

CSSE 220

Day 13

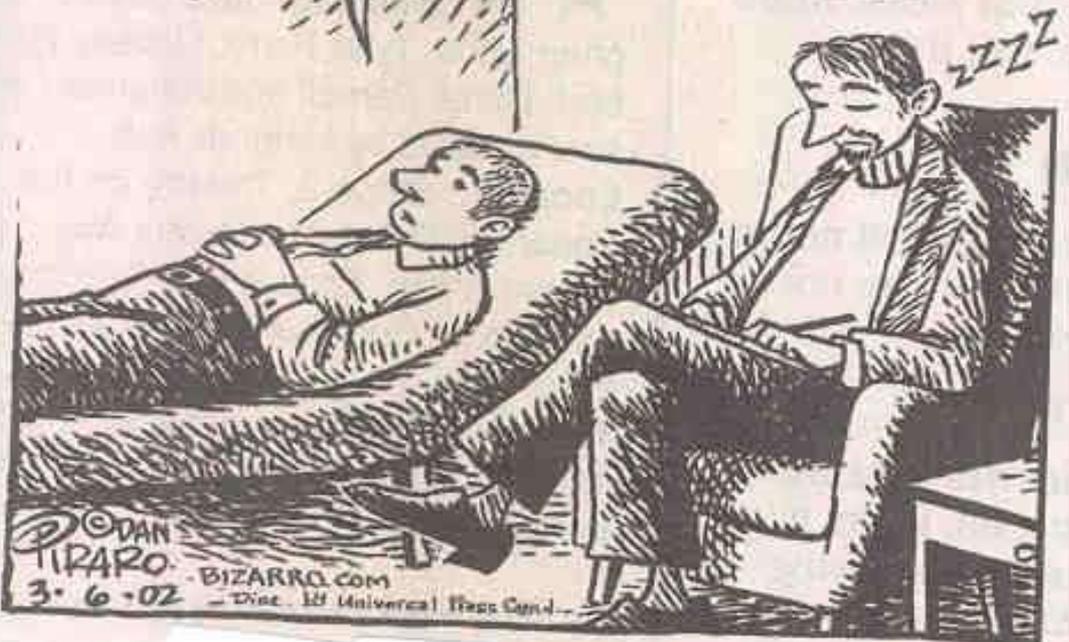
Sierpiński, Recursion and Efficiency, Mutual Recursion

Checkout *Recursion2* project from SVN

Questions?

Bizarro

I have this recurring dream that I'm lying here telling you about a recurring dream about lying here telling you about a recurring dream about...

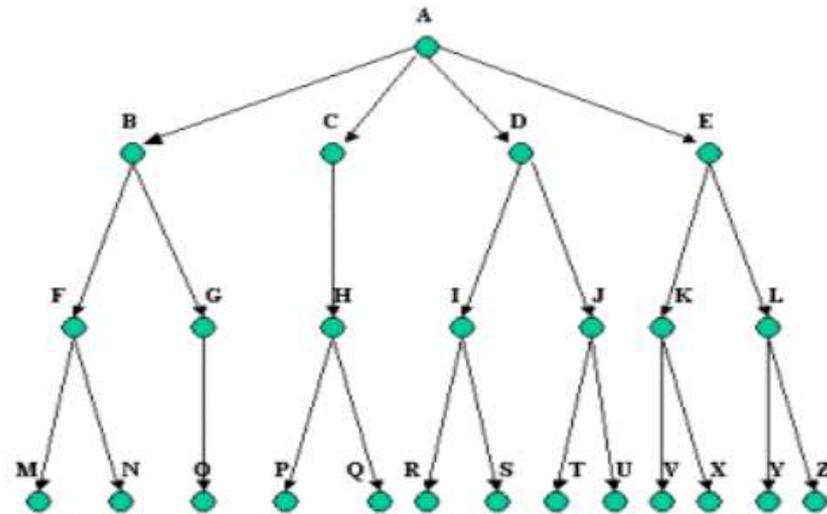


What are recursive methods?

- ▶ Any method that calls itself
 - On a simpler problem
 - So that it makes progress toward completion

When should recursive methods be used?

- ▶ When implementing a recursive definition
- ▶ When implementing methods on recursive data structures



- ▶ Where parts of the whole look like smaller versions of the whole

The pros and cons of recursive methods

▶ The pros

- easy to implement,
- easy to understand code,
- easy to prove code correct

▶ The cons

- takes more space than equivalent iteration
- Why?
 - because of function calls

Recap: Key Rules to Using Recursion

- ▶ Always have a **base case** that **doesn't recurse**
- ▶ Make sure recursive case always makes **progress**, by **solving a smaller problem**
- ▶ **You gotta believe**
 - Trust in the recursive solution
 - Just consider one step at a time

Recursive Functions

- ▶ Factorial:

$$n! = \begin{cases} 1 & \text{if } n \leq 1 \\ n * (n - 1)! & \text{otherwise} \end{cases}$$

