HOMEWORK ASSIGNED 3/14/02

PROBLEM 1

Verify that obtained nodal temperature solutions agree with

$$(9)\quad Q_1 = \frac{sL^2}{2k} + \frac{qL}{k} + T_b$$

$$Q_2 = \frac{3sL^2}{8k} + \frac{qL}{2k} + T_b$$

$$Q_3 = T_b$$

and that boundary heat fluxes agree with

$$(11)\quad -k\frac{dT^N}{dx}\bigg|_{x=L} = sL + q_{in}$$

Reorder

$$(3b)\quad \frac{k}{L/2}Q_2 - \frac{k}{L/2}Q_3 = \frac{sL/2}{2}$$

$$-\frac{k}{L/2}Q_2 + \frac{k}{L/2}Q_3 = \frac{sL/2}{2} + k\frac{dT^N}{dx}$$

to move Q_3 to the right side, $k\frac{dT^N}{dx}$ to the left side, then continue through to obtain a global matrix statement similar to

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \frac{L}{2k}\begin{bmatrix} \frac{sL}{4} + q \\ 2sL/4 \\ \frac{sL}{4} + k\frac{dT^N}{dx} \end{bmatrix}$$

where the column of unknowns has $k\,dT^N/dx$ as the bottom entry.

STARTING WITH

$$(3b)\quad \frac{k}{L/2}Q_2 = \frac{sL/2}{2} + \frac{k}{L/2}Q_3$$

$$\frac{-k}{L/2}Q_2 - \frac{k\,dT^N}{dx} = \frac{sL/2}{2} - \frac{k}{L/2}Q_3$$

$$(3a)\quad \frac{k}{L/2}Q_1 - \frac{k}{L/2}Q_2 = \frac{sL/2}{2} + q$$

$$\frac{-k}{L/2}Q_1 + \frac{k}{L/2}Q_2 = \frac{sL/2}{2}$$

PROBLEM 1

$$\checkmark\ \frac{2k}{L}Q_1 - \frac{2k}{L}Q_2 + 0\,\frac{k\,dT^N}{dx} = \frac{sL}{4} + q$$

$$\checkmark\ -\frac{2k}{L}Q_1 + \frac{2k}{L}Q_2 + 0\,\frac{k\,dT^N}{dx} = \frac{sL}{4} \quad \checkmark$$

$$\checkmark\ 0\,Q_1 + 0\,Q_2 + 0\,\frac{k\,dT^N}{dx} = 0$$

$$\checkmark\ 0\,Q_1 + 0\,Q_2 + 0\,\frac{k\,dT^N}{dx} = 0$$

$$\checkmark\ 0\,Q_1 + \frac{2k}{L}Q_2 + 0\,\frac{k\,dT^N}{d} = \frac{sL}{4} + \frac{2k}{L}Q_3 \quad \checkmark$$

$$0\,Q_1 - \frac{2k}{L}Q_2 - 1\,\frac{k\,dT^N}{dx} = \frac{sL}{4} - \frac{2k}{L}Q_3$$

$$\frac{2k}{L}Q_1 - \frac{2k}{L}Q_2 + 0\,\frac{k\,dT^N}{dx} = \frac{sL}{4} + q$$

$$-\frac{2k}{L}Q_1 + \frac{4k}{L}Q_2 + 0\,\frac{k\,dT^N}{dx} = \frac{sL}{2} + \frac{2k}{L}Q_3$$

$$0\,Q_1 - \frac{2k}{L}Q_2 - 1\,\frac{k\,dT^N}{dx} = \frac{sL}{4} - \frac{2k}{L}Q_3$$

$$\frac{2k}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\Big\} = \frac{sL}{4}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} q_{in} \\ 0 \\ 0 \end{bmatrix} \quad \checkmark \qquad \Big\} = \begin{bmatrix} Q_1 \\ Q_2 \\ k\frac{dT^N}{dx} \end{bmatrix}$$

$$\frac{2k}{L}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & \frac{-L}{2k} \end{bmatrix}\Big\} = \frac{sL}{4}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \frac{2k}{L}\begin{bmatrix} 0 \\ Q_3 \\ -Q_3 \end{bmatrix}$$

$$\frac{2k}{\cancel{L}}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & \frac{-L}{2k} \end{bmatrix}\Big\} = \frac{sL^2}{k\,4(2)}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \frac{\cancel{2k}}{\cancel{L}}\begin{bmatrix} \frac{L}{2k}q_{in} \\ Q_3 \\ -Q_3 \end{bmatrix}$$

