ME 410   Day 27

- Description of Finite Heat Release Function - SI Engines
- Differential Equations to Model Cycle
- Software Implementation in EES
- Questions that can be answered

1. Description of Finite Heat Release Function

This is an attempt to model more realistically the burn rates of fuel in the cylinder during the working cycle. The key is to specify heat addition rate as a function of crank angle.

Naturally, heat addition rate is zero for much of the cycle. We start with an "S" curve or Weibe function which describes the fraction of fuel that has been burned.
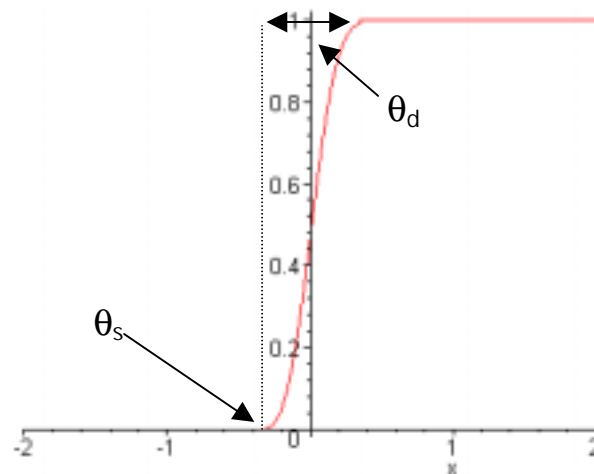
Here is the math function which describes the burned fraction.

$$x_b(\theta) = 1 - \exp\left(-a\left(\frac{\theta - \theta_s}{\theta_d}\right)^n\right)$$

The shape is determined by the parameters a and n. Values of a = 5 and n = 3 have been found to agree well with experiments. See pages 766 to 769 in the Text. Another ref: Internal Combustion Engines by Ferguson and Kirkpatrick, Wiley 2001.

How it can be used.

$$\frac{dQ}{d\theta} = Q^* \frac{dx_b}{d\theta}$$

$$\frac{dQ}{d\theta} = an \frac{Q^*}{\theta_d} (1 - x_b) \left(\frac{\theta - \theta_s}{\theta_d} \right)^{n-1}$$

This is a very useful description of how heat input rate varies with θ during combustion. Naturally, when the combustion process is not occurring, this derivative is zero.

2. Differential Equations to Model Cycle

Start with the Ideal Gas Law

$$PV = mRT$$

and differentiate with respect to q.

$$mR \frac{dT}{d\theta} = P \frac{dV}{d\theta} + V \frac{dP}{d\theta}$$

Next, write the first law of thermo for this closed system in rate form.

$$\delta Q - \delta W = dU$$

and make the following substitutions.

$$\delta W = P \, dV$$

$$dU = m \, c_v dT$$

After some hand waving,

$$\frac{dQ}{d\theta} - P\frac{dV}{d\theta} = \frac{c_v}{R}\left(P\frac{dV}{d\theta} + V\frac{dP}{d\theta}\right)$$

Next solve for the derivative of pressure with respect to θ.

$$\frac{dQ}{d\theta} - P\left(1 + \frac{c_v}{R}\right)\frac{dV}{d\theta} = \frac{c_v}{R}V\frac{dP}{d\theta}$$

$$\frac{dP}{d\theta} = \frac{R}{c_v V}\frac{dQ}{d\theta} - P\frac{R}{c_v V}\left(1 + \frac{c_v}{R}\right)\frac{dV}{d\theta}$$

$$\frac{dP}{d\theta} = \frac{c_p - c_v}{c_v V}\frac{dQ}{d\theta} - \frac{P}{V}\left(\frac{c_p - c_v}{c_v} + 1\right)\frac{dV}{d\theta}$$

$$\frac{dP}{d\theta} = \frac{\gamma - 1}{V}\frac{dQ}{d\theta} - \gamma\frac{P}{V}\frac{dV}{d\theta}$$

Next, we need the following expressions from kinematics. See our text, page 44.

$$V = V_c + \frac{V_c}{2}(r_c - 1)\left[R + 1 + \cos(\theta) - \sqrt{R^2 - \sin^2(\theta)}\right]$$

Now R no longer stands for the gas constant, it now stands for the ratio of connecting rod length to crank radius. V_c is the clearance volume and V the volume of the cylinder at θ.

