# More Notation

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# Tomorrow

- **Required**: Finish leftovers from convo schedule
- **Optional**: HW8 work time
- HW 8 due Thursday after break, 6 a.m.
  - Very challenging for some people

The *loop invariant* is the most important part of proving a loop correct.

**IF:**

```
    //@ assert P && B;
S;
    //@ assert P;
```

**AND**

```
P && !B ==> Q
```

**THEN:**

```
    //@ assert P;
while (B) {
        //@ assert P && B;
    S;
        //@ assert P;
}
    //@ assert P && !B;
    //@ assert Q;
```

**P SHOULD BE TRUE WHENEVER THE LOOP CONDITION IS EVALUATED**

# Another Example

- Prove Correctness:

```
    //@ assert b > 0;
r = 0;
c = b;
while (c > 0)  {
    r = r + a;
    c = c - 1;
}
    //@ assert r == a * b;
```

# Solution

**LOOP INVARIANT:** `c >= 0 && r == (b - c) * a`

```
        //@ assert b > 0;
10:     //@ assert b >= 0 && 0 == (b - b) * a;
    r = 0;
9:      //@ assert b >=0 && r == (b - b) * a;
8:  c' = b; // tick-trick
2:      //@ assert c' >= 0 && r == (b - c') * a;
    while (c > 0)  {
3:          //@ assert c >= 0 && r == (b - c) * a && c > 0;
7:          //@ assert c > 0 && r + a == (b - c) * a + a;
6:      r' = r + a;  // tick-trick
5:          //@ assert c - 1 >= 0 && r' == (b - c + 1) * a;
4:      c' = c - 1;  // tick-trick
1:          //@ assert c' >= 0 && r == (b - c) * a;
    }
11:     //@ assert c >= 0 && r == (b - c) * a && c <= 0;
12:     //@ assert c == 0 && r == (b - c) * a;
        //@ assert r == a * b;
```

# JML provides two clauses that let us record loop invariants.

- **maintaining** clauses
  - Gives one part of the loop invariant
  - Conjoin the parts to get the complete invariant
- **decreasing** clause
  - Gives *bound function* for loop

# Example

```
    //@ assert b > 0;
r = 0;
c = b;
    /*@ maintaining c >= 0;
      @ maintaining r == (b - c) * a;
      @ decreasing c;
      @*/
while (c > 0) {
    r = r + a;
    c = c - 1;
}
    //@ assert r == a * b;
```

