ASSIGNED 3-25-02

DETERMINE METRIC FOR $(k=1)$

$$\{K\}_e \int_{\Omega_e} \{N_k\} \frac{d\{N_k\}}{dx} \frac{d\{N_k\}^T}{dx} d\bar{x}$$

$$\{N_1\} = \begin{Bmatrix} 1 - \bar{x}/l_e \\ \bar{x}/l_e \end{Bmatrix}$$

$$\begin{Bmatrix} -1 \\ +1 \end{Bmatrix} \{-1 \quad 1\} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\frac{d\{N_1\}}{dx} = \frac{1}{l_e} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \qquad \therefore \frac{d\{N_1\}}{dx} \frac{d\{N_1\}^T}{dx} = \frac{1}{l_e^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\frac{1}{l_e^2} \int_{\Omega_e} \{N_1\} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d\bar{x} = \frac{1}{l_e^2} \left(l_e \begin{Bmatrix} 1/2 \\ 1/2 \end{Bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right)$$

$$= \frac{1}{l_e} \begin{bmatrix} \begin{Bmatrix} 1/2 \\ 1/2 \end{Bmatrix} & \begin{Bmatrix} -1/2 \\ -1/2 \end{Bmatrix} \\ \begin{Bmatrix} -1/2 \\ -1/2 \end{Bmatrix} & \begin{Bmatrix} 1/2 \\ 1/2 \end{Bmatrix} \end{bmatrix} \checkmark$$

$\therefore$ metric $= -1$ ✓

$$\mathcal{L}(y) = -EI(x)\frac{d^2}{dx^2}y(x) + M(x) = 0$$

HINTS:
Product rule differentiation
Results in 2 DIFF Terms
[A3011k]
[A3101k]

THE APPROXIMATION

$$y(x) = y^N = \sum_{\alpha=1}^{N} \Psi_\alpha Y_\alpha = \Psi_\alpha Y_\alpha \qquad 1 \le \alpha \le N$$

THE WEAK STATEMENT

$$GWS^N = \int_\Omega \Psi_\beta \mathcal{L}(y)\,dx = 0 \qquad 1 \le \beta \le N$$

$$= \int_\Omega \Psi_\beta \mathcal{L}(Y^N)\,dx = 0 \qquad 1 \le \beta \le N$$

WITH THE FUNCTION

$$GWS^N = \int_\Omega \Psi_\beta \left(-EI(x)\frac{d^2(Y^N)}{dx^2}\right)dx = 0 \qquad 1 \le \beta \le N$$

REARRANGING FOR INTEGRATION

$$= \int_\Omega -\Psi_\beta EI(x)\frac{d^2(Y^N)}{dx^2}\,dx = 0 \qquad 1 \le \beta \le N$$

TERMS FOR INTEGRATION BY PARTS $\qquad \int u\,dv = -\int v\,du + uv\Big|_0^L$

$$u = -\Psi_\beta(x) EI(x)$$

$$du = -EI(x)\frac{d\Psi_\beta}{dx} - E\Psi_\beta(x)\frac{dI}{dx} \qquad \Rightarrow d(ab) = b(da) + a(db)$$

$$dv = \frac{d^2(y^N)}{dx}$$

$$v = \frac{d(y^N)}{dx}$$

RESULT OF INTEGRATION

$$+\int_\Omega M(x)\Psi_\beta\,dx$$

$$GWS^N = E\int \frac{d(Y^N)}{dx}\left(I(x)\frac{d\Psi_\beta}{dx} + \Psi_\beta\frac{dI}{dx}\right)dx - \Psi_\beta EI\frac{d(Y^N)}{dx}\Bigg|_{\partial\Omega}$$

SUBSTITUTING THE APPROXMATION

$$GWS^N = E\int\left(I(x)\frac{d\Psi_\beta}{dx} + \Psi_\beta\frac{dI}{dx}\right)\frac{d\Psi_\alpha}{dx}\,dx\,Y_\alpha + \int \Psi_\beta M(x)dx) - \text{SAME}$$

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DISCRETIZING (RECALL)

$$\psi_\beta = \{N_k\} \qquad \psi_\alpha y_\alpha = S\big(\underbrace{\{N_k\}^T\{y\}_e}_{\text{keep together}}\big) \checkmark$$

what to do?!

$$GWS^h = S_e\left[E\int_{\Omega_e}\left(I(x)\frac{d\{N_k\}}{dx} + \{N_k\}\frac{dI}{dx}\right)\frac{d\{N_k\}^T}{dx}d\bar{x}\,\{y\}_e + \int_{\Omega_e}\{N_k\}\{N_k\}^T d\bar{x}\,\{M\}_e - \{N_k\}EI(x)\frac{dy^N}{dx}\Big|_{\partial\Omega_e}\right] \checkmark$$

INTERPOLATION

$$M(x) = \{M\}_e$$

$(\ )\ (\ )_e\ \{\ \}_e^T\ (\ )\ [\ \ ]\ \{\ \}_e$

- $\{\ \}_e$ → vector unknown / data interpolated Ω_e
- $[\ \ ]$ → finite element master element matrix
- $(\ )$ → metric
- $\{\ \}_e^T$ → vector data interpolated over each Ω_e
- $(\ )_e$ → scalar data averaged on each Ω_e
- $(\ )$ → scalar data constant on all Ω_e