PROBLEM 1

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & -1/2k \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ kdT^N/dx \end{bmatrix} = \frac{sL^2}{8k} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{L}{2k} q \\ T_b \\ -T_b \end{bmatrix}$$

Solve 3 eqn for 3 unknowns $Q_1, Q_2, (kdT^N/dx)$

✓ $Q_1 = \frac{1}{2k}\left[sL^2 + 2Lq + 2T_b k \right] = \left(\frac{sL^2}{2k} + \frac{Lq}{k} + T_b \right)$ ✓

✓ $Q_2 = \frac{1}{8k}\left[3sL^2 + 4Lq + 8T_b k \right] = \left(\frac{3sL^2}{8k} + \frac{Lq}{2k} + T_b \right)$

✓ $k\frac{dT^N}{dx} = -sL - q$

VERIFIED ☺

Woo-hoo!

Assigned 3/14/02

PROBLEM 2 Starting with

linear interpolation basis

$$GWS^h = \sum_{e=1}^{M} GWS_e^h = \sum_{e=1}^{M} \left(\frac{k}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \{Q_e\} - \frac{s l_e}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} - k \frac{dT^N}{dx} \begin{Bmatrix} \delta_{e1} \\ \delta_{eM} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \right)$$

repeat the assembly and solution process for a 3 element (4 node) uniform discretization of the domain (0, L). Include a plot of the exact solution and the M=2 and M=3 finite element results for problem data specified in the text. p 32, problem 1.

Be sure that nodes are connected by linear functions. Comment on solution accuracy and effort required.

APPLYING 3-elements:

$$GWS_1^h = \frac{k}{L/3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} - \frac{s L/3}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} - q \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$GWS_2^h = \frac{k}{L/3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} Q_2 \\ Q_3 \end{Bmatrix} - \frac{s L/3}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} - k \frac{dT^N}{dx} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$GWS_3^h = \frac{k}{L/3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} Q_3 \\ Q_4 \end{Bmatrix} - \frac{s L/3}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} - k \frac{dT^N}{dx} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

✓

LINEAR ALGEBRAIC:

$e=1$

$$\frac{k}{L/3}(Q_1 - Q_2) = \frac{s L/3}{2} + q$$

$$\frac{k}{L/3}(-Q_1 + Q_2) = \frac{s L/3}{2}$$

$e=2$

$$\frac{k}{L/3}(Q_2 - Q_3) = \frac{s L/3}{2}$$

$$\frac{k}{L/3}(-Q_2 + Q_3) = \frac{s L/3}{2}$$

✓

$e=3$

$$\frac{k}{L/3}(Q_3 - Q_4) = \frac{s L/3}{2}$$

$$\frac{k}{L/3}(-Q_3 + Q_4) = \frac{s L/3}{2} + k \frac{dT^N}{dx}$$

PADDING THE EQUATIONS:

$e=1$

$$\frac{k}{L/3}\begin{pmatrix} Q_1 - Q_2 + 0Q_3 + 0Q_4 \\ -Q_1 + Q_2 + 0Q_3 + 0Q_4 \\ 0Q_1 + 0Q_2 + 0Q_3 + 0Q_4 \\ 0Q_1 + 0Q_2 + 0Q_3 + 0Q_4 \end{pmatrix} \Rightarrow \frac{3k}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix}$$

$e=2$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$e=3$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

ASSEMBLY:

$$\therefore \frac{3k}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & (1+1) & -1 & 0 \\ 0 & -1 & (1+1) & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \frac{sL}{6}\begin{Bmatrix} 1 \\ 1+1 \\ 1+1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} q \\ 0 \\ 0 \\ k\,dT^N/dx \end{Bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \frac{L}{3k}\begin{Bmatrix} sL/6 + q \\ sL/3 \\ sL/3 \\ sL/6 + k\,dT^N/dx \end{Bmatrix}$$

IMPOSING DIRICHLET:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} L/3k\,(sL/6 + q) \\ sL^2/9k \\ sL^2/9k \\ T_b \end{Bmatrix}$$

