Lab 7

EXPERIMENTALLY IDENTIFYING THE TIME CONSTANT AND CONVECTION COEFFICIENT OF A THERMOCOUPLE

OBJECTIVES

At the conclusion of this experiment, students should be able to:

- Estimate the time constant of a first-order system using three methods.
- Explain the log-incomplete response method of determining time constants, explain the performance index method of determining time constants, and describe the differences between the two methods.
- Experimentally determine the convection coefficient of a thermocouple.

DELIVERABLES

The deliverables of this experiment are:

- The lab worksheet. Fill in the blanks and answer the questions in a more than superficial manner.
- A plot of the experimental step response, $T_m(t)$, showing 1-τ, 2-τ, and 3-τ estimates of the time constant.
- A plot of the log-incomplete response, $Z(t)$, with the linear least-squares curve-fit showing the slope.
- A plot of the experimental data and the two performance index fits on the same graph.
- A plot comparing the experimental step response, $T_m(t)$, to the three theorectical responses: 1) using τ from the 1-τ estimate, 2) using τ from the log-incomplete response, and 3) using τ from the performance index.

NOMENCLATURE

A	bead surface area	T_0	initial bead temperature
h	convective heat transfer coefficient	T_{SS}	steady-state bead temperature
$\dot{Q}$	rate of heat transfer	T_∞	fluid temperature
ρ	bead density	τ	system time constant
T	bead temperature	$\not{V}$	bead volume
T_m	measured bead temperature		

INTRODUCTION

Identification is a process in which experimental measurements are used to draw inferences about the characteristics of a system by comparing experimental results to predictions from a mathematical model. In *system identification* (or *system ID*), the inferences involve system-level characteristics such as time constants, steady-state gains, natural frequencies, or damping ratios. In *parameter ID*, the inferences involve system parameters or coefficients such as spring constants, motor torque- and voltage-constants, damping coefficients, or, as in this experiment, convection coefficients.

In this experiment, a thermocouple is subjected to a step temperature input. The response is measured and the data are manipulated to obtain an estimate of the system's time constant. This process is an example of *system ID*. From this time constant an estimate is made of the convective heat transfer coefficient between the surface of the thermocouple bead and the fluid in which it is immersed. This process is an example of *parameter ID*. The convective heat transfer coefficient is compared to published values.

THEORY

The model of a thermocouple bead has been derived in class. As illustrated in Fig. 1, the system is the bead, the principle is the conservation of energy, and the assumptions are that conduction through the wire leads and radiation heat transfer are negligible, and that the temperature T of the bead is uniform (lumped capacitance assumption).

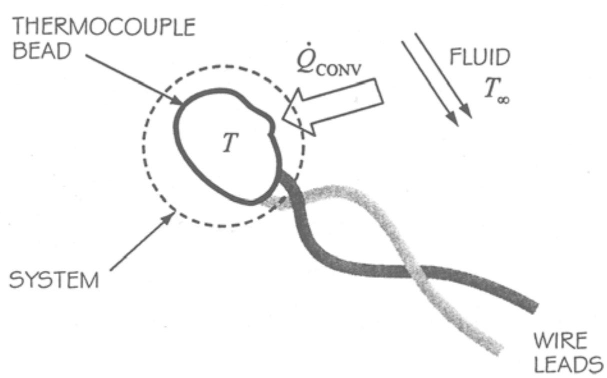

Fig. 1: Convective heat-transfer model of a thermocouple.

Model

Applying the conservation of energy to this system, the model is given by

$$\rho C_v \not{V} \dot{T} = hA\left(T_\infty - T\right) \tag{1}$$

where the bead properties are density ρ, thermal capacitance C_v, volume $\not{V}$ and surface area A, and where h is the unknown convective heat transfer coefficient. Rewriting (1) in standard form yields the time constant given by

$$\tau = \frac{\rho C_v \forall}{hA} \tag{2}$$

By obtaining a time-constant estimate from the experimental step response, and knowing the bead properties, the convective heat transfer coefficient *h* is determined from (2).

The known step-response solution to (1), a first-order ODE, is given by

$$T(t) = T_{SS} + \left(T_0 - T_{SS}\right)e^{-t/\tau} \tag{3}$$

where T_0 is the initial temperature of the thermocouple and T_{SS} is its steady-state value.

Log-incomplete response

The theory underlying the log-incomplete response was developed in a previous handout. Applying this theory to (3), the log-incomplete response function $Z(t)$ is given by

$$Z(t) = \ln\left(\frac{T_m(t) - T_{ss}}{T_0 - T_{ss}}\right) = -\frac{t}{\tau} \tag{4}$$

where $T_m(t)$ is the experimental response. It follows from (4) that the time constant is the negative inverse of the slope of $Z(t)$.

