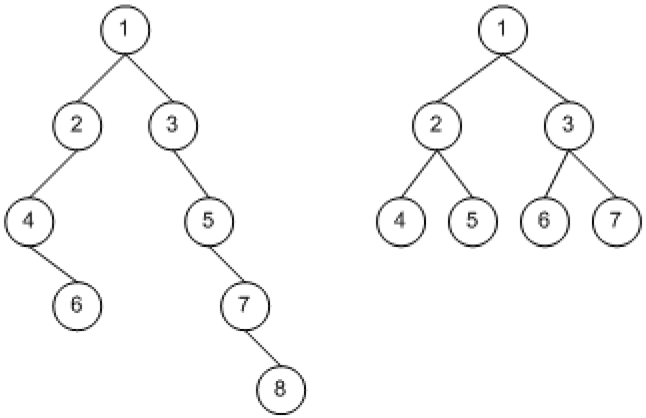

(a) (b)

CSSE 230 Day 10

Size vs height in a Binary Tree

http://i.msdn.microsoft.com/dynimg/IC71494.gif

Announcements

- Today:
  - Size vs height of trees: patterns and proofs
  - Q/A and worktime for BSTs

- BST deadline extended to Friday night
  - No late days
- HW4 deadline extended to Monday night
  - Cut from 8 to 3 problems
  - No late days

- Exam: next Wednesday, 15 Jan, 7–9 pm:
  - See me today if an impossible time conflict
  - Written (~50%):
    - big $O/\theta/\Omega$: true/false, using definition, code analysis
    - Choosing an ADT to solve a given problem
    - Implementing one ADT using another ADT
  - Programming (~50%):
    - Binary Trees / Binary Search Trees

Questions?

Size and Height of Binary Trees

- Notation:
  - Let T be a tree
  - Write **h(T)** for the height of the tree, and
  - **N(T)** for the size (i.e., number of nodes) of the tree

- Given h(T), what are the bounds on N(T)?
  - N(T) < _______ and N(T) > _______

- Given N(T), what are the bounds on h(T)?
  - Solve each inequality for h(T) and combine

Extreme Trees

- A tree with the maximum number of nodes for its height is a **full** tree.
  - Its height is O(log N)
- A tree with the minimum number of nodes for its height is essentially a ______ ______
  - Its height is O(N)
- Height matters!
  - Recall that the algorithms for search, insertion, and deletion in a binary search tree are O(h(T))

To prove recursive properties (on trees), we use a technique called mathematical induction

- Actually, we use a variant called *strong induction* :

The former governor of California

Strong Induction

- To prove that p(n) is true for all $n >= n_0$:
  - Prove that $p(n_0)$ is true (base case), and
  - For all $k > n_0$, prove that if we assume p(j) is true for $n_0 \leq j < k$, then p(k) is also true
- An analogy for those who took MA275:
  - Regular induction uses the previous domino to knock down the next
  - Strong induction uses all the previous dominos to knock down the next!
- Warmup: prove the arithmetic series formula
- Actual: prove the formula for N(T)

Current assignment

Questions and answers

Worktime