SOLVE FOR NODAL VALUES:

$$Q_1 = \frac{1}{2k}\left[sL^2 + 2Lq + 2T_b k\right] = \frac{sL^2}{2k} + \frac{Lq}{k} + T_b$$

$$Q_2 = \frac{1}{9k}\left[4sL^2 + 6Lq + 9T_b k\right] = \frac{4}{9}\frac{sL^2}{k} + \frac{2}{3}\frac{Lq}{k} + T_b$$

$$Q_3 = \frac{1}{18k}\left[5sL^2 + 6Lq + 18T_b k\right] = \frac{5}{18}\frac{sL^2}{k} + \frac{1}{3}\frac{Lq}{k} + T_b$$

$$Q_4 = T_b$$

p.32 data

Determine actual nodal temperatures for

$q(0) = 1500\ Btu/h\cdot ft^2$

$s = -1400\ Btu/ft^3\cdot h$

$k = 4\ Btu/h\cdot °F\cdot ft$

$T_b(L) = 0°F$

SOLVE FOR $\frac{k\,dT^N}{dx}$:

$$-Q_3 + Q_4 = \frac{L}{3k}\left(\frac{sL}{6} + k\,\frac{dT^N}{dx}\right)$$

$$k\,\frac{dT^N}{dx} = -sL - q$$

w/ given data

$Q_1 = 200°F$

$Q_2 = 94.4°F$

$Q_3 = 27.8°F$

$Q_4 = 0°F$

Plot needed w/ comparison to 3 node solution

Comments: When doing so much by hand its easy to miss a sign or write/type something incorrectly & much more difficult to find the ~~error~~ mistake. imagine doing a 20 element grid!

In other news, for this function, increasing the number of nodes decreases the error by a noticable (see plot) amount, while doubling the effort req'd to obtain it. Makes me think that there must be another way to do it.

Excellent Job!

```
[ >
```

3/14/02 Problem 1 (Done)

```
> restart;
> e1:=Q1-Q2=s*L^2/8/k+L/2/k*q;
  e2:=-Q1+2*Q2=2*s*L^2/8/k+Tb;
  e3:=-Q2-L/2/k*k_dTN_dx=s*L^2/8/k-Tb;
```

$$e1 := Q1 - Q2 = \frac{1}{8}\frac{s\,L^2}{k} + \frac{1}{2}\frac{L\,q}{k}$$

$$e2 := -Q1 + 2\,Q2 = \frac{1}{4}\frac{s\,L^2}{k} + Tb$$

$$e3 := -Q2 - \frac{1}{2}\frac{L\,k\_dTN\_dx}{k} = \frac{1}{8}\frac{s\,L^2}{k} - Tb$$

```
> solve({e1,e2,e3},{Q1,Q2,k_dTN_dx});
```

$$\{Q2 = \frac{1}{8}\frac{3\,s\,L^2 + 4\,L\,q + 8\,Tb\,k}{k},\ Q1 = \frac{1}{2}\frac{s\,L^2 + 2\,L\,q + 2\,Tb\,k}{k},\ k\_dTN\_dx = -s\,L - q\}$$

```
[ >
```

3/14/02 Problem 2 (Done)

```
> restart;
> e1:=Q1-Q2=L/3/k*(s*L/6+q);
  e2:=-Q1+2*Q2-Q3=s*L^2/9/k;
  e3:=-Q2+2*Q3-Q4=s*L^2/9/k;
  e4:=Q4=Tb;
```

$$e1 := Q1 - Q2 = \frac{1}{3}\frac{L\left(\frac{1}{6}s\,L + q\right)}{k}$$

$$e2 := -Q1 + 2\,Q2 - Q3 = \frac{1}{9}\frac{s\,L^2}{k}$$

$$e3 := -Q2 + 2\,Q3 - Q4 = \frac{1}{9}\frac{s\,L^2}{k}$$

$$e4 := Q4 = Tb$$

```
> sol1:=solve({e1,e2,e3,e4},{Q1,Q2,Q3,Q4});
```

$$sol1 := \{Q4 = Tb,\ Q3 = \frac{1}{18}\frac{5\,s\,L^2 + 6\,L\,q + 18\,Tb\,k}{k},\ Q2 = \frac{1}{9}\frac{4\,s\,L^2 + 6\,L\,q + 9\,Tb\,k}{k},$$