APPROXIMATION

$$I(x) = I^N = \psi_\alpha I_\alpha \checkmark$$
$$= S_e\left(\{N_k\}^T\{I\}_e\right)$$

$$\frac{dI(x)}{dx} = S_e\left(\frac{d\{N_k\}^T}{dx}\{I\}_e\right)$$

Because it is a scalar ...

$$\underset{\text{assembly}}{I(x)} = S_e\left(\{I\}^T\cdot\{N_k\}\right) \checkmark$$

$$\frac{dI(x)}{dx} = S_e\big(\underbrace{\frac{d\{N_k\}}{dx}\{I\}^T}_{\text{keep together}}\big) \checkmark$$

So, I <u>THINK</u> OUR DISCRETIZED FUNCTION BECOMES

$$GWS^h = S_e\Big(E\{I\}^T\int\{N_k\}\frac{d\{N_k\}}{dx}\frac{d\{N_k\}^T}{dx}d\bar{x}\,\{y\}_e \checkmark$$
$$+ E\{I\}^T\int\frac{d\{N_k\}}{dx}\{N_k\}\frac{d\{N_k\}^T}{dx}d\bar{x}\,\{y\}_e$$
$$+ \int\{N_k\}\{N_k\}^T d\bar{x}\,\{M\}_e - \{N_k\}E\,\cancel{I(x)}\,\frac{dy^N}{dx}\Big|_{\partial\Omega_e}\Big)$$

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Solving for metrics, we know from previous homework that

$$\{N_k\}\frac{d\{N_k\}}{dx}\frac{d\{N_k\}^T}{dx} \Rightarrow \frac{1}{l_e}\begin{bmatrix} \begin{Bmatrix}1/2\\1/2\end{Bmatrix} & \begin{Bmatrix}-1/2\\-1/2\end{Bmatrix} \\ \begin{Bmatrix}-1/2\\-1/2\end{Bmatrix} & \begin{Bmatrix}1/2\\1/2\end{Bmatrix}\end{bmatrix}$$

metric $\Rightarrow (-1)$

$$\frac{d\{N_k\}^T}{dx} = \frac{1}{l_e}\{-1 \quad 1\} \checkmark$$

$$\{N_k\}\frac{d\{N_k\}^T}{dx} = \frac{1}{l_e}\begin{Bmatrix}1-\bar{x}/l_e \\ \bar{x}/l_e\end{Bmatrix}\{-1 \quad 1\}$$

$$= \frac{1}{l_e}\begin{bmatrix}\bar{x}/l_e - 1 & 1-\bar{x}/l_e \\ -\bar{x}/l_e & \bar{x}/l_e\end{bmatrix}$$

$$\frac{d\{N_k\}}{dx}\{N_k\}\frac{d\{N_k\}^T}{dx} = \frac{1}{l_e^2}\begin{Bmatrix}-1\\1\end{Bmatrix}\begin{bmatrix}\bar{x}/l_e - 1 & 1-\bar{x}/l_e \\ -\bar{x}/l_e & \bar{x}/l_e\end{bmatrix}$$

evaluating integral $= \frac{1}{l_e^2}\begin{Bmatrix}-1\\1\end{Bmatrix}\begin{bmatrix}l_e(\frac{1}{2}-1) & l_e(1-\frac{1}{2}) \\ l_e(-1/2) & l_e(1/2)\end{bmatrix}$

$$= \frac{1}{l_e}\begin{Bmatrix}-1\\1\end{Bmatrix}\begin{bmatrix}-1/2 & 1/2 \\ -1/2 & 1/2\end{bmatrix}$$

$$= \frac{1}{l_e}\begin{bmatrix} \begin{Bmatrix}1/2\\-1/2\end{Bmatrix} & \begin{Bmatrix}-1/2\\1/2\end{Bmatrix} \\ \begin{Bmatrix}1/2\\-1/2\end{Bmatrix} & \begin{Bmatrix}-1/2\\1/2\end{Bmatrix}\end{bmatrix}$$

metric $\Rightarrow (-1)$ ✓

great job!

So, putting our function into syntax...

$$[\text{DIFF}1]_e = E\{I\}_e^T \int \{N_k\}\frac{d\{N_k\}}{dx}\frac{d\{N_k\}^T}{dx}d\bar{x}$$

$$= (E)(\ )_e\{I\}_e^T(-1)[A3011k]\{\ \}_e \checkmark$$

$$[\text{DIFF}2]_e = E\{I\}_e^T \int \frac{d\{N_k\}}{dx}\{N_k\}\frac{d\{N_k\}^T}{dx}d\bar{x}$$

$$= (E)(\ )_e\{I\}_e^T(-1)[A3101k]\{\ \}_e \checkmark$$

$$\{Mom\}_e = \int \{N_k\}\{N_k\}^T d\bar{x}\{M\}_e \checkmark$$

$$= (\ )(\ )_e\{\ \}_e^T(1)[A200k]\{M\}_e$$

$$\{BSLP\} = \{N_k\} EI(x)\frac{dy^N}{dx}\Big|_{\partial\Omega_e} \checkmark$$

which gives eqn...

$$\underbrace{([DIFF1] + [DIFF2])}_{A}\underbrace{\{y\}_e}_{x} = \underbrace{-\{Mom\}_e + \{BSLP\}}_{b}$$

$$A \quad x = b$$

Awesome!

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