Transformers – Part 2

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With assistance from Claude Sonnet 4.5

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Transformer Blocks

- The self-attention calculation lies at the core of what is called a **transformer block**.
- In addition to the self-attention layer, it includes feedforward layers, residual connections, and normalizing layers.

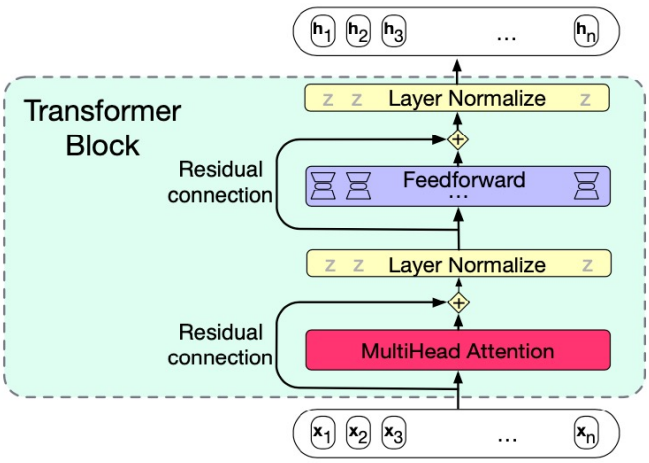

Image source: TBA

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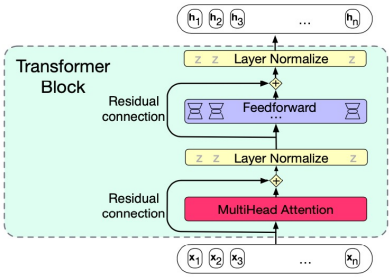
Transformer Blocks



- **Feedforward layer**: It contains *N* position-wise networks, one at each position.

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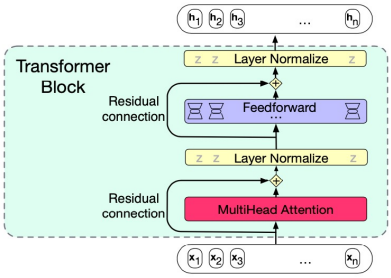
Transformer Blocks



- **Residual connections**: They pass information from a lower layer to a higher layer without going through the intermediate layer.
- **Layer normalization (Layer norm)**.
  - Summed vectors are normalized.
  - It is used to improve training performance in deep neural networks.
  - It keeps the values of a hidden layer in a range that facilitates gradient-based training.

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Residual Stream View of the Transformer

- A better view of a transformer is to look at the residual stream.
- The feedforward layers and the multi-head attention layers are added to a stream.
- The Multi Head Attention layer reads information from the other residual streams.
- The feedforward layer processes just one token.

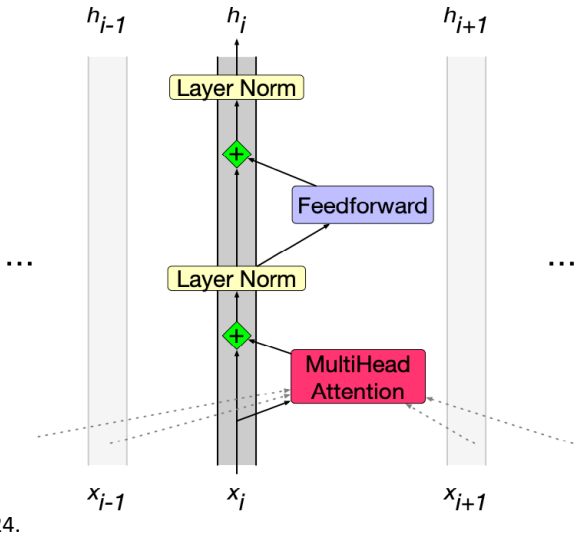

Image source: Chapter 9, Jurafsky and Martin, Aug. 20, 2024.

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Moving Information

- The attention head can move information from token A's residual stream into token B's residual stream.

