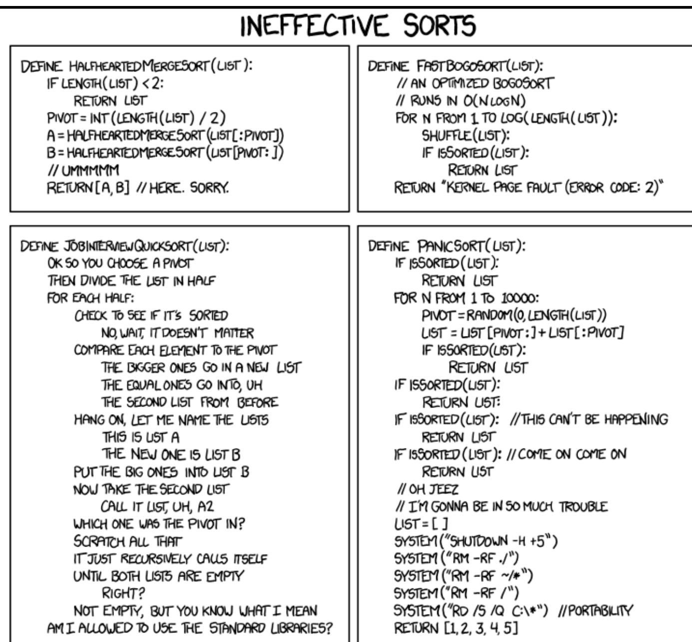
CSSE 230
Day 23

Quicksort algorithm
Average case analysis



http://www.xkcd.com/1185/

Stacksort connects to StackOverflow, searches for "sort a list", and downloads and runs code snippets until the list is sorted.

Review: The Master Theorem works for divide-and-conquer recurrence relations only … but works well!

Q1-3

- For any recurrence relation of the form:

$$T(N) = aT(\frac{N}{b}) + f(N)$$

with $a \geq 1, b > 1,$ and $f(N) = O(N^k)$

- The solution is:

$$T(N) = \begin{cases} O(N^{\log_b a}) & \text{if } a > b^k \\ O(N^k \log N) & \text{if } a = b^k \\ O(N^k) & \text{if } a < b^k \end{cases}$$

- Note: Replace O with θ everywhere in the theorem!

Theorem 7.5 in Weiss

Sorting Demos

- Check out now:
  - www.sorting-algorithms.com
- http://maven.smith.edu/~thiebaut/java/sort/demo.html
- http://www.cs.ubc.ca/~harrison/Java/sorting-demo.html

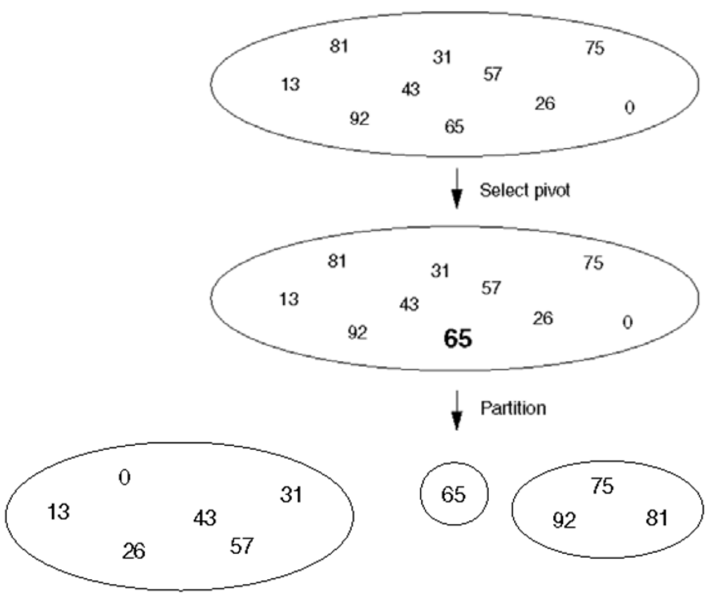
QuickSort (a.k.a. "partition-exchange sort")

- Invented by C.A.R. "Tony" Hoare in 1961*
- Very widely used
- Somewhat complex, but fairly easy to understand
  - Like in basketball, it's all about planting a good pivot.

A quote from Tony Hoare:
> There are two ways of constructing a software design: One way is to make it so simple that there are obviously no deficiencies, and the other way is to make it so complicated that there are no obvious deficiencies. The first method is far more difficult.

