# CSSE 230 Day 22

Recurrence Relations

Sorting overview

# More on Recurrence Relations

A technique for analyzing recursive algorithms

# Recap: Recurrence Relation

- An equation (or inequality) that relates the $n^{th}$ element of a sequence to certain of its predecessors (recursive case)
- Includes an initial condition (base case)
- **Solution:** A function of n.

- Similar to differential equations, but discrete instead of continuous
- Some solution techniques are similar to diff. eq. solution techniques

# Solve Simple Recurrence Relations

- One strategy: **guess and check**
- Examples:
  - $T(0) = 0$, $T(N) = 2 + T(N-1)$
  - $T(0) = 1$, $T(N) = 2\ T(N-1)$
  - $T(0) = T(1) = 1$, $T(N) = T(N-2) + T(N-1)$
  - $T(0) = 1$, $T(N) = N\ T(N-1)$
  - $T(0) = 0$, $T(N) = T(N-1) + N$
  - $T(1) = 1$, $T(N) = 2\ T(N/2) + N$
    (just consider the cases where $N=2^k$)

# Another Strategy

- **Substitution**
- $T(1) = 1$, $T(N) = 2\ T(N/2) + N$
  (just consider $N=2^k$)
- Suppose we substitute N/2 for N in the recursive equation?
  - We can plug the result into the original equation!

# Solution Strategies for Recurrence Relations

- Guess and check
- Substitution
- Telescoping and iteration
- The "master" method

# Selection Sort

```
public static void selectionSort(int[] a) {
    //Sorts a non-empty array of integers.

    for (int last = a.length-1; last > 0; last--) {
        // find largest, and exchange with last
        int largest = a[0];
        int largePosition = 0;

        for (int j=1; j<=last; j++)
            if (largest < a[j]) {
                largest = a[j];
                largePosition = j;
        }
        a[largePosition] = a[last];
        a[last] = largest;
    }
}
```

What’s N?

# Another Strategy: Telescoping

- Basic idea: tweak the relation somehow so successive terms cancel

- Example: T(1) = 1, T(N) = 2T(N/2) + N
  where N = $2^k$ for some k
- Divide by N to get a "piece of the telescope":

$$\begin{aligned} T(N) &= 2T(\frac{N}{2}) + N \\ \Longrightarrow \frac{T(N)}{N} &= \frac{2T(\frac{N}{2})}{N} + 1 \\ \Longrightarrow \frac{T(N)}{N} &= \frac{T(\frac{N}{2})}{\frac{N}{2}} + 1 \end{aligned}$$

# A Fourth Strategy: Master Theorem

- For Divide-and-conquer algorithms
  - Divide data into two or more parts
  - Solve problem on one or more of those parts
  - Combine "parts" solutions to solve whole problem
- Examples
  - Binary search
  - Merge Sort
  - MCSS recursive algorithm we studied last time

**Theorem 7.5 in Weiss**

Start 8

# Divide and Conquer Recurrence

$$T(N) = aT(\frac{N}{b}) + f(N)$$

$$a \geq 1, b > 1, \text{ and } f(N) = O(N^k)$$

- b = number of parts we divide into
- a = number of parts we solve
- f(N) = overhead of dividing and combining
- Binary Search: b =__ , a =___ , k = ___.
- Merge sort: b =__ , a =___ , k = ___.

# The Master Theorem is convenient, but only works for divide and conquer recurrences

9, finish 8

- For any recurrence relation *in the form*:

$$T(N) = aT(\frac{N}{b}) + f(N)$$

with $a \geq 1, b > 1, \text{ and } f(N) = O(N^k)$

- The solution is:

$$T(N) = \begin{cases} O(N^{\log_b a}) & \text{if } a > b^k \\ O(N^k \log N) & \text{if } a = b^k \\ O(N^k) & \text{if } a < b^k \end{cases}$$

Theorem 7.5 in Weiss

# Summary: Recurrence Relations

- Analyze code to determine relation
  - Base case in code gives base case for relation
  - Number and “size” of recursive calls determine recursive part of recursive case
  - Non-recursive code determines rest of recursive case
- Apply one of four strategies
  - Guess and check
  - Substitution (a.k.a. iteration)
  - Telescoping
  - Master theorem

