More Inference Rules

Curt Clifton
Rose-Hulman Institute of Technology

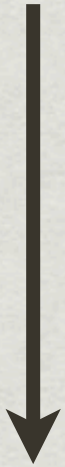

Hmm,
elephants

Last Time

- Assignment
- Sequencing (a.k.a., composition)

Assignment Rule

WHATEVER IS TRUE ABOUT *e* BEFORE IS TRUE ABOUT *v* AFTER



```
    //@ assert P(e);
v = e;
    //@ assert P(v);
```

WHAT MUST BE TRUE ABOUT *e* BEFORE IF WE WANT SOME PROPERTY TO BE TRUE ABOUT *v* AFTER?

Composition Rule

IF:

```
    //@ assert P1;
S1;
    //@ assert Q1;
```

AND

```
    //@ assert P2;
S2;
    //@ assert Q2;
```

AND

$Q_1 \Longrightarrow P_2$

THEN:

```
    //@ assert P1;
S1;
    //@ assert Q1;
    //@ assert P2;
S2;
    //@ assert Q2;
```

IMPORTANT: IMPLICATIONS GO DOWN

More Bites

- If-then-else
- If-then
- While

If-Then-Else Rule

IF:

```
    //@ assert P1 && B;
S1;
    //@ assert Q;
```

AND

```
    //@ assert P2 && !B;
S2;
    //@ assert Q;
```

AND

$P \&\& B \Longrightarrow P_1$

AND

$P \&\& !B \Longrightarrow P_2$

THEN:

```
    //@ assert P;
if (B) {
    S1;
} else {
    S2;
}
    //@ assert Q;
```

Another Bite of Elephant–What's P?

```
    //@ assert P ;
if (x > 0) {
    y = x;
} else {
    y = -x;
}
    //@ assert y >= 0;
```

What's P?

```
        //@ assert P;
    if (x > 0) {
5:          //@ assert P && x > 0;  // from above
4:          //@ assert x >= 0;  // P1
        y = x;
2:          //@ assert y >= 0;
    } else {
6:          //@ assert P && x <= 0;  // from above
3:          //@ assert -x >= 0;  // P2
        y = -x;
1:          //@ assert y >= 0;
    }
        //@ assert y >= 0;
```

NEED:

$$((P \;\&\&\; x > 0) \Longrightarrow x >= 0) \;\&\&$$
$$((P \;\&\&\; x <= 0) \Longrightarrow -x >= 0)$$

WEAKEST PRECONDITION: P IS true

Q1

If-Then-Else Rule

IF:

```
    //@ assert P1 && B;
S1;
    //@ assert Q;
```

AND

```
    //@ assert P2 && !B;
S2;
    //@ assert Q;
```

AND

$P \&\& B \Longrightarrow P_1$

AND

$P \&\& !B \Longrightarrow P_2$

THEN:

```
    //@ assert P;
if (B) {
    S1;
} else {
    S2;
}
    //@ assert Q;
```

USEFUL WHEN WORKING BACKWARDS:
IN GENERAL P IS $(B \Longrightarrow P_1) \&\& (!B \Longrightarrow P_2)$

Another Example

FIND THE WEAKEST PRE-CONDITION, P

```
    //@ assert P;
if (i % 2 != 0) {
    i = i / 2;
} else {
    i = i + 1;
}
    //@ assert i >= 0;
```

RECALL A ==> B IS EQUIVALENT TO B || !A

Another Example

```
10:       //@ assert i >= 0;
9:        //@ assert (i >= 0 || i%2 == 0) && (i >= -1 || i%2 != 0);
          // ==> by arithmetic
8:        //@ assert (i/2 >= 0 || i%2 == 0) && (i+1 >= 0 || i%2 != 0);
          // ==> by defn. of implication
7:        //@ assert (i%2 != 0 ==> i/2 >= 0) && (i%2 == 0 ==> i+1 >= 0);
      if (i % 2 != 0) {
5:            //@ assert P && i % 2 != 0;
4:            //@ assert i / 2 >= 0;              // <----- P1
          i = i / 2;
2:            //@ assert i >= 0;
      } else {
6:            //@ assert P && i % 2 == 0;
3:            //@ assert i + 1 >= 0;              // <----- P2
          i = i + 1;
1:            //@ assert i >= 0;
      }
          //@ assert i >= 0;
```

