474 HW 08 problems (highlighted problems are the ones to turn in)

8.1acegklptuz
(#1)

8.1 b

8.1d

8.1h

8.1i

8.1j

8.1n

(#2 - #7)
3 pts. each

1. For each of the following languages L, state whether L is regular or not and prove your answer:

a. $\{a^i b^j : i, j \geq 0 \text{ and } i + j = 5\}$.

b. $\{a^i b^j : i, j \geq 0 \text{ and } i - j = 5\}$.

c. $\{a^i b^j : i, j \geq 0 \text{ and } |i - j| \equiv_5 0\}$.

d. $\{w \in \{0, 1, \#\}^* : w = x \# y, \text{ where } x, y \in \{0, 1\}^* \text{ and } |x| \cdot |y| \equiv_5 0\}$.

e. $\{a^i b^j : 0 \leq i < j < 2000\}$.

f. $\{w \in \{Y, N\}^* : w \text{ contains at least two Y's and at most two N's}\}$.

g. $\{w = xy : x, y \in \{a, b\}^* \text{ and } |x| = |y| \text{ and } \#_a(x) \geq \#_a(y)\}$.

h. $\{w = xyzy^R x : x, y, z \in \{a, b\}^*\}$.

i. $\{w = xyzy : x, y, z \in \{0, 1\}^+\}$.

j. $\{w \in \{0, 1\}^* : \#_0(w) \neq \#_1(w)\}$.

k. $\{w \in \{a, b\}^* : w = w^R\}$.

l. $\{w \in \{a, b\}^* : \exists x \in \{a, b\}^+ \ (w = x\, x^R x)\}$.

m. $\{w \in \{a, b\}^*$: the number of occurrences of the substring ab equals the number of occurrences of the substring ba$\}$.

n. $\{w \in \{a, b\}^* : w$ contains exactly two more b's than a's$\}$.

o. $\{w \in \{a, b\}^* : w = xyz, |x| = |y| = |z|$, and $z = x$ with every a replaced by b and every b replaced by a$\}$. Example: abbbabbaa $\in L$, with $x =$ abb, $y =$ bab, and $z =$ baa.

p. $\{w : w \in \{a - z\}^*$ and the letters of w appear in reverse alphabetical order$\}$. For example, spoonfeed $\in L$.

q. $\{w : w \in \{a - z\}^*$ every letter in w appears at least twice$\}$. For example, unprosperousness $\in L$.

r. $\{w : w$ is the decimal encoding of a natural number in which the digits appear in a non-decreasing order without leading zeros$\}$.

s. $\{w$ of the form: $<integer_1> + <integer_2> = <integer_3>$, where each of the substrings $<integer_1>, <integer_2>$, and $<integer_3>$ is an element of $\{0 - 9\}^*$ and $integer_3$ is the sum of $integer_1$ and $integer_2\}$. For example, 124+5=129 $\in L$.

t. $L_0{}^*$, where $L_0 = \{ba^i b^j a^k, j \geq 0, 0 \leq i \leq k\}$.

u. $\{w : w$ is the encoding of a date that occurs in a year that is a prime number$\}$. A date will be encoded as a string of the form $mm/dd/yyyy$, where each m, d, and y is drawn from $\{0\text{–}9\}$.

v. $\{w \in \{1\}^* : w$ is, for some $n \geq 1$, the unary encoding of $10^n\}$. (So $L = \{1111111111, 1^{100}, 1^{1000}, \ldots\}$.)

6. Prove *by construction* that the regular languages are closed under:

a. intersection.

b. set difference.

8.7a
(#8) 9

7. Prove that the regular languages are closed under each of the following operations:

a. $pref(L) = \{w : \exists x \in \Sigma^* (wx \in L)\}$.

b. $suff(L) = \{w : \exists x \in \Sigma^* (xw \in L)\}$.

c. $reverse(L) = \{x \in \Sigma^* : x = w^R \text{ for some } w \in L\}$.

d. letter substitution (as defined in Section 8.3).