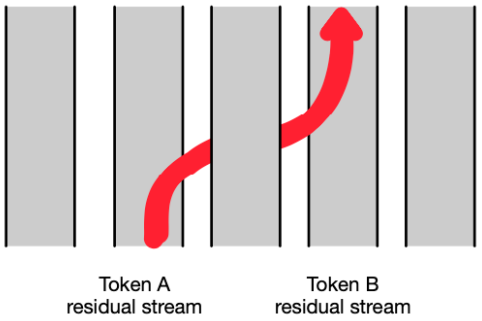

Image source: Chapter 9, Jurafsky and Martin, Aug. 20, 2024.

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Transformer Block: Layer Normalization

- Typical: z-score $$z = \frac{x - \mu}{\sigma}$$
- X = observed value
- μ = mean of sample
- σ = standard deviation of the sample

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Transformer Block: Layer Normalization

- Let x = [2.0, 4.0, 6.0, 8.0]
- Mean μ = (2 + 4 + 6 + 8) / 4 = 5.0
- Variance σ^2 = [(2-5)² + (4-5)² + (6-5)² + (8-5)²] / 4 =
  [9 + 1 + 1 + 9] / 4 = 5.0
- Let z = (x - μ) / √(σ^2 + ε), where ε = 1e-5 (small constant for numerical stability)
  = [(2-5)/√5, (4-5)/√5, (6-5)/√5, (8-5)/√5] ≈
  [-1.34, -0.45, 0.45, 1.34]
- Mean: 0. Variance: 1

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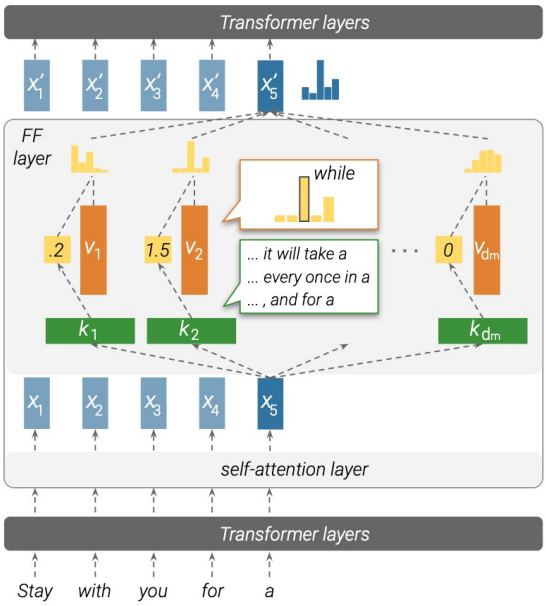
Feedforward layers

- The FF layers are not a classifier as in FFnets or CNNs.
- Instead, they perform key-value look-up.



Source: Transformer Feed-Forward Layers are Key-Value Memories. Geva et al.

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Memory in LSTMs

- They are similar in spirit to the context vector in LSTMs.
- **Long-term memory storage:**
  - **LSTM cell state (c_t)**: Explicitly maintained memory that accumulates information via the input gate
- **Memory retrieval/access:**
  - **LSTM output gate**: Controls what gets read from cell state and exposed to the output

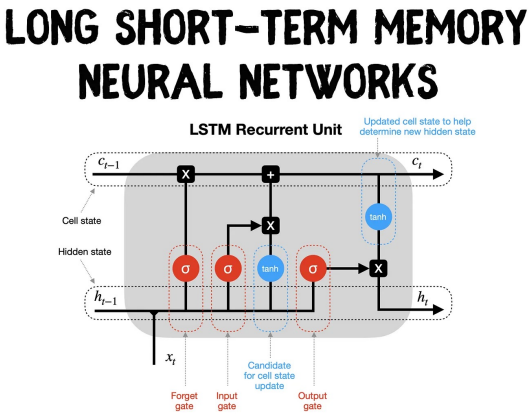

Image source: https://towardsdatascience.com/lstm-recurrent-neural-networks-how-to-teach-a-network-to-remember-the-past-55e54c2ff22e