P

(B ==> P_1) && (!B ==> P_2)

Cartoon of the Day

If-Then Rule

IF:

```
    //@ assert P1;
S;
    //@ assert Q;
```

WHERE

P && B ==> P_1 **AND**

P && !B ==> Q

THEN:

```
    //@ assert P;
if (B) {
    S;
}
    //@ assert Q;
```

USEFUL WHEN WORKING BACKWARDS:
IN GENERAL P IS (B ==> P_1) && (!B ==> Q)

Example:
Prove Correctness

```
    //@ assert true;
if (x % 2 != 0) {
    x = x - 1;
}
    //@ assert x % 2 == 0;
```

Example:
Prove Correctness

```
        //@ assert true;
    if (x % 2 != 0) {
2:          //@ assert true && x % 2 != 0;
4:          //@ assert (x - 1) % 2 == 0;
3:      x' = x - 1;  // tick trick
1:          //@ assert x' % 2 == 0;
    }
        //@ assert x % 2 == 0;
```

Quote of the Day

EVERY PROGRAM HAS AT LEAST ONE BUG AND CAN BE SHORTENED BY AT LEAST ONE INSTRUCTION — FROM WHICH, BY INDUCTION, IT IS EVIDENT THAT EVERY PROGRAM CAN BE REDUCED TO ONE INSTRUCTION THAT DOES NOT WORK

– KEN ARNOLD

The *loop invariant* is the most important part of proving a loop correct.

IF:

```
    //@ assert P && B;
S;
    //@ assert P;
```

AND

```
P && !B ==> Q
```

THEN:

```
    //@ assert P;
while (B) {
        //@ assert P && B;
    S;
        //@ assert P;
}
    //@ assert P && !B;
    //@ assert Q;
```

P SHOULD BE TRUE WHENEVER THE LOOP CONDITION IS EVALUATED

Prove Correctness

```
    //@ assert true;
x = 0;
while (x < 20)  {
    x = x + 2;
}
    //@ assert x == 20;
```

Bad Guess

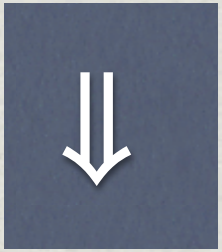
GUESS LOOP INVARIANT: x <= 20

```
        //@ assert true;
4:      //@ assert 0 <= 20;
    x = 0;
1:      //@ assert x <= 20;
    while (x < 20)  {
3:          //@ assert x <= 20 && x < 20;
7:          ??? can't make this leap, bad loop invariant guess
6:          //@ assert x + 2 <= 20;
5:      x' = x + 2;  // tick-trick
2:          //@ assert x' <= 20;
    }
        //@ assert x == 20;
```

Better Guess

LOOP INVARIANT: `x <= 20 && x % 2 == 0`

```
        //@ assert true;
4:      //@ assert 0 <= 20 && 0 % 2 == 0;
    x = 0;
1:      //@ assert x <= 20 && x % 2 == 0;
    while (x < 20)  {
3:          //@ assert x <= 20 && x % 2 == 0 && x < 20;
7:          //@ assert x <= 18 && x % 2 == 0;
6:          //@ assert x + 2 <= 20 && (x + 2) % 2 == 0;
5:      x' = x + 2;  // tick-trick
2:          //@ assert x' <= 20 && x' % 2 == 0;
    }
8:      //@ assert x <= 20 && x % 2 == 0 && x >= 20;
        //@ assert x == 20;
```



IMPLICATIONS HOLD READING DOWN THE PAGE