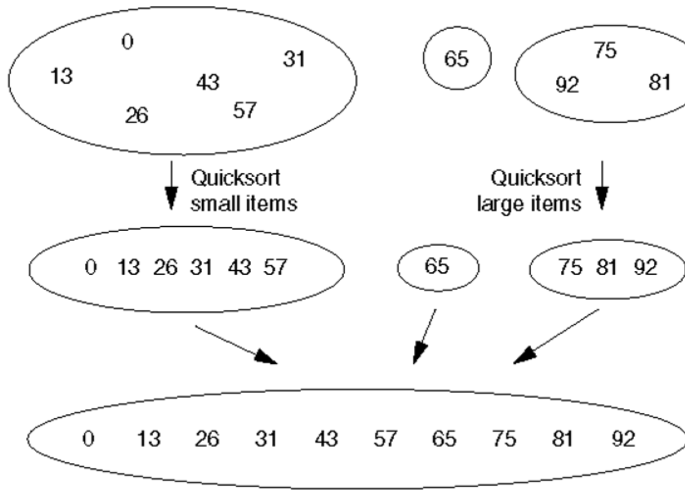
Image from http://www.ultimate-youth-basketball-guide.com/pivot-foot.html.

Partition: split the array into 2 parts: smaller than pivot and greater than pivot



Quicksort then recursively calls itself on the partitions

Q4



Partition: efficiently move small elements to the left of the pivot and greater ones to the right

Q5

```
// Assume min and max indices are low and high
pivot = a[low]
i = low+1, j = high
while (true) {
  while (a[i] < pivot) i++
  while (a[j] > pivot) j--
  if (i >= j) break
  swap(a, i, j)
}
swap(a, low, j) // moves the pivot to the
             // correct place
return j
```

Q6-7

QuickSort Average Case

- Running time for partition **of N elements** is $\Theta(N)$
- Quicksort Running time:
  - call partition. Get two subarrays of sizes N_L and N_R (what is the relationship between N_L, N_R, and N?)
  - Then Quicksort the smaller parts
  - $T(N) = N + T(N_L) + T(N_R)$
- Quicksort Best case: write and solve the recurrence
- Quicksort Worst case: write and solve the recurrence
- average: a little bit trickier
  - We have to be careful how we measure

Average time for Quicksort

- Let T(N) be the average # of comparisons of array elements needed to quicksort N elements.
- What is T(0)? T(1)?
- Otherwise T(N) is the sum of
  - time for partition
  - average time to quicksort left part: $T(N_L)$
  - average time to quicksort right part: $T(N_R)$
- $T(N) = N + T(N_L) + T(N_R)$

We need to figure out for each case, and average all of the cases

- Weiss shows how ***not*** to count it:
- What if half of the time we picked the smallest element as the partitioning element and the other half of the time we picked the largest?
- Then on the average, $N_L = N/2$ and $N_R = N/2$,
  - but that doesn't give a true picture of these worst-case scenarios.
  - In every case, either $N_L = N-1$ or $N_R = N-1$

We assume that all positions for the pivot are equally likely

Q8

- We always need to make some kind of "distribution" assumptions when we figure out Average case
- When we execute

  `k = partition(pivot, i, j)`,

  all positions i..j are equally likely places for the pivot to end up
- Thus N_L is equally likely to have each of the values 0, 1, 2, … N–1
- $N_L+N_R = N-1$; thus N_R is also equally likely to have each of the values 0, 1, 2, … N–1
- Thus $T(N_L) = T(N_R) =$

Q9-10

Continue the calculation

- T(N) =
- Multiply both sides by N
- Rewrite, substituting N-1 for N
- Subtract the equations and forget the insignificant (in terms of big-oh) -1:
  - $NT(N) = (N+1)T(N-1) + 2N$
- Can we rearrange so that we can telescope?

Q11-13

Continue continuing the calculation

- $NT(N) = (N+1)T(N-1) + 2N$
- Divide both sides by N(N+1)
- Write formulas for T(N), T(N-1),T(N-2) …T(2).
- Add the terms and rearrange.
- Notice the familiar series
- Multiply both sides by N+1.

Recap

- Best, worst, average time for Quicksort
- What causes the worst case?

Improvements to QuickSort

- Avoid the worst case
  - Select pivot from the middle
  - Randomly select pivot
  - Median of 3 pivot selection.
  - Median of k pivot selection
- "Switch over" to a simpler sorting method (insertion) when the subarray size gets small

  Weiss's code does Median of 3 and switchover to insertion sort at 10.
  - Linked from schedule page

  What does the official Java Quicksort do? See the source!