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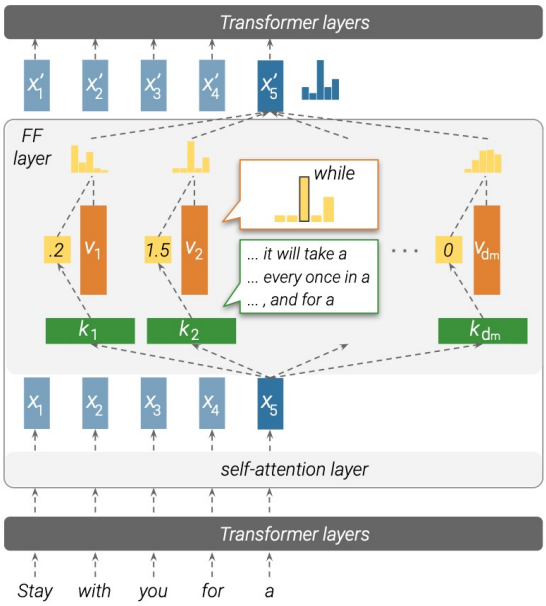
Memory in Transformers

- **Long-term memory storage:**
  - **Transformer FFN layers**: Implicitly store learned knowledge in their parameters—this is where the model's "long-term memory" of patterns, facts, and transformations lives
- **Memory retrieval/access:**
  - **Transformer attention**: Controls what information gets retrieved and aggregated from across the sequence (much more flexible and powerful)



Source: Transformer Feed-Forward Layers are Key-Value Memories. Geva et al.

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Memory in LSTMs and Transformers

- In **both** architectures, individual neurons can specialize to track specific features like sentiment:
- **LSTM**: A single dimension in the cell state vector can learn to track sentiment
- **Transformer**: A single neuron in an FFN layer can also learn to track sentiment (this has been demonstrated empirically with neuron activation studies)

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Memory in LSTMs and Transformers

- The real difference is architectural:
- **LSTM**: Has a designated memory vector (cell state) that's separately maintained and updated
- **Transformer**: Has the residual stream where all information flows through, getting iteratively refined by attention (gathering info) and FFN layers (transforming it)

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Memory in LSTMs and Transformers

- LSTM cell state ↔ Transformer residual stream
  - Both are the primary information highways
  - Both get iteratively refined (LSTM through time steps, Transformer through layers)
  - Both accumulate and transform information progressively
- The key operations on each:
  - **LSTM**: Gates modulate what's added/kept/read from cell state
  - **Transformer**: Attention + FFN add transformations to the residual stream
- The scale difference is massive:
  - LSTM hidden states: typically hundreds to ~2000 dimensions
  - Transformer residual streams: 4096, 12288, or even larger dimensions
  - Orders of magnitude more parameters in the FFN layers
- That scale is what gives Transformers their power—vastly more capacity to represent and refine information, plus the flexibility of attention to route it.

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Information Flow through Transformer Architecture

Input embeddings
↓
[Layer 1: Head 1 + Head 2 + ... + Head 8 → add to stream]
↓
[Layer 1: MLP → add to stream]
↓
[Layer 2: Head 1 + Head 2 + ... + Head 8 → add to stream]
↓
[Layer 2: MLP → add to stream]
↓
... (many layers)
↓
Final contextualized embedding

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Memory in Transformers

- Recent research suggests MLPs act as **key-value memories**.
- They store factual associations like:
  - "Eiffel Tower" → [Paris, France, landmark, tall, iron, 1889, ...]
  - "photosynthesis" → [plants, chlorophyll, sunlight, CO_2, oxygen, ...]
- When the attention mechanism has assembled ”Eiffel tower" in the residual stream, the MLP can "look it up" and add rich semantic information.

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Memory in Transformers

- Unlike attention (which mixes tokens), MLPs process each token position **independently**.
- They can:
  - Amplify or suppress certain features
  - Perform pattern matching ("if I see these features together, activate this concept")
  - Add semantic enrichment based on what attention has already assembled
- **The division of labor:**
  - **Attention heads:** "Let me gather relevant context from other positions"
    - "Oh, 'bank' appears near 'river' → probably the geographic meaning"
    - "This 'bank' is near 'deposit' → probably the financial meaning"
  - **MLPs:** "Now that I know the context, let me add my knowledge"
    - Takes the contextualized "bank (financial)" representation
    - Adds: [money, institution, savings, loans, interest, checking account, ...]