$$Q1 = \frac{1}{2}\frac{s\,L^2 + 2\,L\,q + 2\,Tb\,k}{k}\}$$

```
> assign(sol1);
```

```
> e5:=-Q3+Q4=L/3/k*(s*L/6+k_dTN_dx);
```

$$e5 := -\frac{1}{18}\frac{5\,s\,L^2 + 6\,L\,q + 18\,Tb\,k}{k} + Tb = \frac{1}{3}\frac{L\left(\frac{1}{6}s\,L + k\_dTN\_dx\right)}{k}$$

```
> sol2:=solve(e5,k_dTN_dx);
```

$$sol2 := -s\,L - q$$

```
> q:=1500;s:=-1400;k:=4;Tb:=0;L:=1;
```

$$q := 1500$$

$$s := \text{-}1400$$

$$k := 4$$

$$Tb := 0$$

$$L := 1$$

```
> Q1;
```

$$200$$

```
> evalf(Q2);
```

$$94.44444444$$

```
> evalf(Q3);
```

$$27.77777778$$

```
> Q4;
```

$$0$$

```
> eq1:=s*L^2/2/k*(1-(x/L)^2)+q*L/k*(1-(x/L))+Tb;
```

$$eq1 := 200 + 175\,x^2 - 375\,x$$

```
> with(plots):
> p0:=plot(eq1,x=0..1,color=black):
> p1:=pointplot({[0,Q1],[L/3,Q2],[2*L/3,Q3],[L,Q4]},color=blue,sy
mbol=box):p2:=pointplot({[0,s*L^2/2/k+q*L/k+Tb],[L/2,3*s*L^2/8/
k+q*L/2/k+Tb],[L,Tb]},color=red,symbol=circle):
```