Performance Index

Given a mathematical model of a system response and an estimate of the time constant τ, the predicted values for temperature $T(t)$ can be computed over a range of time values *t*. The predicted temperature is compared to the measured temperature at each measured time step. The error between the theoretical and experimental values at each time step is squared and added over the entire time domain to create a performance index, *J*. "Tuning" the model is the process of varying τ until *J* is minimized. The performance index, *J*, is defined by

$$J(\tau) = \sum_{t_0}^{t_f}\left[T_m(t) - T(t,\tau)\right]^2 \tag{5}$$

where T_m is the measured temperature, *T* is the temperature predicted by the model based on a selected value of τ, and t_0 and t_f are initial and final values of time.

APPARATUS

A schematic of the experimental setup is shown in Fig. 2. A thermocouple is taken from an ice bath, near 0ºC, and is quickly placed in a beaker of hot water near 100ºC. This change in fluid temperature closely approximates a step input to the thermocouple. The thermocouple wire leads are connected to a computer-based data acquisition system, which records time in seconds and the transient response in Volts.

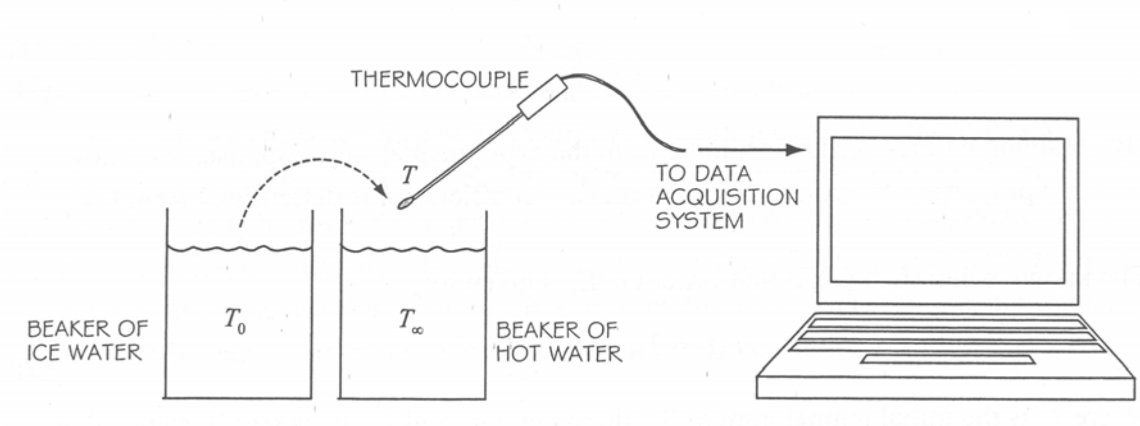

Fig. 2: Apparatus for imposing a step input on a thermocouple and measuring the response.

Procedure

- The water in the beaker is brought to a boil using a hot plate.
- Calibrate the thermocouple using the ice water bath and the boiling water. Take a steady-state voltage reading in the ice water bath (~0 °C) and another steady-state voltage reading in the boiling water (~100 °C). Use these two data points to linearly relate the thermocouple voltage reading to temperature.
- With the thermocouple at steady-state in ice water, the data acquisition is started. Real-time results are displayed using the data acquisition system with the computer.
- A step input to the system is created by quickly changing T_∞ from a low temperature (ice water, near 0ºC) to a high temperature (boiling water near 100ºC).
- Data acquisition is stopped after the temperature reaches steady-state.

Preliminary data reduction

The data file generated by the data acquisition system will be in a comma separated value (CSV) format. The data acquisition system records elapsed time and the thermocouple voltage output. The following preliminary data reduction needs to be completed before moving on to the detailed analysis:

- Convert the voltage measurements into temperature using your calibration.
- Delete the data points prior to the step input so that the first time measurement is at the beginning of the step input.
- Subtract a constant Δt from the measured time values so that the time vector starts at $t = 0$.
- The response reaches 98% of its final value over an interval of four time constants. Delete, or better yet, keep but ignore, data after approximately 4τ so that your log-incomplete response data is not corrupted by noise.

You can perform these steps by opening the *.csv file in Excel.

DETERMINING THE SYSTEM TIME CONSTANT

Method 1: Time constant from the step-response graph

Discussion

After you complete the preliminary data reduction in the previous section, save it as an Excel spreadsheet so that you may work with the data (perform calculations and make plots) and determine the model characteristics and system parameters.

