Pheromone Modification Strategies for Ant Algorithms applied to Dynamic TSP

Helen
Magdeline

Ant algorithms

- For Traveling Salesperson Problem (TSP) where an instance may change through deletion/insertion of a city
- 2 ways to handle change of problem instance
  1. Restart an ant algorithm after change has occurred
  2. Transfer knowledge from old optimization run to new run
     a. Assuming change of problem is relatively small, new optimum will be related to the old one
     b. If too much information is transferred, run basically starts and is stuck at a local optimum

Ant algorithm used

- An ant at city *i* selects the next city *j* from a set S of cities using
  - pheromone information, denoted by τ_{ij}
  - heuristic information, denoted by $\eta_{ij} = 1 / d_{ij}$, where d_{ij} is the distance between city i and city j
  - Goal: Maximise $[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}$, where α and β are constants that determine the relative influence of heuristic values and pheromone values on the decision of the ant

$$p_{ij} = \frac{[\tau_{ij}]^{\alpha} [\eta_{ij}]^{\beta}}{\sum_{h \in S} [\tau_{ih}]^{\alpha} [\eta_{ih}]^{\beta}}$$

Ant algorithm used

- pheromone information, denoted by τ_{ij}
- Before doing global pheromone update, some old pheromone is evaporated on all edges

$$\tau_{ij} \mapsto (1 - \rho) \cdot \tau_{ij}$$

p = evaporation rate
i = city i
j = successor of i in the so far best found tour

- Elitist strategy
  - Elitist ant is ant along the best solution found
  - Elitist ant updates pheromone by adding some amount of pheromone to element (i,j) of pheromone matrix
  - Amount of pheromone added is p/4

Pheromone Modification Strategies

When there is a insertion/deletion of a city, the pheromone information needs to be modified to maintain enough old information but also flexible enough to find a new solution.

There are 3 strategies:

- Restart-Strategy
- ŋ(eta)-Strategy
- τ(tau)-Strategy

Restart-Strategy

Reinitialize all the pheromone values by the same degree. It does not take into account the information of where the change happens

All strategies work distributing a reset-value $\gamma_i \in [\bar{0} : 1]$, then the pheromone from city i to city j is reinitialized according to the following equation

$$\tau_{ij} \mapsto (1 - \gamma_i)\tau_{ij} + \gamma_i \frac{1}{n-1}$$

The Restart-Strategy will assign the same reset value to all city, it will have a strategy specific parameter $\lambda_R \in [0,1]$ where $\gamma_i = \lambda_R$.

ŋ-Strategy

Uses the heuristic based information (distance)

γ_i is proportional to the distance from the insert/deleted city j

The distance is drive from η_{ij} so that they are proportional and scaling the ŋ value has no effect

$$d_{ij}^{\eta} = 1 - \frac{\eta_{avg}}{\lambda_E \cdot \eta_{ij}}$$

Where $\eta_{avg} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{k \neq i} \eta_{ik}$ and λ_E is a value between 0 and infinity

The reset value is

$$\gamma_i = \max(0, d_{ij}^{\eta}).$$

τ-Strategy

Use the pheromone based information

The pheromone distance d_{ij}^{τ} between city i and j is the maximum pheromone value of all path from i to j.

The reset-value is

$$\gamma_i = min(1, \lambda_T \cdot d_{ij}^{\tau})$$

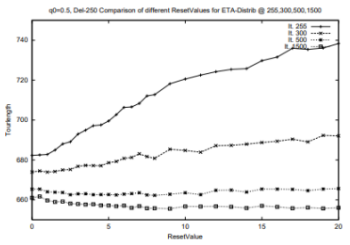
where $\lambda_T \in [0, \infty)$,

Empirical evaluation

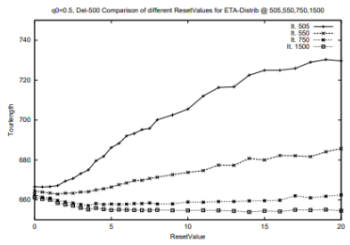
- Insertion and deletion of a city were evaluated after 250 iterations (del250, ins250) or after 500 iterations (del500, ins500)
  - Total of 1500 iterations
  - α=1 and β=5
  - Elitist ant dropped when insertion/deletion occurred and redetermined in the first iteration thereafter
- Average row-/column- entropy, E

$$E = \frac{1}{n \log n} \sum_{i=1}^{n} \sum_{j=1}^{n} -\tau_{ij} \log(\tau_{ij})$$