8.7a Do this by construction, i.e., produce an algorithm that takes as input a DFSM $M = (K, \Sigma, \delta, s, A)$ that accepts L, and produces a DFSM $M' = (K', \Sigma', \delta', s', A')$ that accepts $pref(L)$. Describe how to get from M to M'

Hint: M' will have a lot of its elements in common with M, but it takes a somewhat complex calculation (based on M) to determine exactly what has to be changed.

On the main HW8 assignment document, I posted **the author's solutions to the other three parts of problem 8.7**, so that you will have more examples.

8.2ac
(#9)

2. For each of the following languages L, state whether L is regular or not and prove your answer:

a. $\{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^* : \text{in each prefix } x \text{ of } w, \#_\mathsf{a}(x) = \#_\mathsf{b}(x) = \#_\mathsf{c}(x))\}$.

b. $\{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^* : \exists \text{ some prefix } x \text{ of } w\ (\#_\mathsf{a}(x) = \#_\mathsf{b}(x) = \#_\mathsf{c}(x))\}$.

c. $\{w \in \{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^* : \exists \text{ some prefix } x \text{ of } w\ (x \neq \varepsilon \text{ and } \#_\mathsf{a}(x) = \#_\mathsf{b}(x) = \#_\mathsf{c}(x))\}$.

8.3
(#10)

3. Define the following two languages:

$L_\mathsf{a} = \{w \in \{\mathsf{a}, \mathsf{b}\}^* : \text{in each prefix } x \text{ of } w, \#_\mathsf{a}(x) \geq \#_\mathsf{b}(x)\}$.

$L_\mathsf{b} = \{w \in \{\mathsf{a}, \mathsf{b}\}^* : \text{in each prefix } x \text{ of } w, \#_\mathsf{b}(x) \geq \#_\mathsf{a}(x)\}$.

a. Let $L_1 = L_\mathsf{a} \cap L_\mathsf{b}$. Is L_1 regular? Prove your answer.

b. Let $L_2 = L_\mathsf{a} \cup L_\mathsf{b}$. Is L_2 regular? Prove your answer.

8.4a
(#11) 6

8.4b
(#12)

4. For each of the following languages L, state whether L is regular or not and prove your answer:

a. $\{uww^\mathrm{R}v : u, v, w \in \{\mathsf{a}, \mathsf{b}\}^+\}$.

b. $\{xyzy^\mathrm{R}x : x, y, z \in \{\mathsf{a}, \mathsf{b}\}^+\}$.

8.7
(#13)

7. Prove that the regular languages are closed under each of the following operations:

a. $\mathit{pref}(L) = \{w : \exists x \in \Sigma^* (wx \in L)\}$.

b. $\mathit{suff}(L) = \{w : \exists x \in \Sigma^* (xw \in L)\}$.

c. $\mathit{reverse}(L) = \{x \in \Sigma^* : x = w^\mathrm{R} \text{ for some } w \in L\}$.

d. letter substitution (as defined in Section 8.3).

8. Using the defintions of *maxstring* and *mix* given in Section 8.6, give a precise definition of each of the following languages:

a. $\mathit{maxstring}(\mathrm{A}^n\mathrm{B}^n)$.

b. $\mathit{maxstring}(\mathsf{a}^i \mathsf{b}^j \mathsf{c}^k, 1 \leq k \leq j \leq i)$.

c. $\mathit{maxstring}(L_1L_2)$, where $L_1 = \{w \in \{\mathsf{a}, \mathsf{b}\}^* : w \text{ contains exactly one } \mathsf{a}\}$ and $L_2 = \{\mathsf{a}\}$.

d. $\mathit{mix}((\mathsf{aba})^*)$.

e. $\mathit{mix}(\mathsf{a}^*\mathsf{b}^*)$.

9. Prove that the regular languages are not closed under *mix*.