Lab 6: Tank-Draining Experiment

Initial-condition response, parameter identification, and model validation

Objectives

The goals of this experiment are:

- To measure the initial-condition response of a single draining tank, as shown in Figure 1.
- To compare this experimental response to a model simulation and to use this comparison to obtain a best estimate the value of the discharge coefficient C_d, for each of two tanks.
- To use the best-estimate single-tank models to predict the behavior of the two-tank system shown in Figure 2, and to compare and contrast this prediction to the experimental initial-condition response of the two-tank system.

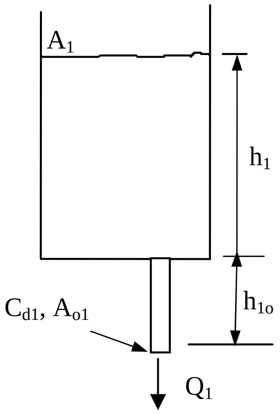


Figure 1: Single tank.

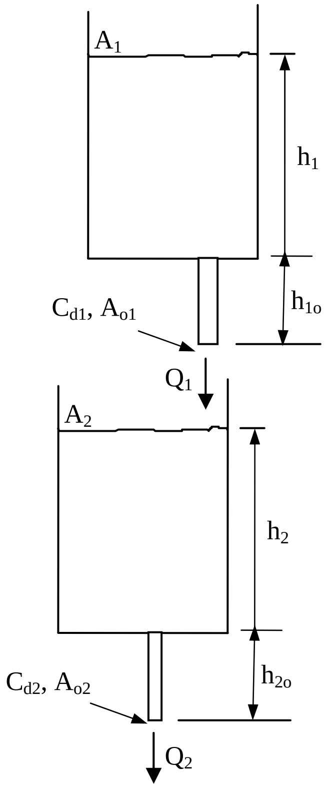


Figure 2: Cascading tanks

Location

Seven tank-draining experimental stations are located in the Controls Laboratory, Room C116. The lock combination is 451. Bring a memory stick/thumb drive. This lab may only be used for work related to this class and other specified classes, so do not tell anyone else the lock combination. ES205 students are permitted to use the lab 24/7 except when another class is using the room.

A picture of the two tank system and the computer used for data acquisition is shown in Figure 3.

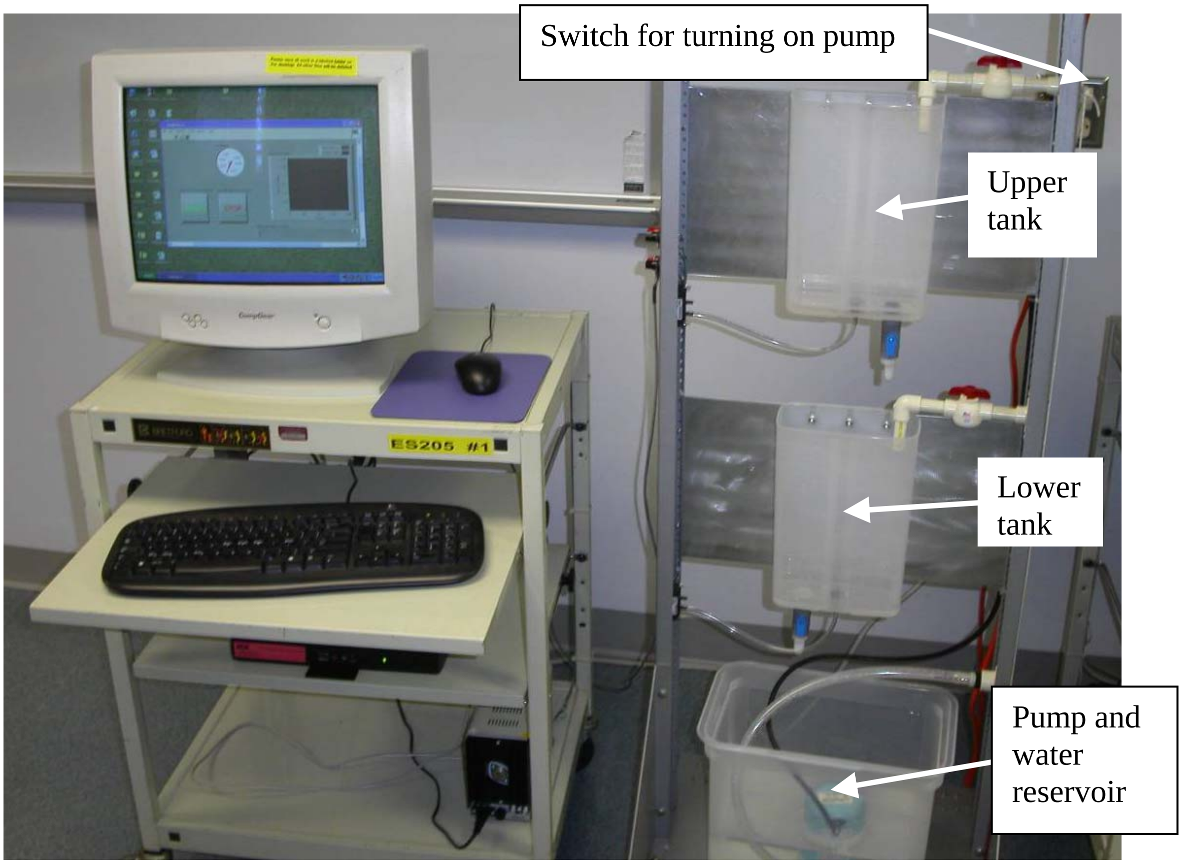


Figure 3: Tank system in C116.