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Memory in Transformers

- **Factual knowledge** is primarily stored in MLP weights
- **Linguistic patterns** and attention patterns are in attention weights
- **Attention** is like your working memory - gathering and organizing relevant information
- **MLPs** are like your long-term memory - vast stores of facts and patterns that get activated based on what's in working memory.
- In GPT-style models with billions of parameters, the majority of those parameters are in the MLPs - that's where "most of the model" actually lives!

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A Standing Example: Attention

- Let's begin with word embeddings with positional info.
- Not much semantic richness yet.
  - "The": [0.2, 0.1, 0.0, 0.3, …]
  - "Eiffel": [0.5, 0.3, 0.1, 0.0, …]
  - "Tower": [0.4, 0.2, 0.2, 0.1, …]
  - "is": [0.1, 0.0, 0.3, 0.2, …]
  - "in": [0.0, 0.2, 0.1, 0.1, …]
  - "Paris": [0.6, 0.1, 0.0, 0.4, ...]
- Layer 1: Multi-Head Attention
- Head 1: Adjacency detector might compute:
  - "Eiffel" attends strongly to "Tower" (they're adjacent)
  - Adds a vector to "Eiffel" that says "I'm followed by Tower", say: [+0.1, +0.2, 0.0, +0.1, ...]
- Head 2: Compound noun detector:
  - Recognizes "Eiffel Tower" is a compound term
  - Adds feature to both tokens indicating they form a unit
  - For "Tower": [+0.2, 0.0, +0.3, 0.0, ...]
- The residual stream for "Tower" now contains:
  - Original: [0.4, 0.2, 0.2, 0.1, ...]
  - + Head 1: [+0.1, +0.2, 0.0, +0.1, ...]
  - + Head 2: [+0.2, 0.0, +0.3, 0.0, ...]
  - = [0.7, 0.4, 0.5, 0.2, ...]

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A Standing Example: Key Layer of MLP

- The MLP has learned patterns during training.
- Think of it as having memorized rules like:
  - IF I see pattern [high value in dim 0, high in dim 2, nearby "Eiffel"]
  - THEN add [landmark features + Paris features + France features + tourism features + architecture features]
- Each neuron in the first layer of the MLP is like a "key" - it activates when it recognizes a specific pattern.
- Continuing with "Tower" in the context we've built:
- Suppose neuron 2847 has weights that make it fire strongly for "Eiffel Tower" specifically:
  - It's checking: "Do I see the pattern that indicates Eiffel Tower?"
  - Input: [0.7, 0.4, 0.5, 0.2, ...]
  - This neuron's weights: [0.8, 0.1, 0.9, -0.1, ...] (learned during training)
  - Dot product: 0.7×0.8 + 0.4×0.1 + 0.5×0.9 + ... = **1.2** (high activation!)
- Suppose neuron 5432, with different pattern:
  - Checking for "generic tower"
  - Its weights: [0.1, 0.8, -0.2, 0.3, ...]
  - Dot product: 0.7×0.1 + 0.4×0.8 + ... = **0.4** (moderate activation)

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A Standing Example: Value layer of MLP

- The second layer of MLP (Key-Value Memory)
- The “value” layer
- Output dimension 42 (let's say "Paris association"):
  = 1.2 × (weight from neuron 2847) +
  0.4 × (weight from neuron 5432) + ...
  = 1.2 × 0.9 + 0.4 × 0.1 + ...
  = 1.12 (strong "Paris" feature added!)
- Output dimension 73 ("landmark"):
  = 1.2 × 0.8 + 0.4 × 0.7 + ...
  = 1.24 (strong "landmark" feature!)
- Output dimension 158 ("French culture"):
  = 1.2 × 0.6 + 0.4 × 0.05 + ...
  = 0.74 (moderate association)
- After MLP, the residual stream for "Tower" becomes:
  Before MLP: [0.7, 0.4, 0.5, 0.2, ...]
  + MLP output: [0.1, 0.05, 1.12, 0.3, ...]
  = [0.8, 0.45, 1.62, 0.5, ...]
- The stream now contains:
  - Original "Tower" embedding
  - From Attention:
    - Information that it's adjacent to "Eiffel"
    - Recognition it's a compound noun
  - From MLP:
    - Paris association (dim 42: 1.12)
    - Landmark feature (dim 73: 1.24)
    - French culture (dim 158: 0.74)

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Positions

- While the order in which the N tokens are inserted represents word order, this is not sufficient.
- Recall that attention heads can move tokens around.
- We wish to associate with each word the order in which it appeared in the text.
- We will create position embeddings.
- For example, just as we have an embedding for the word *fish*, we will have an embedding for position 3.