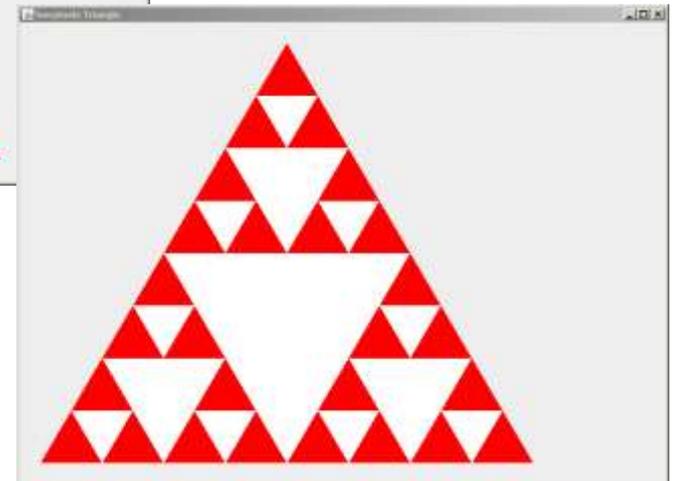
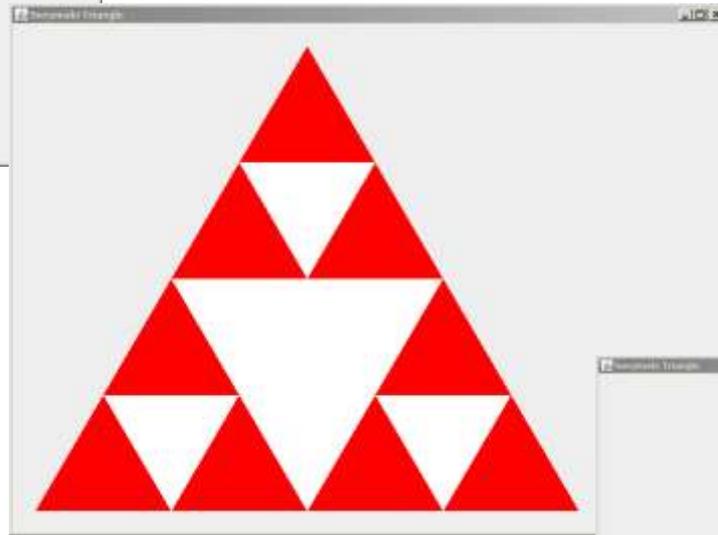
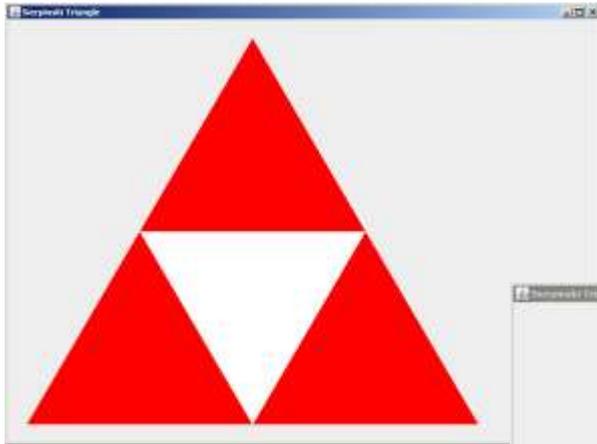
Base Case

Recursive step

- ▶ Ackermann function:

$$A(m, n) = \begin{cases} n + 1 & \text{if } m = 0 \\ A(m - 1, 1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m - 1, A(m, n - 1)) & \text{otherwise} \end{cases}$$

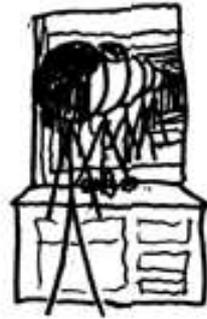
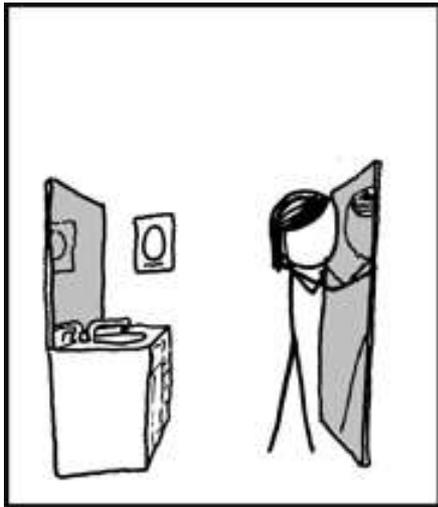
HW: Sierpinski



Work Time

»» HW 12 & 13: Sierpinski Triangle

Two Mirrors



If you actually do this, what really happens is Douglas Hofstadter appears and talks to you for eight hours about strange loops.

What the Fib?

- ▶ Why does recursive Fibonacci take so long?!?
- ▶ Can we fix it?