Outline of main tasks

In this lab, you will accomplish the following tasks.

1. Calibrate the pressure transducers for each tank.
2. Experimentally obtain the initial-condition response for the lower tank, and by comparing the experimental response to the predicted response of your mathematical model, determine a best estimate of the orifice discharge coefficient.
3. Experimentally obtain the initial-condition response of a two-tank system.
4. Using the initial-condition response from the upper tank determine a best estimate of the upper tank's discharge coefficient.
5. Using the identified discharge coefficients compare and contrast the experimental response of the two-tank system to the results of a Simulink model.

MODELS

Derive the nonlinear differential equation describing the height, h_1, of the liquid for the upper tank assuming the liquid level is originally $h_1(0)$ before the orifice is opened and that $Q_i = 0$. This equation should be in terms of $h_{1o}, A_1, A_{o1}, C_{d1}, t$ and the gravitational constant g. If you assume the tank has a constant cross sectional area the differential equation can be solved to obtain an exact solution for $h_1(t)$. Repeat for the lower tank assuming $Q_1 = 0$. Remember that the flow though an orifice will depend on the height of fluid above the orifice, so you will need to consider the lengths of the nozzles in your model (*e.g.*, $h + h_{1o}$ for the upper tank).

Make sure your nonlinear differential equations for the two-tank system and Simulink model for solving the equations of motion you made in the pre-lab are correct. The model should allow you to set a different initial condition (initial water height) for each tank. The simulation results (outputs) are the water heights in each tank as a function of time.

EXPERIMENTAL PROCEDURE

The tanks stations are numbered 1 to 7. Record your station and identify it in your memo. A ruler is provided in the lab to take measurements necessary to determine the tank areas A_1 and A_2, and the orifice cross-sectional areas, A_{o1} and A_{o2}.

Some notes on operating the equipment

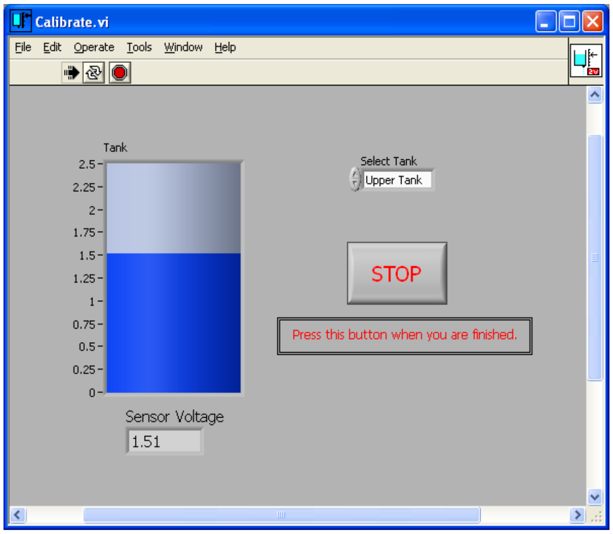
1. Before turning power on check the valve positions, drain connections, and so forth.
2. The pump can be turned on by using the switch on the side of the apparatus (see Figure 3).
3. The pressure transducers measure the pressure at the bottom of each tank.
4. When finished with the apparatus, close all programs, remove your files from the computer hard-drive, and turn off (or log off) the computer.

Calibration

Calibrate the pressure transducers using the program TankCalibrate.exe located on the desktop. The program icon is shown in Figure 4. A sensor calibration requires a relationship between the sensor input (physical variable) and the sensor output (electrical signal). Hopefully this relationship is linear or easily understood so that the collected electrical data can be converted back to a physical variable.

Figure 4: Program used fom calibration

Power up the equipment. Fill the tank to about 9 inches of water. Run TankCalibrate. There are markings on the side of the tanks to determine the height of water in the tank above the bottom of the tank. The voltage corresponding to this height is shown on the computer screen as shown in Figure 5. Gradually let out the water and collect data in a table similar to the one shown in Table 1. Select the upper tank or lower tank as necessary using the tank selection contol shown in Figure 5. Take as many data points as you think are necessary. The minimum height measured should be about 1 inch of water . From your calibration graph, obtain the slope (m) and y-intercept (b) of the linear curve relating measured voltage to water height. Repeat this procedure for the 2nd tank.