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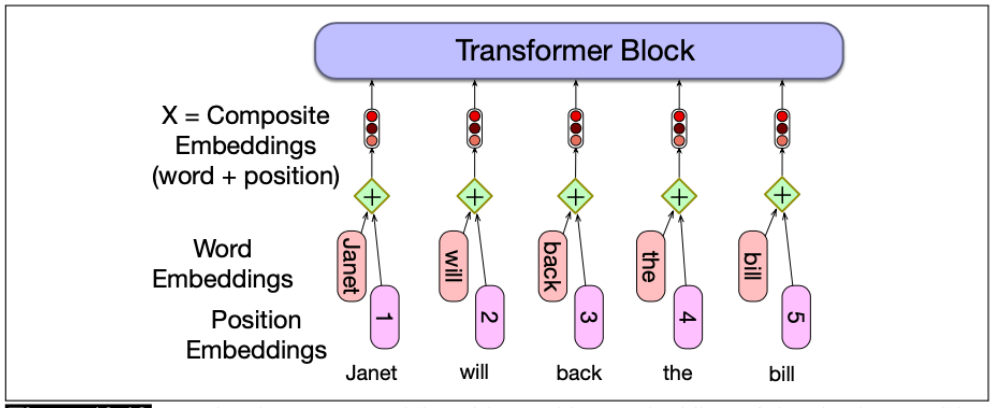
Positions



Combining word embeddings with positions.

Image source: Chapter 9, Jurafsky and Martin, Aug. 20, 2024.

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Positions

- **The classic approach** from the original "Attention is All You Need" paper uses sinusoidal functions with different frequencies for each dimension:
  - PE(pos, 2i) = sin(pos / 10000^(2i/d))
  - PE(pos, 2i+1) = cos(pos / 10000^(2i/d))
- This gives each position a unique "signature" that the model can learn to interpret.

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Example of Positional Encoding

- **Setup:**
- d_model = 4 (embedding dimension - keeping it small for clarity)
- We have 3 tokens in a sequence: ["The", "cat", "sat"]
- Positions: 0, 1, 2

- **The positional encoding formulas:**
- PE(pos, 2i) = sin(pos / 10000^(2i/d_model))
- PE(pos, 2i+1) = cos(pos / 10000^(2i/d_model))
- Where i is the dimension index (0, 1, 2, 3 in our case).

- **Position 1:**
- For i=0:
  - PE(1, 0) = sin(1 / 10000^0) = sin(1) ≈ **0.841**
  - PE(1, 1) = cos(1 / 10000^0) = cos(1) ≈ **0.540**
- For i=1:
  - PE(1, 2) = sin(1 / 10000^0.5) = sin(1/100) ≈ **0.010**
  - PE(1, 3) = cos(1 / 10000^0.5) = cos(1/100) ≈ **1.000**
- Position 1 encoding: **[0.841, 0.540, 0.010, 1.000]**
- Say "cat" has word embedding: [0.5, 0.3, -0.2, 0.8]
- The actual input to the transformer becomes: [0.5, 0.3, -0.2, 0.8] + [0.841, 0.540, 0.010, 1.000] = **[1.341, 0.840, -0.190, 1.800]**

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Alternatives: Learned Positional Embeddings

- Trainable embedding vectors
- One for each position up to a maximum sequence length.
- The model learns optimal position representations during training.
- They're typically initialized randomly.

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Alternatives: Relative Positional Embeddings

- Encode the distance between tokens rather than absolute positions
- Distances beyond a maximum threshold are often clipped (e.g., all distances > 128 use the same embedding).
- They're initialized randomly

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The Language Modelling Head

- Language models are word predictors.
- For example, if the preceding context is “Thanks for all the” and we want to know how likely the next word is “fish” we would compute:

  $P(fish|Thanks\ for\ all\ the)$

- The language modeling head takes the output of the final transformer layer.
- It looks at the last token N.
- Predicts the upcoming word at position N + 1.