Memoization

- ▶ Save every solution we find to sub-problems
- ▶ Before recursively computing a solution:
 - Look it up
 - If found, use it
 - Otherwise do the recursive computation

Classic Time–Space Trade Off

- ▶ A deep discovery of computer science
- ▶ In a wide variety of problems we can tune the solution by varying the amount of storage space used and the amount of computation performed
- ▶ Studied by “Complexity Theorists”
- ▶ Used everyday by software engineers

Mutual Recursion

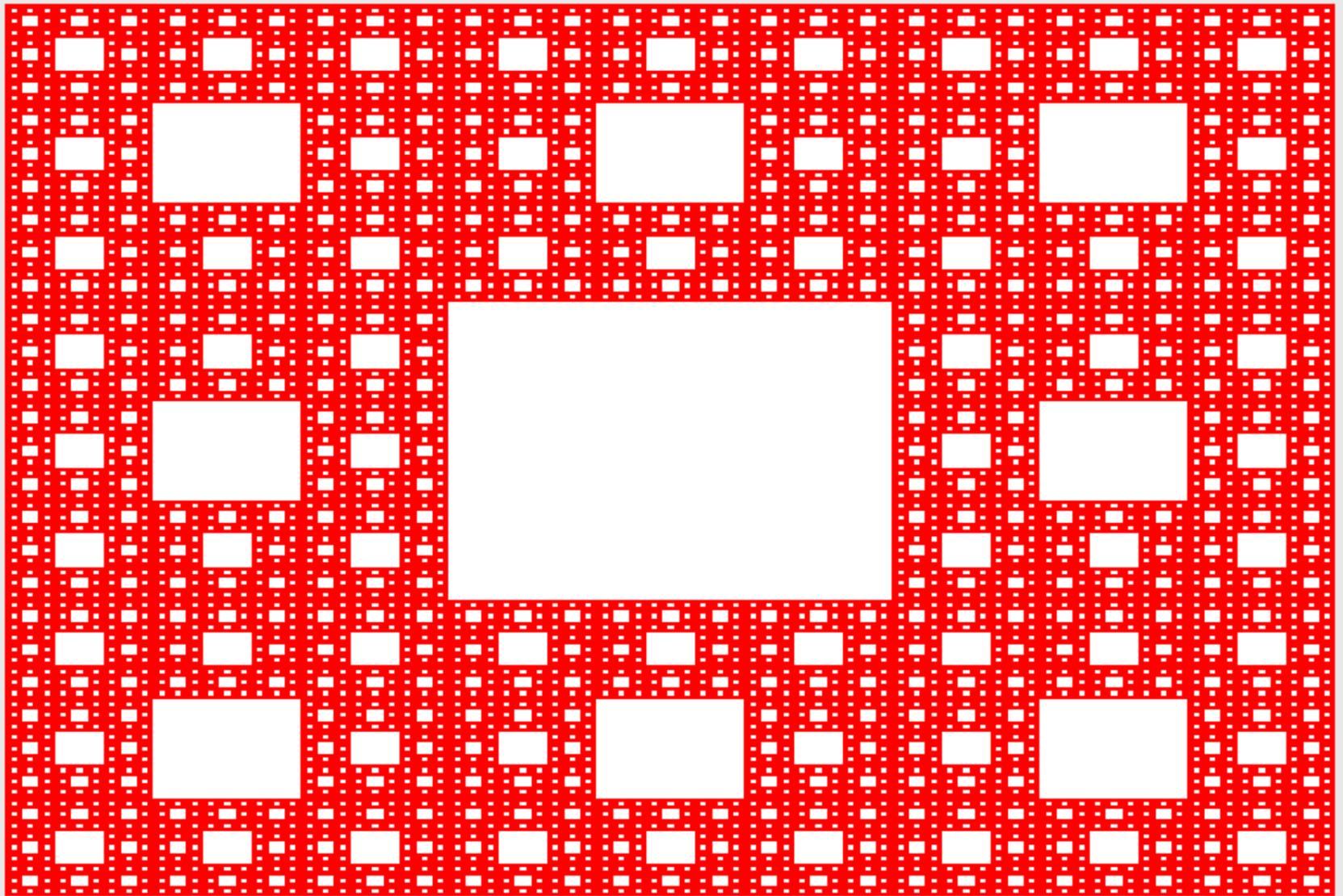
- ▶ 2 or more methods call each other repeatedly
 - E.g., Hofstadter Female and Male Sequences

$$F(n) = \begin{cases} 1 & \text{if } n = 0 \\ n - M(F(n - 1)) & \text{if } n > 0 \end{cases}$$

$$M(n) = \begin{cases} 0 & \text{if } n = 0 \\ n - F(M(n - 1)) & \text{if } n > 0 \end{cases}$$

- Burning Questions for you to figure out **now** by coding:
 - How often are the sequences different in the first 50 positions? first 500? first 5,000? first 5,000,000?

Sierpinski Carpet



Work Time

»» HW 13: Sierpinski Carpet