LOOP INVARIANT

```
c >= 0 && r == (b - c) * a
```

TERMINATION PROOF:
BOUND FUNCTION C
HAS A LOWER BOUND AND
IS STRICTLY DECREASING

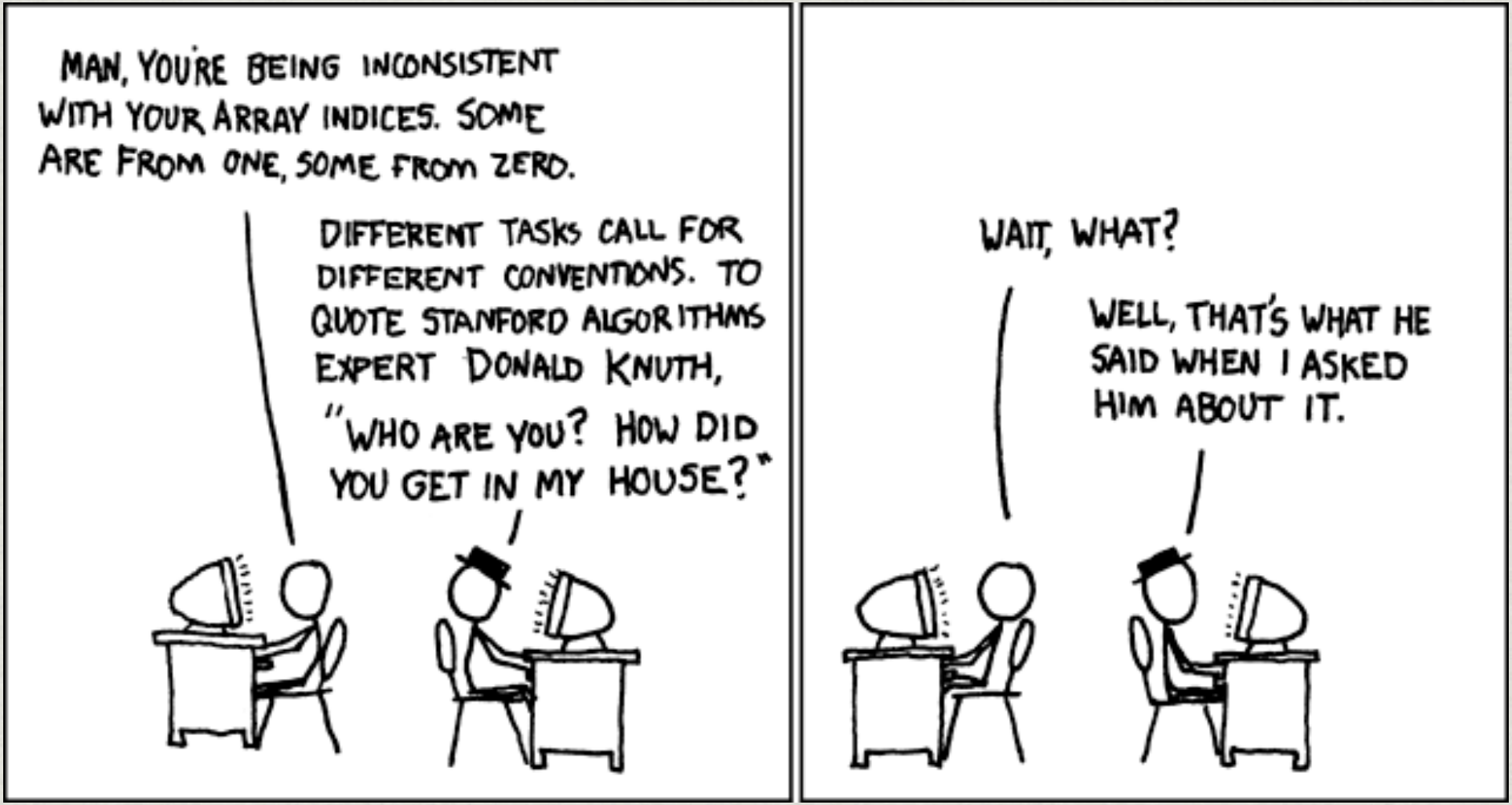
# Termination Example

```
year = 1980;
  //@ decreasing days;
while (days > 365) {
    if (IsLeapYear(year)) {
        if (days > 366) {
            days -= 366;
            year += 1;
        }
    } else {
        days -= 365;
        year += 1;
    }
}
```

- Initially **days** equals elapsed days since Jan. 1, 1980
- Prove termination:
  - Bound function?
  - Lower bound?
  - Strictly decreasing?

Q1,2

# Cartoon of the Day




**HIS BOOKS WERE KINDA INTIMIDATING; REPELLING DOWN THROUGH HIS SKYLIGHT SEEMED LIKE THE BEST OPTION.**

# Quantification in JML

- **(\forall int k; 0 < k && k < a.length; a[k-1] <= a[k])**
  - "The array **a** is sorted."
  - Vacuously **true** if range predicate is **false**
- **(\exists int j; 0 <= j && j < b.length; isPrime(b[j]))**
  - "Some element in the array **b** is prime"
  - Assuming **isPrime()** does primality testing
  - Vacuously **false** if range predicate is **false**

# More Quantification

- **(\max int k; lo <= k && k <= hi; a[k])**
  - "The largest element in **a[lo]**, …, **a[hi]**."
  - Or the smallest value of the correct type if the range is empty
  - **\min** expressions are similar

# Summing Up

- **(\sum int j; 0 <= j && j < b.length; Math.abs(b[j]))**
  - "The sum of the absolute values of the elements array **b**."
  - Zero if the range predicate is **false**

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