Procedure

1. Load the Excel file containing your data and create a plot of Temperature vs. Time. Figure 3 shows and example of this step.
2. Get a hardcopy of the graph you created by printing this figure.
3. Estimate the initial condition T_0 and the steady-state value T_{SS}. Record these values on the lab worksheet.
4. Use the graph to estimate an average time constant using values at approximately τ, 2τ, 3τ, and so forth. Show your work on the graph, by hand. Record your average time constant on the lab worksheet.

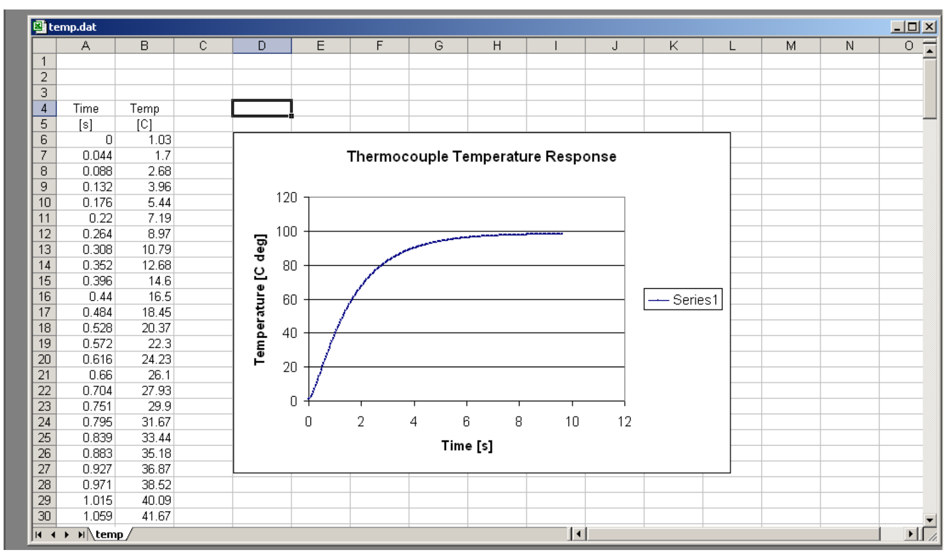

Fig. 3 Plot of thermocouple data file, `temp.dat`.

Method 2: Time constant from the log-incomplete response plot

Discussion

You are to manipulate the data using the Excel spreadsheet so that you will be able to plot the incomplete response curve and use it to find the time constant, τ, of the thermocouple system. From a plot of $Z(t)$ vs. time you will be able to determine the slope of the linear-least-squares curve, from which you can obtain an estimate of the time constant. Recall that the incomplete response only uses data points when time is less than $4\,\tau$. Once the incomplete response curve is found this will be compared to the actual data set.

Procedure

1. Estimate the initial value of the Temperature response, T_0.
2. Estimate the steady state value of the temperature response data, T_{ss}.
3. Estimate what time corresponds to $4\,\tau$.
4. Set up a column in Excel to calculate the incomplete response

$$Z(t) = \ln\left(\frac{T_m(t) - T_{SS}}{T_0 - T_{SS}}\right) = -\frac{t}{\tau}$$

   Use only the data points which fall below $4\,\tau$.
5. Create a plot of $Z(t)$ vs. t and use a least squares fit to determine the slope and intercept of the line. Select the option which forces the curve to pass through the origin. An example is provided in Figure 4.
6. From the slope determine the estimate of the time constant, τ. Record this value on your worksheet.
7. Explore the consequences of varying T_0 and T_{ss}. Can you obtain a better curve-fit? When the linear least-squares curve fit is as good as you can get it (by comparing the R^2 values), print the resulting figure and record the resulting value of time constant, τ, on the worksheet. Also record the final values used for T_0 and T_{ss}.

Time [s]	Temp [C]	Incomplete Z(t)
0	1.03	0
0.044	1.7	-0.00691
0.088	2.68	-0.01711
0.132	3.96	-0.03059
0.176	5.44	-0.04641
0.22	7.19	-0.06544
0.264	8.97	-0.08517
0.308	10.79	-0.10576
0.352	12.68	-0.1276
0.396	14.6	-0.15028
0.44	16.5	-0.17325
0.484	18.45	-0.19739
0.528	20.37	-0.22173
0.572	22.3	-0.24681
0.616	24.23	-0.27254
0.66	26.1	-0.29812
0.704	27.93	-0.3238

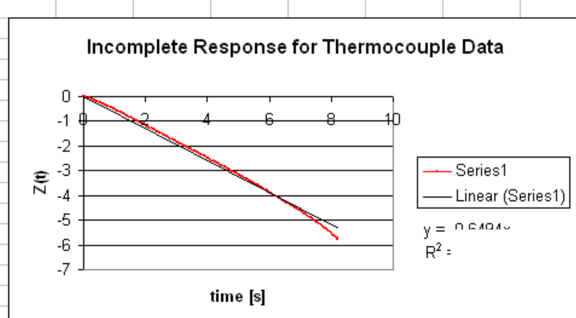

Fig 4. Incomplete response plot with best fit.