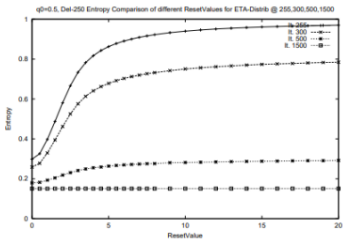
ŋ-Strategy



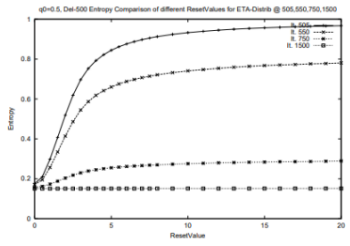
(a) del250, best solution



(b) del500, best solution

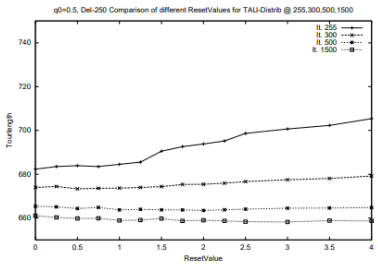


(c) del250, entropy

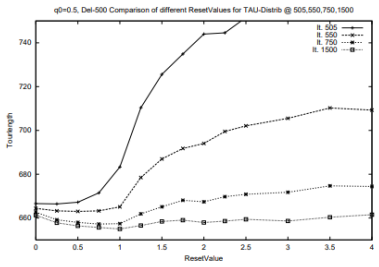


(d) del500, entropy

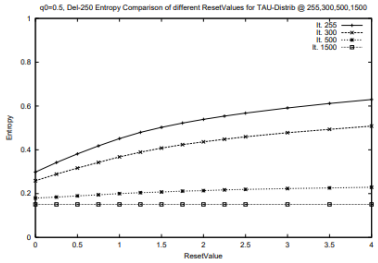
τ-Strategy



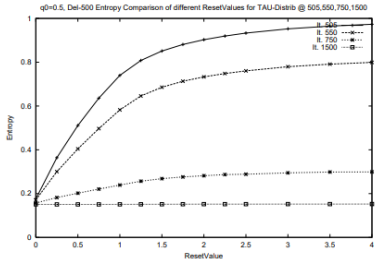
(a) del250, best solution



(b) del500, best solution

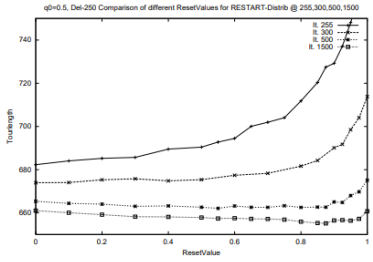


(c) del250, entropy

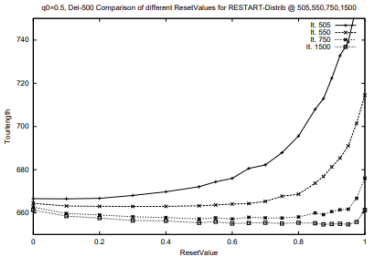


(d) del500, entropy

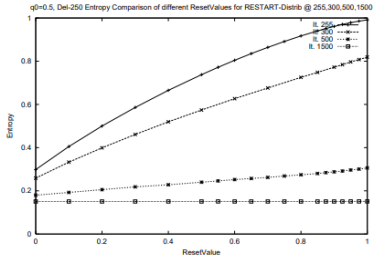
Restart-Strategy



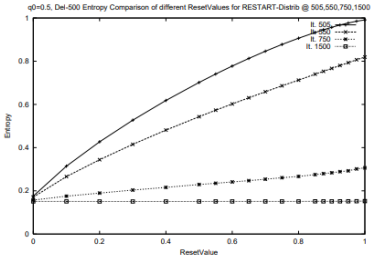
(a) del250, best solution



(b) del500, best solution



(c) del250, entropy



(d) del500, entropy

Conclusion

- Tested influence of λ parameter which influence height of reset-values on all 3 strategies
  - good values of λ_E are not too large (≤8)
  - λ_T approximately = 1
  - λ_r approximately = 1
- ŋ-Strategy and Restart-strategies perform best, closely followed by τ-Strategy
- Hypotheses that Restart-Strategy performs well due to singular insertion/deletion performed

Questions?