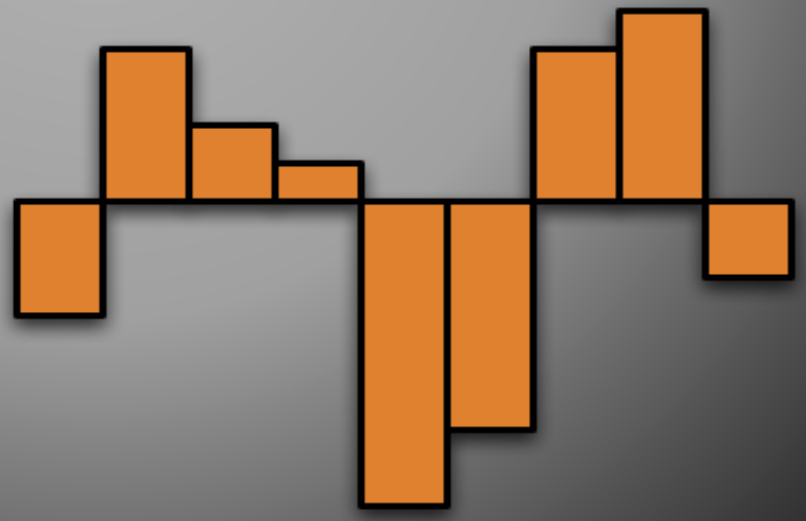
6. a^n is $O(b^n)$ and is $o(b^n)$; b^n is $\Omega(a^n)$ and $\omega(a^n)$
   Hint: rewrite as $(a/b)^n$. Because $a < b$, $a/b < 1$, and when $x < 1$, x^n approaches 0 as n goes to infinity.

Q1

Maximum Contiguous Subsequence Sum

A deceptively deep problem with a surprising solution.

{−3, 4, 2, 1, −8, −6, 4, 5, −2}

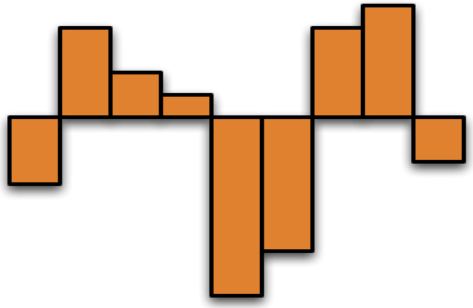


Why do we look at this problem?

- It's interesting
- Analyzing the obvious solution is instructive
- We can make the program more efficient

A Nice Algorithm Analysis Example

- **Problem**: Given a sequence of numbers, find the maximum sum of a contiguous subsequence.



- **Consider**:
  - What if all the numbers were positive?
  - What if they all were negative?
  - What if we left out “contiguous”?

Q2-4

Formal Definition: Maximum Contiguous Subsequence Sum

Problem definition: Given a non-empty sequence of n (possibly negative) integers $A_1, A_2, \ldots, A_n$, find the maximum consecutive subsequence $S_{i,j} = \sum_{k=i}^{j} A_k$, and the corresponding values of i and j.

- Quiz questions:
  - In {−2, **11, −4, 13**, −5, 2}, $S_{2,4} = ?$
  - In {1, −3, 4, −2, −1, 6}, what is MCSS?
  - If every element is negative, what's the MCSS?

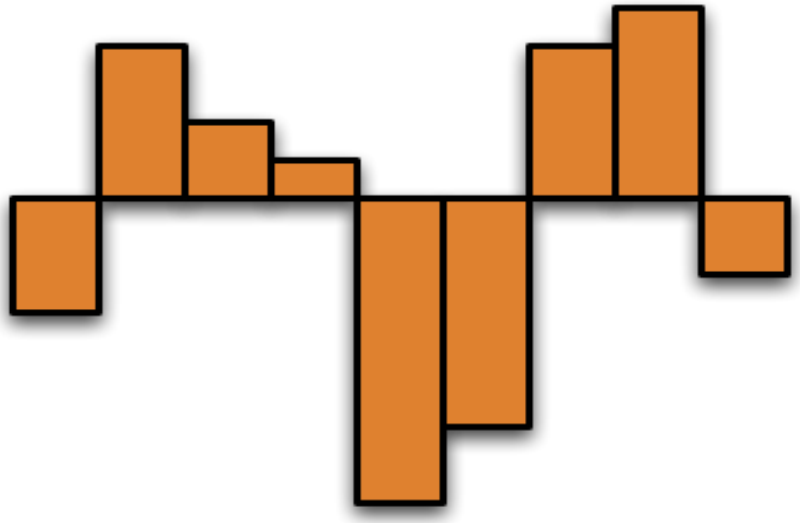
1-based indexing. We'll use when analyzing b/c easier

