# CSSE 230 Day 24

Sorting Overview

## Agenda

- Student questions
- Exam preview
- Master Theorem examples
- Sorting grand tour
- Quicksort
- Scrabble work time

# Exam 2 Tuesday at 7 PM (O267-269)

- Format same as Exam 1
  - One 8.5x11 sheet of paper (2-sided) for written part
  - Same resources as before for programming part
- Topics: weeks 1-8 (no Scrabble).
  - Reading, programs, in-class, written assignments.
  - Especially
    - OO programming, using various data structures (lists, stacks, queues, sets, maps, priority queues)
    - Binary trees, including AVL, rank, and threaded trees
      - Traversals and iterators, numeric properties
    - Graphs
    - PQs, Heaps and heapsort, other sorting methods.
    - Issues in Hash table implementation
    - Exhaustive search and the Queens problem
    - File compression and Huffman trees
    - Mathematical induction
    - Simple recurrence relations

## The Master Theorem works for divide-and-conquer recurrence relations … and works well!

Q1-3

- For any recurrence relation of the form:

$$T(N) = aT(\frac{N}{b}) + f(N)$$

with $a \geq 1, b > 1, \text{ and } f(N) = O(N^k)$

- The solution is:

$$T(N) = \begin{cases} O(N^{\log_b a}) & \text{if } a > b^k \\ O(N^k \log N) & \text{if } a = b^k \\ O(N^k) & \text{if } a < b^k \end{cases}$$

Theorem 7.5 in Weiss

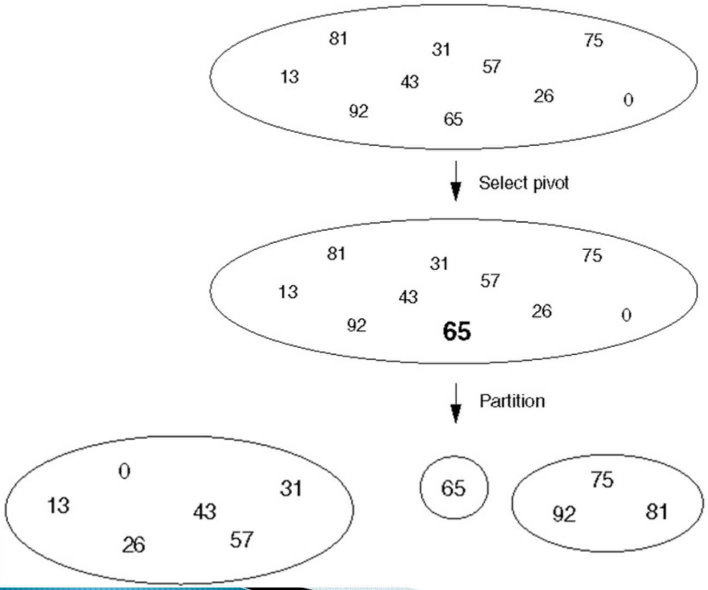
# Elementary Sorting Methods

- Name as many as you can
- How does each work?
- Running time for each (sorting N items)?
  - best
  - worst
  - average
  - extra space requirements
- Spend 10 minutes with a group of three, answering these questions. Then we will summarize

# QuickSort (a.k.a. "partition-exchange sort")

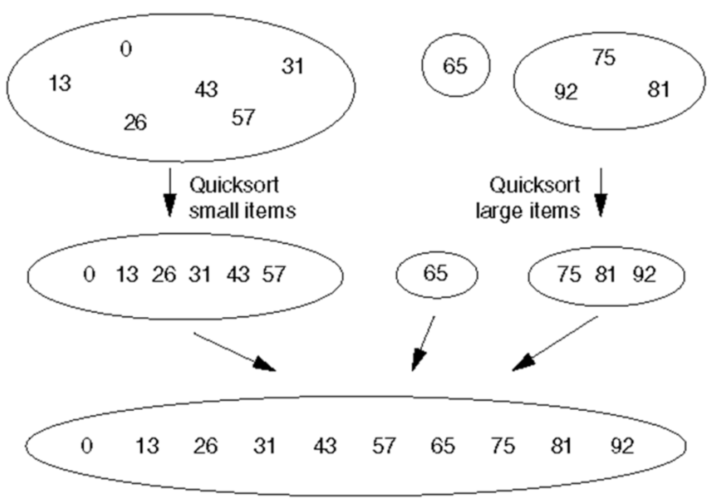
- Invented by C.A.R. Hoare in 1961
- Very widely used
- Somewhat complex, but fairly easy to understand