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The Language Modelling Head

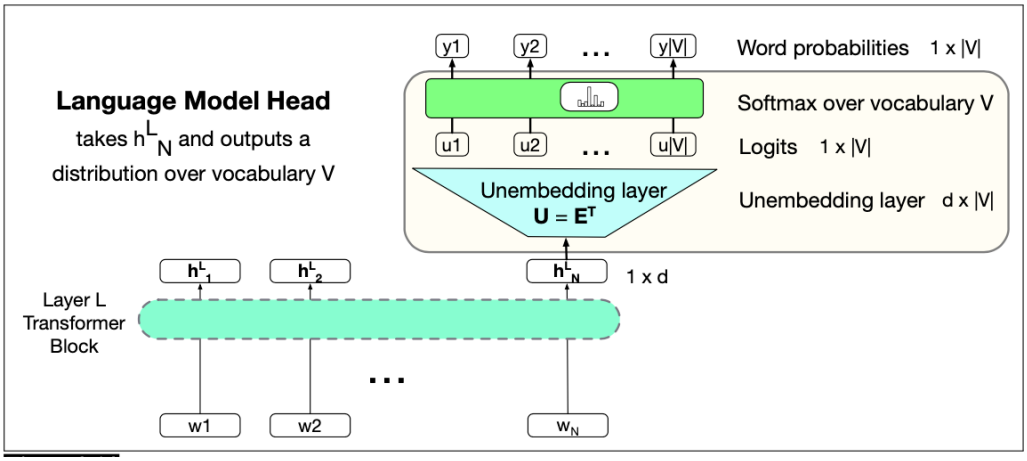

Image source: Chapter 9, Jurafsky and Martin, Aug. 20, 2024.

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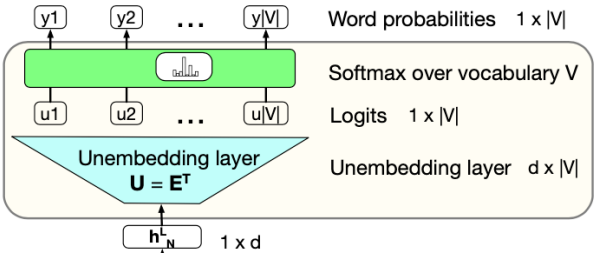
The Language Modelling Head

The unembedding layer takes as input a vector of size d.

It needs to produce an output of size |V|.

Conveniently, the input embeddings are of size |V|

Let's take the transpose of the input embeddings matrix.



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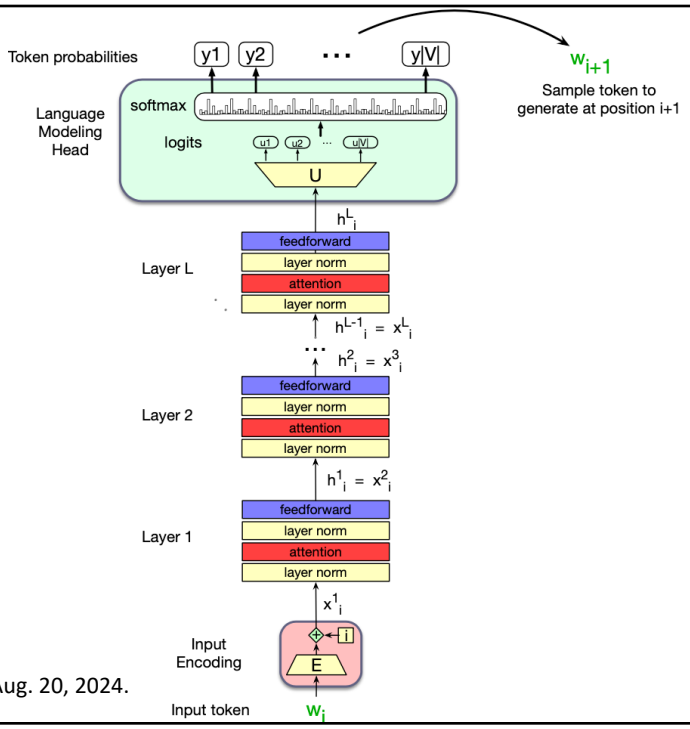
Encoder Architecture



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