Method 3: Time constant using a cost function

Discussion

A value for the time constant may also be found by computing and minimizing the value of a performance index, J, which is based on the sum of the squared errors. This method compares the known form of the analytical solution using different values of τ with the experimental data until the performance index has reached a minimum. The performance index is given by

$$J(\tau) = \sum_{t_0}^{t_f} \left[T_m(t) - T(t,\tau) \right]^2 ,$$

where $T_m(t)$ is the experimental data and $T(t, \tau)$ is the theoretical temperature given by

$$T(t,\tau) = T_{SS} + \left(T_0 - T_{SS} \right) \; e^{-t/\tau}$$

Procedure

1. Select an initial guess for the value time constant, τ. Set it up as a variable in the Excel spreadsheet.
2. Set up a new column in Excel which calculates the temperature predicted by the theoretical solution to the DE.
3. Set up another column which computes each individual squared-errors in the performance index.

   $$\left[T_m(t) - T(t,\tau) \right]^2 \qquad \text{for } i = 1 \text{ to } N$$

4. Compute the sum of this column to get the performance index, $J(\tau)$.
5. Use the "Solver" function to minimize J with respect to τ only.
6. Now use the "Solver" function again to minimize J, but this time allow the solver to vary τ, T_0, and T_{ss}.
7. Create plots of the experimental data, the theoretical response using the τ value from step 5 (minimizing J by changing τ only), and the theoretical response using the τ value from step 6 (minimizing J by changing τ, T_0, and T_{ss}) versus time all together on the same plot.

Comparing results of three methods of determining the time constant

1. On the lab worksheet, indicate your best estimate of the time constant for each of the three methods by placing an '*' next to it in each section.
2. Create one last plot which shows the temperature vs. time plot for the three different line fits provided by each of the three different "best" time constants that were found. These are to be all shown on the same graph along with the original temperature data set. Adhere to the graphics standards, add a legend, and use different line-types (not colors).
3. Print out a copy and comment on your results.

ESTIMATING THE CONVECTION COEFFICIENT

Assume the copper-constantan thermocouple bead has the following properties:

density	ρ	=	8920 kg/m^3,
specific heat	C_v	=	410 J/kg·K at 100ºC,
diameter	d	=	0.5 mm,
volume/area	$\cancel{V} / A$	=	d/6.

Using equation (2) and your range of best estimates of the time constant, compute a range of values for the convection coefficient h. Show your calculations on the worksheet and record your values of h.

For free convection in liquids, the convection coefficient h is generally in the range of 50 to 1000 W/m^2·K. Compare your results to these published values.

WRITE-UP AND DISCUSSION

Fill in the worksheet blanks. Answer the worksheet questions thoughtfully, thoroughly, and wherever possible, quantitatively. Use precise technical vocabulary. Turn in the worksheet with your figures attached.

ACKNOWLEDGEMENTS

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REFERENCES

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[3] Incropera, F.P. and DeWitt, D.P., 1985, *Introduction to Heat Transfer*, Wiley: NY.
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GLOSSARY

correlation coefficient Measure of how well a curve fits a set of data. A value of 1.0 indicates a perfect relationship and a value of 0.0 indicates no relationship. Be cautious about ascribing too much virtue to values of the correlation coefficient close to 1.0. Always plot the data and the curve-fit to obtain a visual check of the behavior. If the data points do indeed hug the least-squares curve, then correlation coefficient close to 1.0 is indicative of a good correlation.

data acquisition Capture of information from real-world sources such as sensors and transducers. Often automated using a printed circuit board installed in a computer with dedicated software to sample the measurement and store it.

identification Drawing inferences about system characterization from experimental data.

order of a system Order of the differential equation representing the dynamic behavior of a system.

parameter Numerical value defining some property of a system.

parameter identification Identifying, from experimental data, system parameters or coefficients.

system identification Identifying, from experimental data, system-level characteristics such as time constants, steady-state gains, natural frequencies, or damping ratios.

thermocouple Temperature sensor consisting of the junction of two dissimilar metals. The output voltage produced is a function of the difference in temperature between the hot and cold junctions of the two metals.

time constant Usually used for first-order systems. It is a characteristic time of a system indicating how fast the system reaches steady state when subjected to a step input. It is defined as the time that the output reaches 63.2% of its final value.