Since $V_d = (r_c - 1)V_c$ we can write this as

$$V = \frac{V_d}{(r_c - 1)} + \frac{V_d}{2}\left[R + 1 + \cos(\theta) - \sqrt{R^2 - \sin^2(\theta)}\right]$$

The rate at which volume is swept out as a function of crank angle, we get by differentiating with respect to θ.

$$\frac{dV}{d\theta} = \frac{V_d}{2}\sin(\theta)\left[1 + \frac{\cos(\theta)}{\sqrt{R^2 - \sin^2(\theta)}}\right]$$

Finally, we recall the definition of work gives us

$$\frac{dW}{d\theta} = P\frac{dV}{d\theta}$$

3. Software Implementations

We conceptualize this problem mathematically as 4 simultaneous differential equations in the independent variable q and having dependent variables V, P, T and W.

These equations must then be solved numerically using a time stepping scheme.

The "integral" function in EES invokes the time stepping scheme which is described in the software help.

The EES file is included in these notes.

"Finite Heat Release Calculations"

"Reference: Internal Combustion Engines by Colin Ferguson and Allan Kirkpatrick, 2nd Edition. John Wiley 2001. Pages 39 44."

"The heat release occurs over a finite time given by a Weibe function. The heat release rate is calculated and used to write the first law of thermo in rate form. (Rate of change with respect to crank angle) The ideal gas law is used to relate pressure and temperature rates. The kinematics of the engine give volumetric rates. The result is a set of differential equations which may be integrated to describe the cycle."

"Function to calculate burn fraction vs. crank angle. Crank angle varies from -pi at start of cycle to 0 at TC to pi at the end of the cycle. Exhaust processes are not modeled."

```
function xb(theta)
{ theta_s is start of combustion in radians, theta_d is the duration, a and n are the
  Weibe parameters.}
$common  theta_s, theta_d,a,n
    if (theta < theta_s)  then
         xb := 0
    else
         if (theta > theta_s + theta_d) then
              xb := 1
         else
              xb := 1-exp(- a*((theta-theta_s)/theta_d)^n)
         endif
    endif
end
```

"Function to calculate heat release rate"

```
function dQdtheta(theta)
{ theta_s is start of combustion in radians, theta_d is the duration, a and n are the
  Weibe parameters. Q is the total heat release }
$common  theta_s, theta_d,a,n,Q
```

```
    if (theta < theta_s) then
        dQdtheta := 0
    else
        if (theta > theta_s+theta_d) then
            dQdtheta := 0
        else
            dQdtheta := n*a*Q/theta_d*(1-xb(theta))*((theta-
theta_s)/theta_d)^(n-1)
        endif
    endif
end
```

"Function to calculate volume rate wrt crank angle"

```
function dVdtheta(theta)
$common V_d, r_c, R
dVdtheta := V_d/2 * sin(theta)*(1+cos(theta)/sqrt(R^2 -
sin(theta)^2))
end
```

"Function to calculate volume at a given crank angle"

```
function V(theta)
$common V_d, r_c, R
V := V_d/(r_c-1) + V_d/2 *(R + 1 - cos(theta) - sqrt(R^2-
sin(theta)^2))
end
```

"Combustion Data"

```
theta_s = -pi/9
theta_d = 4*pi/9
a = 5
n = 3
Q = 1.8
```

"Engine Data"

```
bore = 0.1
stroke = 0.1
```

```
P_1 = 101.3
r_c = 10
V_d = pi /4 * bore^2 * stroke
L = 0.150
R = 2*L/stroke
gamma = 1.4
T_1 = 300
V_1 = V(theta_1)
mR = P_1*V_1 / T_1
Mw = 29
theta_1 = -pi
theta_f = pi

vv = V(theta)
```

"The derivatives of Pressure and Temperature, Work and Vol with respect to crank angle"

```
dPdtheta = -gamma*P/V(theta) * dVdtheta(theta) + (gamma-1)/V(theta)*dQdtheta(theta)
dTdtheta = 1/mR * ( P * dVdtheta(theta) + V(theta)*dPdtheta )
dWdtheta = P * dVdtheta(theta)
dVoldtheta = dVdtheta(theta)
```

"We can now integrate the above differential equations."

```
P = P_1 + integral(dPdtheta,theta,theta_1,theta_f)
T = T_1 + integral(dTdtheta,theta,theta_1,theta_f)
W =       integral(dWdtheta,theta,theta_1,theta_f)
Vol = V_1 + integral(dVoldtheta,theta,theta_1,theta_f)

$IntegralTable theta:0.1,P,T,W,Vol
```

"Performance Calculations"

```
eta_f = W / Q
imep = W / V_d
```

4. Questions that can be answered

This model is useful in the sense that it allows us to study the effects of spark timing on engine performance.