DEFINITION

PREDICATE CALCULUS SYMBOLS

The alphabet that makes up the symbols of the predicate calculus consists of:

1. The set of letters, both upper- and lowercase, of the English alphabet.
2. The set of digits, 0, 1, …, 9.
3. The underscore, _.

Symbols in the predicate calculus begin with a letter and are followed by any sequence of these legal characters.

Legitimate characters in the alphabet of predicate calculus symbols include

a R 6 9 p _ z

Examples of characters not in the alphabet include

% @ / & " "

Legitimate predicate calculus symbols include

George fire3 tom_and_jerry bill XXXX friends_of

Examples of strings that are not legal symbols are

3jack "no blanks allowed" ab%cd ***71 duck!!!

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DEFINITION

SYMBOLS and TERMS

Predicate calculus symbols include:

1. *Truth symbols* true and false (these are reserved symbols).
2. *Constant symbols* are symbol expressions having the first character lowercase.
3. *Variable symbols* are symbol expressions beginning with an uppercase character.
4. *Function symbols* are symbol expressions having the first character lowercase. Functions have an attached arity indicating the number of elements of the domain mapped onto each element of the range.

A *function expression* consists of a function constant of arity n, followed by n terms, t_1, t_2, …, t_n, enclosed in parentheses and separated by commas.

A predicate calculus *term* is either a constant, variable, or function expression.

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DEFINITION

PREDICATES and ATOMIC SENTENCES

Predicate symbols are symbols beginning with a lowercase letter.

Predicates have an associated positive integer referred to as the *arity* or "argument number" for the predicate. Predicates with the same name but different arities are considered distinct.

An atomic sentence is a predicate constant of arity n, followed by n terms, $t_1, t_2, \ldots, t_n$, enclosed in parentheses and separated by commas.

The truth values, true and false, are also atomic sentences.

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DEFINITION

PREDICATE CALCULUS SENTENCES

Every atomic sentence is a sentence.

1. If s is a sentence, then so is its negation, $\neg$ s.
2. If s_1 and s_2 are sentences, then so is their conjunction, $s_1 \wedge s_2$.
3. If s_1 and s_2 are sentences, then so is their disjunction, $s_1 \vee s_2$.
4. If s_1 and s_2 are sentences, then so is their implication, $s_1 \rightarrow s_2$.
5. If s_1 and s_2 are sentences, then so is their equivalence, $s_1 \equiv s_2$.
6. If X is a variable and s a sentence, then $\forall$ X s is a sentence.
7. If X is a variable and s a sentence, then $\exists$ X s is a sentence.

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DEFINITION

INTERPRETATION

Let the domain D be a nonempty set.

An *interpretation* over D is an assignment of the entities of D to each of the constant, variable, predicate, and function symbols of a predicate calculus expression, such that:

1. Each constant is assigned an element of D.
2. Each variable is assigned to a nonempty subset of D; these are the allowable substitutions for that variable.
3. Each function f of arity m is defined on m arguments of D and defines a mapping from D^m into D.
4. Each predicate p of arity n is defined on n arguments from D and defines a mapping from D^n into {T, F}.

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DEFINITION

TRUTH VALUE OF PREDICATE CALCULUS EXPRESSIONS

Assume an expression E and an interpretation I for E over a nonempty domain D. The truth value for E is determined by:

1. The value of a constant is the element of D it is assigned to by I.
2. The value of a variable is the set of elements of D it is assigned to by I.
3. The value of a function expression is that element of D obtained by evaluating the function for the parameter values assigned by the interpretation.
4. The value of truth symbol "true" is T and "false" is F.
5. The value of an atomic sentence is either T or F, as determined by the
6. The value of the negation of a sentence is T if the value of the sentence is F and is F if the value of the sentence is T.
7. The value of the conjunction of two sentences is T if the value of both sentences is T and is F otherwise.

8.–10. The truth value of expressions using $\vee$, $\rightarrow$, and $\equiv$ is determined from the value of their operands as defined in Section 2.1.2.

Finally, for a variable X and a sentence S containing X:

11. The value of $\forall$ X S is T if S is T for all assignments to X under I, and it is F otherwise.
12. The value of $\exists$ X S is T if there is an assignment to X in the interpretation under which S is T; otherwise it is F.

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DEFINITION

FIRST-ORDER PREDICATE CALCULUS

First-order predicate calculus allows quantified variables to refer to objects in the domain of discourse and not to predicates or functions.

DEFINITION

SATISFY, MODEL, VALID, INCONSISTENT

For a predicate calculus expression X and an interpretation I:

If X has a value of T under I and a particular variable assignment, then I is said to *satisfy* X.

If I satisfies X for all variable assignments, then I is a *model* of X.

X is *satisfiable* if and only if there exist an interpretation and variable assignment that satisfy it; otherwise, it is *unsatisfiable*.

A set of expressions is *satisfiable* if and only if there exist an interpretation and variable assignment that satisfy every element.

If a set of expressions is not satisfiable, it is said to be *inconsistent*.

If X has a value T for all possible interpretations, X is said to be *valid*.

DEFINITION

PROOF PROCEDURE

A *proof procedure* is a combination of an inference rule and an algorithm for applying that rule to a set of logical expressions to generate new sentences.

We present proof procedures for the *resolution* inference rule in Chapter 12.

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DEFINITION

LOGICALLY FOLLOWS, SOUND, and COMPLETE

A predicate calculus expression X *logically follows* from a set S of predicate calculus expressions if every interpretation and variable assignment that satisfies S also satisfies X.

An inference rule is *sound* if every predicate calculus expression produced by the rule from a set S of predicate calculus expressions also logically follows from S.

An inference rule is *complete* if, given a set S of predicate calculus expressions, the rule can infer every expression that logically follows from S.

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