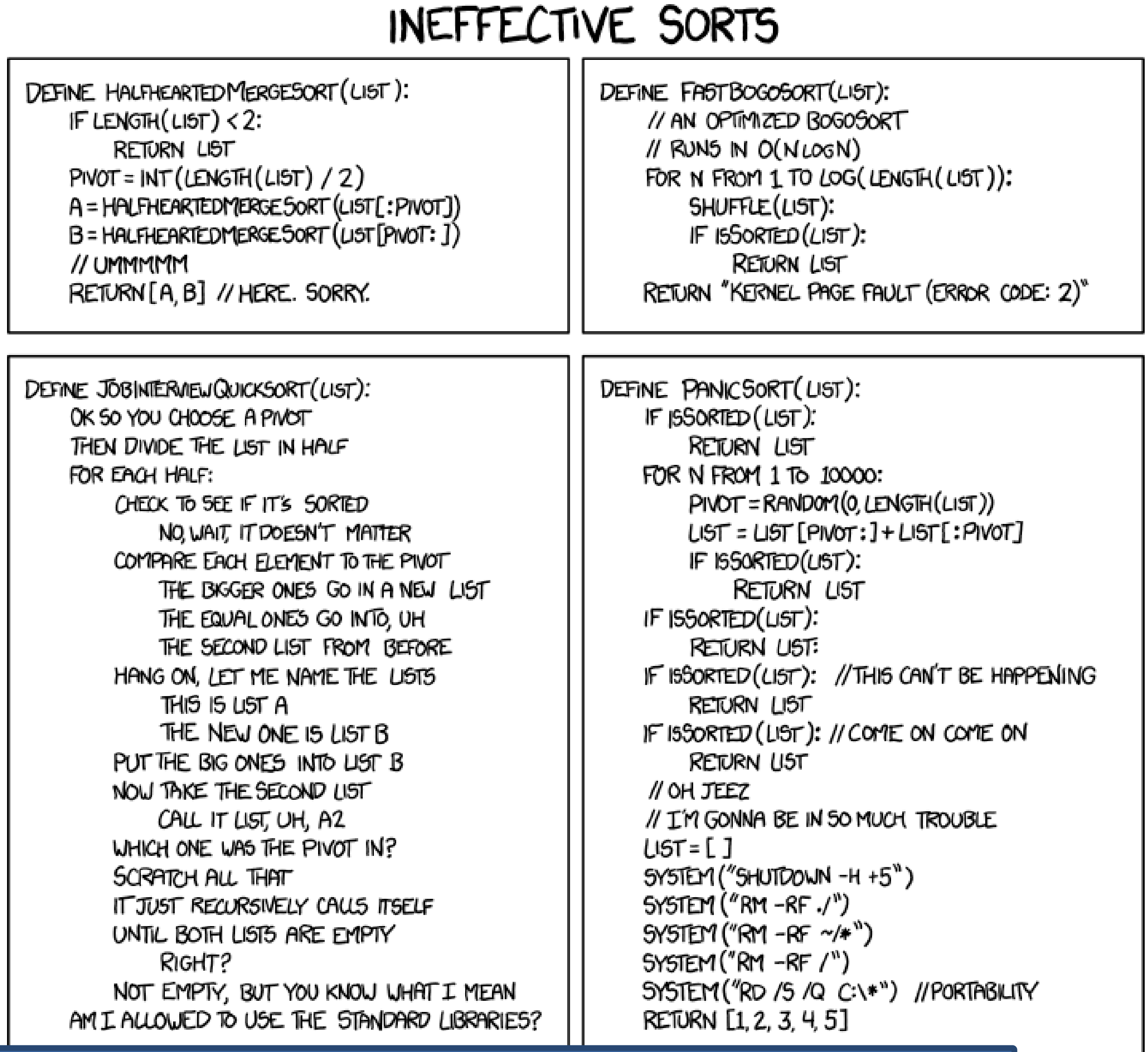
# Sorting overview

- Quick look at several sorting methods
- Focus on quicksort
- Quicksort average case analysis

# Elementary Sorting Methods

- Name as many as you can
- How does each work?
- Running time for each (sorting N items)?
  - best
  - worst
  - average
  - extra space requirements
- Spend 10 minutes with a group of three, answering these questions. Then we will summarize

Put list on board




http://www.xkcd.com/1185/

Stacksort connects to StackOverflow, searches for “sort a list”, and downloads and runs code snippets until the list is sorted.