## Partition: split the array into 2 parts: smaller than pivot and greater than pivot




## Quicksort then recursively calls itself on the partitions

Q4



Partition: efficiently move small elements to the left of the pivot and greater ones to the right

Q5

```
// Assume min and max indices are low and high
pivot = a[low]
i = low+1, j = high
while (true) {
  while (a[i] < pivot) i++
  while (a[j] > pivot) j--
  if (i >= j) break
  swap(a, i, j)
}
swap(a, low, j) // moves the pivot to the
             // correct place
```

Q6-7

# QuickSort Average Case

- Running time for partition **of N elements** is $\Theta(N)$
- Quicksort Running time:
  - call partition. Get two subarrays of sizes $N_L$ and $N_R$ (what is the relationship between $N_L$, $N_R$, and N?)
  - Then Quicksort the smaller parts
  - $T(N) = N + T(N_L) + T(N_R)$
- Quicksort Best case: write and solve the recurrence
- Quicksort Worst case: write and solve the recurrence
- average: a little bit trickier
  - We have to be careful how we measure

# Average time for Quicksort

- Let T(N) be the average # of comparisons of array elements needed to quicksort N elements.
- What is T(0)? T(1)?
- Otherwise T(N) is the sum of
  - time for partition
  - average time to quicksort left part: $T(N_L)$
  - average time to quicksort right part: $T(N_R)$
- $T(N) = N + T(N_L) + T(N_R)$

## We need to figure out for each case, and average all of the cases

- Weiss shows how **not** to count it:
- What if we picked as the partitioning element the smallest element half of the time and the largest half of the time?
- Then on the average, $N_L = N/2$ and $N_R = N/2$,
  - but that doesn't give a true picture of this worst-case scenario.
  - In every case, either $N_L = N-1$ or $N_R = N-1$

## We assume that all positions for the pivot are equally likely

Q8a

- We always need to make some kind of "distribution" assumptions when we figure out Average case
- When we execute

  **`k = partition(pivot, i, j),`**

  all positions i..j are equally likely places for the pivot to end up
- Thus $N_L$ is equally likely to have each of the values 0, 1, 2, … N−1
- $N_L+N_R$ = N−1; thus $N_R$ is also equally likely to have each of the values 0, 1, 2, … N−1
- Thus $T(N_L)= T(N_R)$ =

Q9-10

## Continue the calculation

- T(N) =
- Multiply both sides by N
- Rewrite, substituting N-1 for N
- Subtract the equations and forget the insignificant (in terms of big-oh) -1:
  - NT(N) = (N+1)T(N-1) + 2N
- Can we rearrange so that we can telescope?

Q11-15

## Continue continuing the calculation

- $NT(N) = (N+1)T(N-1) + 2N$
- Divide both sides by $N(N+1)$
- Write formulas for $T(N)$, $T(N-1)$,$T(N-2)$ …$T(2)$.
- Add the terms and rearrange.
- Notice the familiar series
- Multiply both sides by $N+1$.

## Improvements to QuickSort

- Avoid the worst case
  - Select pivot from the middle
  - Randomly select pivot
  - Median of 3 pivot selection.
  - Median of k pivot selection
- "Switch over" to a simpler sorting method (insertion) when the subarray size gets small.

# Other Sorting Demos

- http://maven.smith.edu/~thiebaut/java/sort/demo.html
- http://www.cs.ubc.ca/~harrison/Java/sorting-demo.html