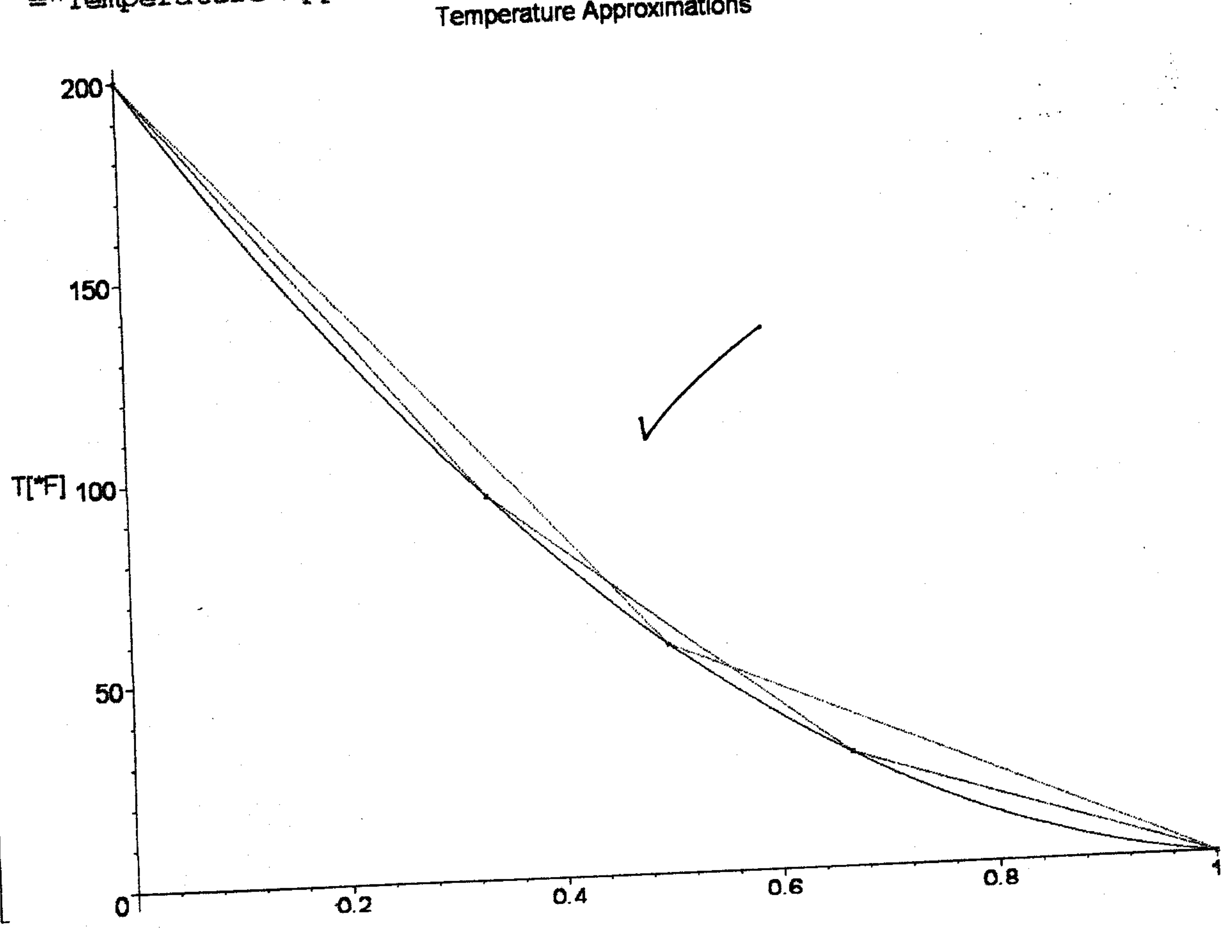
y = mx + b
m[redacted]=(y1-y2)/(x1-x2)
y2 = (y1-y2)x2/(x1-x2) +b
b = y2 - x2(y1-y2)/(x1-x2)
y = (y1-y2)/(x1-x2) x + y2 - x2(y1-y2)/(x1-x2)
y (x1-x2) - y2 (x1-x2) = x (y1-y2) - x2 (y1-y2)
y (x1-x2) - y2 (x1-x2) = (x - x2) (y1-y2)
y - y2 = (x-x2) (y1-y2)/(x1-x2)
y = (x-x2) * (y1-y2)/(x1-x2) + y2

```
> piece1:=plot(piecewise(x<1/3,
  (x-1/3)*
  (Q1-Q2)/(0-1/3)
 +Q2,x<2/3,
  (x-2/3)*
```

```
(Q2-Q3)/(1/3-2/3)
+Q3,x<1,
(x-1)*
(Q3-Q4)/(2/3-1)
+Q4),x=0..1
,color=blue):
piece2:=plot(piecewise(x<1/2,
(x-1/2)*
(s*L^2/2/k+q*L/k+Tb-(3*s*L^2/8/k+q*L/2/k+Tb))/(0-1/2)
+(3*s*L^2/8/k+q*L/2/k+Tb),x<1,
(x-1)*
(3*s*L^2/8/k+q*L/2/k+Tb-Tb)/(1/2-1)
+Tb),x=0..1):
```

provide plot of points for M=2 and M=3 with eq1

```
> display({p0,p1,p2,piece1,piece2},labels=["x[ft]","T[*F]"],title
  ="Temperature Approximations");
```



[>

3/15/02 Problem 1 is done, by hand.
3/15/02 Problem 2 is done, but not fully understood, by hand.

[>

HOMEWORK ASSIGNED 3/15/02

PROBLEM 1

DETERMINE MASTER MATRICES [A200L] AND [A211L]

A200L: 1-dimensional matrix
2- bases in integral
(neither is differentiated) ✓
L - linear basis function

$$\int_{\Omega_e} \{N_1\}\{N_1\}^T d\bar{x}$$

$$\underset{2\times1}{\{N_1\}} = \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} 1-\bar{x}/l_e \\ \bar{x}/l_e \end{Bmatrix} \qquad \underset{1\times2}{\{N_1\}^T} = \{\delta_1 \;\; \delta_2\} = \{1-\bar{x}/l_e \;\; \bar{x}/l_e\}$$ ✓

$$\int_{\Omega_e} \begin{bmatrix} (1-\bar{x}/l_e)^2 & \bar{x}/l_e(1-\bar{x}/l_e) \\ \bar{x}/l_e(1-\bar{x}/l_e) & (\bar{x}/l_e)^2 \end{bmatrix} d\bar{x}$$ ✓

$$\int \begin{bmatrix} 1-2\,\bar{x}/l_e+(\bar{x}/l_e)^2 & \bar{x}/l_e-(\bar{x}/l_e)^2 \\ \bar{x}/l_e-(\bar{x}/l_e)^2 & (\bar{x}/l_e)^2 \end{bmatrix} d\bar{x}$$

