474 Notes on Day 3 slides:

Slide 3: Boolean (Propositional) Logic Wffs

Note that $P \rightarrow Q$ is an abbreviation for $\neg P \vee Q$. What does $P \leftrightarrow Q$ abbreviate?

Slide 6: Inference riles

More on soundness and completeness later

Slide 9: First-order logic

Note that the definition is recursive, so proofs about wffs are likely to be by induction.

On board:

Example of a ternary predicate:

Pythagorean(a, b, c) is true iff $a^2 + b^2 = c^2$.

Pythagorean(5, 12, 13) has no free variables, Pythagorean(x, y, 13) has free variables

For last bullet, consider: $\exists x\ (\exists y\ (x \in \mathbb{N} \wedge y \in \mathbb{N} \wedge \text{Pythagorean}(x, y, 13)))$. x and y are bound by the $\exists$ quantifier here.

We can abbreviate this $\exists x, y \in \mathbb{N}$ (Pythagorean(x, y, 13))

Slide 10: Sentences

The first is a sentence, if we assume that Smokey is a constant

True

True

False

True (if we assume that "exists" is not temporal)

Slide 11: interpretations and models

An interpretation of the sentence on this page is the integers, with < assigned to the normal < predicate.

Note that we use infix x < y instead of the formal <(x, y).

What about the sentence $\exists x\ (\forall y\ (x*y = 0))$? A model for this sentence is the integers with the normal meanings of =, 0, and *.

Note that this involves assigning a value to the constant 0 in the expression.

Slide 12 Examples

First one is valid, independent of the values of P, Q, and Smokey

Second is invalid

Third depends on D,I. Example: satisfied by (integers, <=), but not (integers, <)

Slide 23: Fibonacci Running time

Point out that the initial formula for C is given by a recurrence relation

Use induction.

Base cases, N=3, N=4

Assume by induction that if N>=3, then C_N and C_{N+1} are the right things. Show that C_{N+2} is the right thing.

$C_{N+2} = 1 + C_N + C_{N+1} = (F_{N+2} + F_{N-1} – 1) + (F_{N+3} + F_N – 1) + 1 = F_{N+4} + F_{N+1} – 1 = F_{N+2+2} + F_{N+2-1} – 1$