Write a simple correct algorithm now

Q5

- Must be easy to explain
- Correctness is KING. Efficiency doesn't matter yet.
- 3 minutes

- Examples to consider:
  - {-3, 4, 2, 1, -8, -6, 4, 5, -2}
  - {5, 6, -3, 2, 8, 4, -12, 7, 2}



First Algorithm

Find the sums of *all* subsequences

```
public final class MaxSubTest {
    private static int seqStart = 0;
    private static int seqEnd = 0;

    /* First maximum contiguous subsequence sum algorithm.
     * seqStart and seqEnd represent the actual best sequence.
     */
    public static int maxSubSum1( int [ ] a ) {
        int maxSum = 0;
            //In the analysis we use "n" as a shorthand for "a.length
        for( int i = 0; i < a.length; i++ )   "
            for( int j = i; j < a.length; j++ ) {
                int thisSum = 0;

                for( int k = i; k <= j; k++ )
                    thisSum += a[ k ];

                if( thisSum > maxSum ) {
                    maxSum   = thisSum;
                    seqStart = i;
                    seqEnd   = j;
                }
            }
        return maxSum;
    }
```

i: beginning of subsequence

j: end of subsequence

k: steps through each element of subsequence

Where will this algorithm spend the most time?

How many times (exactly, as a function of N = a.length) will that statement execute?

Q6, Q7

Analysis of this Algorithm

- What statement is executed the most often?
- How many times?

```
//In the analysis we use "n" as a shorthand for "a.length "
for( int i = 0; i < a.length; i++ )
    for( int j = i; j < a.length; j++ ) {
        int thisSum = 0;

        for( int k = i; k <= j; k++ )
            thisSum += a[ k ];
```

Interlude

- Computer Science is no more about computers than astronomy is about _____.

Donald Knuth

Interlude

- Computer Science is no more about computers than astronomy is about telescopes.

Donald Knuth

Where do we stand?

- We showed MCSS is $O(n^3)$.
  - Showing that a **problem** is O(g(n)) is relatively easy – just analyze a known algorithm.
- Is MCSS $\Omega(n^3)$?
  - Showing that a **problem** is Ω (g(n)) is much tougher. How do you prove that it is impossible to solve a problem more quickly than you already can?
  - Or maybe we can find a faster algorithm?

f(n) is O(g(n)) if $f(n) \leq cg(n)$ for all $n \geq n_0$
- So O gives an upper bound

f(n) is Ω(g(n)) if $f(n) \geq cg(n)$ for all $n \geq n_0$
- So Ω gives a lower bound

f(n) is θ(g(n)) if $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$
- So θ gives a tight bound
- f(n) is θ(g(n)) if it is both O(g(n)) **and** Ω(g(n))

What is the main source of the simple algorithm's inefficiency?

```
    //In the analysis we use "n" as a shorthand for "a.length "
for( int i = 0; i < a.length; i++ )
    for( int j = i; j < a.length; j++ ) {
        int thisSum = 0;

        for( int k = i; k <= j; k++ )
            thisSum += a[ k ];
```

- The performance is bad!

Eliminate the most obvious inefficiency…

```
for( int i = 0; i < a.length; i++ ) {
     int thisSum = 0;
     for( int j = i; j < a.length; j++ ) {
          thisSum += a[ j ];

          if( thisSum > maxSum ) {
               maxSum = thisSum;
               seqStart = i;
               seqEnd   = j;
          }
     }
}
```

This is Θ(?)

MCSS is $O(n^2)$

- Is MCSS $\Omega(n^2)$?
  - Showing that a problem is $\Omega(g(n))$ is much tougher. How do you prove that it is impossible to solve a problem more quickly than you already can?
  - Can we find a yet faster algorithm?

$f(n)$ is $O(g(n))$ if $f(n) \leq cg(n)$ for all $n \geq n_0$
- So O gives an upper bound

$f(n)$ is $\Omega(g(n))$ if $f(n) \geq cg(n)$ for all $n \geq n_0$
- So Ω gives a lower bound

$f(n)$ is $\theta(g(n))$ if $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$
- So θ gives a tight bound
- $f(n)$ is $\theta(g(n))$ if it is both $O(g(n))$ **and** $\Omega(g(n))$

Q8, Q9

Can we do even better?

Tune in next time for the exciting conclusion!

Think about the 7,2 on the other side of the -12:

{5, 6, -3, 2, 8, 4, -12, 7, 2}