$$\left.\begin{bmatrix} \bar{x}-\bar{x}^2/l_e+\bar{x}^3/3l_e^2 & \bar{x}^2/2l_e-\bar{x}^3/3l_e^2 \\ \bar{x}^2/2l_e-\bar{x}^3/3l_e^2 & \bar{x}^3/3l_e^2 \end{bmatrix}\right|_{\Omega_e=0,\,l_e}$$

$$\begin{bmatrix} \bar{x}\left(1-\bar{x}/l_e+\tfrac{1}{3}(\bar{x}/l_e)^2\right) & \bar{x}\left(\tfrac{1}{2}(\bar{x}/l_e)-\tfrac{1}{3}(\bar{x}^2/l_e^2)\right) \\ \bar{x}\left(\tfrac{1}{2}(\bar{x}/l_e)-\tfrac{1}{3}(\bar{x}/l_e)^2\right) & \bar{x}\left(\tfrac{1}{3}(\bar{x}/l_e)^2\right) \end{bmatrix}$$

$$l_e\begin{bmatrix} 1-1+\tfrac{1}{3}(1) & \tfrac{1}{2}(1)-\tfrac{1}{3}(1) \\ \tfrac{1}{2}(1)-\tfrac{1}{3}(1) & \tfrac{1}{3}(1) \end{bmatrix}$$

$$l_e\begin{bmatrix} \tfrac{1}{3} & \tfrac{1}{6} \\ \tfrac{1}{6} & \tfrac{1}{3} \end{bmatrix} \Rightarrow \text{A200L}$$ ✓

Rad!

PROBLEM 1

AZ11L:

1- dimensional matrix ($k=1$)
2- bases in integral
(both differentiated)
L- linear basis function

$$\int_{\Omega_e} \frac{d\{N_1\}}{dx} \frac{d\{N_1\}^T}{dx} d\bar{x}$$ ✓

$$\{N_1\} = \begin{matrix} \delta_1 \\ \delta_2 \end{matrix} = \begin{matrix} 1-\bar{x}/l_e \\ \bar{x}/l_e \end{matrix} \qquad \frac{d}{dx} = \begin{matrix} -1/l_e \\ 1/l_e \end{matrix}$$

$$\begin{pmatrix} -1/l_e \\ 1/l_e \end{pmatrix} \begin{pmatrix} -1/l_e & 1/l_e \end{pmatrix} = \begin{matrix} 1/l_e^2 & -1/l_e^2 \\ -1/l_e^2 & 1/l_e^2 \end{matrix}$$

$$\frac{1}{l_e^2}\int_{\Omega_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d\bar{x} \checkmark = \frac{1}{l_e^2}\begin{bmatrix} \bar{x} & -\bar{x} \\ -\bar{x} & \bar{x} \end{bmatrix}\Bigg|_{l_e} = \frac{1}{l_e^2}\begin{bmatrix} l_e & -l_e \\ -l_e & l_e \end{bmatrix}$$ ✓

$$AZ11L \Rightarrow \frac{1}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ ✓

Gnarly!

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
AMPAD

ASSIGNED 3/15/02

PROBLEM 2:

Apply recipie steps 1-6, ie. eq (12)

$$GWS^h = S_e\, GWS_e^h$$

$$= S_e\left(\int_{\Omega_e} \frac{d\{N_k\}}{dx} k(x) \frac{d\{N_k\}^T}{dx} d\bar{x} \{Q\}_e - \int_{\Omega_e} \{N_k\} s(x) d\bar{x} - k(x)\frac{dT^N}{dx}\begin{Bmatrix}\delta_{e1}\\ \delta_{eM}\end{Bmatrix} = \{0\}\right)$$

for the following ODE & BC set

(a) $\mathcal{L}(h) = \dfrac{d}{dx}\left(\dfrac{1}{Pa}\dfrac{dh(x)}{dx}\right) - uh(x) = 0 \qquad \Omega \in (0, L)$

(b) $l(h) = \dfrac{dh(x)}{dx} = 0 \qquad \partial\Omega \in L$

(c) $h = h_0 \qquad \partial\Omega \in 0$

---

(1) $h(x) \approx h^N(x) = \sum_{\alpha=1}^{N} \Psi_\alpha(x) Q_\alpha$ $\qquad \int_0^L u\,dv = -\int_0^L v\,du + uv\Big|_0^L$

(2) $e^N(x) = h(x) - h^N(x)$

$\mathcal{L}\{e^N\} = \mathcal{L}\{h\} - \mathcal{L}\{h^N\} = -\mathcal{L}\{h^N\}$

$WS^N = \int_\Omega \Phi_\beta \mathcal{L}(h^N)\,dx \qquad GWS^N = \int_\Omega \Psi_\beta \mathcal{L}(h^N)\,dx$

$$GWS^N = \int_\Omega \Psi_\beta \left[\frac{d}{dx}\left(\frac{1}{Pa}\frac{dh(x)}{dx}\right) - uh(x)\right] dx = 0$$

Substitution from (1)

$$= \int_\Omega \Psi_\beta \left[\frac{d}{dx}\left(\overset{\text{const}}{\frac{1}{Pa}}\frac{d\Psi_\alpha Q_\alpha}{dx}\right) - u\Psi_\alpha Q_\alpha\right] dx = 0$$

Integrate by parts

$u = \Psi_\beta \qquad dv = \dfrac{1}{Pa}\dfrac{d}{dx}\left(\dfrac{d\Psi_\alpha Q_\alpha}{dx}\right)$

$du = \dfrac{d\Psi_\beta}{dx} \qquad v = \dfrac{1}{Pa}\left(\dfrac{d\Psi_\alpha Q_\alpha}{dx}\right)$

$$= \int_\Omega \left[-\frac{1}{Pa}\left(\frac{d\Psi_\alpha Q_\alpha}{dx}\right)\frac{d\Psi_\beta}{dx} - u\Psi_\alpha Q_\alpha\right] dx + \Psi_\beta \frac{1}{Pa}\left(\frac{\partial \Psi_\alpha Q_\alpha}{dx}\right)\Bigg|_{\partial\Omega}^{L} = 0$$

PREVIOUSLY

$$\int_{\Omega} \left[-\frac{d\Psi_\beta}{dx}\left(\frac{1}{Pa}\ \frac{d\Psi_\alpha}{dx}\right) - u\Psi_\alpha \right] dx\, Q_\alpha + \Psi_\beta \frac{1}{Pa}\left(\underbrace{\frac{d\Psi_\alpha Q_\alpha}{dx}}_{\frac{dh^N}{dx}}\right)\Bigg|_0^L$$

DESCRITIZATION $\Rightarrow$ $(\Psi_\alpha Q_\alpha)_e = \{N_K(\mathcal{Y}_i)\}^T \{Q\}_e$

$(\Psi_\beta)_e = \{N_K(\mathcal{Y}_i)\}$ ✓

FOR ASSEMBLY $\Rightarrow$ $\Psi_\alpha Q_\alpha = S_e\left(\{N_K(\mathcal{Y}_i)\}^T \{Q\}_e\right)$

$\quad$; $GWS^h = S_e\, GWS_e^h$

$$GWS_e^h = \int_{\Omega_e} \left[-\frac{d\{N_K\}}{dx}\left(\frac{1}{Pa}\ \frac{d\{N_K\}^T}{dx}\right)\right] d\bar{x}\{Q\}_e \ + \checkmark$$

$$\int_{\Omega_e} \left[-u\{N_K\}^T \right] d\bar{x}\,\{Q\}_e \ + \checkmark$$

$$\left\{ \quad \frac{1}{Pa}\frac{dh^N}{dx}\begin{Bmatrix}\delta_{e1}\\ \delta_{em}\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix} \right.$$

this I don't understand

$$\therefore\ GWS^h = S_e\left[-\int_{\Omega_e} \frac{d\{N_K\}}{dx}\frac{1}{Pa}\frac{d\{N_K\}^T}{dx} d\bar{x}\{Q\}_e - \int_{\Omega_e} u\{N_K\}^T d\bar{x}\,\{Q\}_e + \frac{1}{Pa}\frac{dh^N}{dx}\begin{Bmatrix}\delta_{e1}\\ \delta_{em}\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix}\right] \checkmark$$

do you see it now?

Awesome!