Select Tank: Upper Tank

Press this button when you are finished.

Sensor Voltage: 1.51

Figure 5: Screen shot from TankCalibrate.exe

Table 1: Calibration data

Data point no. *i*	Water height from bottom (in)	Transducer output (V)
1	9.0	
2	⋮	
⋮	⋮	
n		

Initial condition response of lower tank

Use the data acquisition software TankData.exe to collect data for the transient IC response of the lower tank. Use the shortcut icon, shown in Figure 6, on the computer desktop to lauch the program. When TankData launches you should see a screen as shown Figure 7. To obtain the initial condition response follow the following steps:

Figure 6: Program used to colled IC resonse data

1) Fill the tank to about 9 inches of water
2) Set the file location and name by selecting the control shown in Figure 7. <u>Save the file as a .csv file!</u>
3) Select the "start" button and open the valve at the bottom of the tank. The voltage values are written to a data file named according to what you specified in step 2. These data are in comma-delimited columns.
4) When the tank is sufficiently drained hit "stop".
5) Open the data file in Excel. You will notice that there will be three columns of data. These correspond to the time and the voltages from the pressure transducers at the bottom of the upper and lower tanks. Be sure you know which column corresponds to which tank. There is also a lot of junk that is also recorded in this file. Delete all the zeros or -5's at the end of columns 1, 2 and 3.

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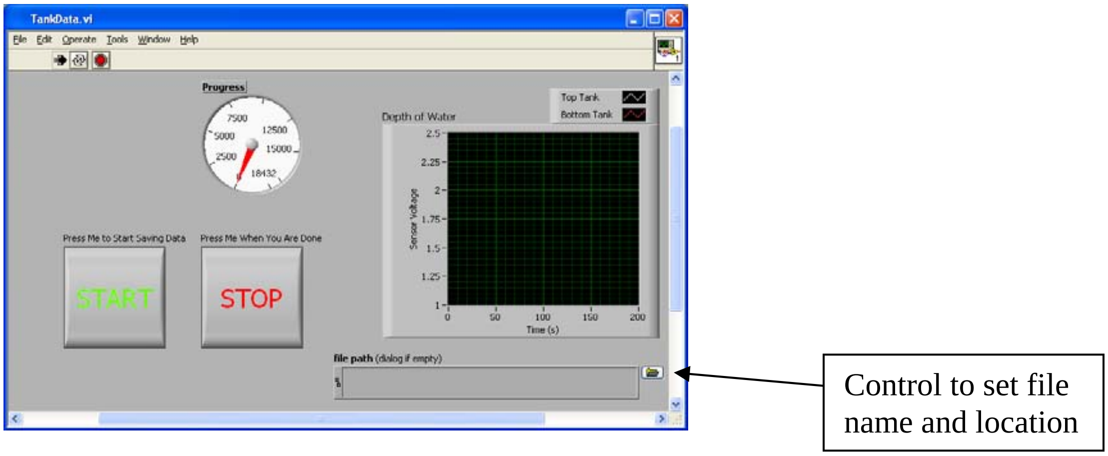

Control to set file name and location

Figure 7: Snapshot from TankData.exe

Two-tank experiment

Fill the upper tank to a fluid height of approximately 9 inches and the lower tank to a fluid height of approximately 4 inches. The upper tank drains into the lower tank. Both orifices are to be opened at the same time. Using TankData, record the height of fluid in the the tanks as a function of time until the lower tank water height has dropped below one inch.

When you are finished

Save your data files to your own memory stick or floppy disk and delete the files from the hard-drive of the data-acquisition computer.

ANALYSIS AND SIMULATION

Parameter identification: determine C_d for each tank

For your initial analysis assume the discharge coefficient C_d is approximately equal to 0.7. Substitute this value of C_d and the parameters you measured into your analytical expression for $h_i(t)$ (from your exact solution of the nonlinear differential equation). Using this model, compute the predicted response for $h_i(t)$ for the same time span as the experiment. Plot this prediction and the experimental data on the same figure. Use a continuous line for the theoretical prediction and individual symbols for the experimental data. When identifying C_d for the upper tank use the initial condition response from the two-tank experiment. When identifying C_d for the lower tank use the initial condition response from the single-tank experiment.

Tuning the model

Generally, analytical solutions and experimental measurements differ. The theoretical curve can be made to more closely (though usually not exactly) predict the experimental measurements by adjusting the model parameters (in this case, the discharge coefficient C_d). A measure of experimental/theoretical closeness is the *performance index J* given by

$$J = \frac{1}{n}\sum_{i=1}^{n}(h_i^{\text{model}} - h_i^{\text{data}})^2 \tag{1}$$

where:

h_i^{model} = water height predicted by the model at the i^{th} point in time

h_i^{data} = water height experimentally determined at the i^{th} point in time

n = number of data points used in comparing experiment to theory

Your task is to find the value of C_d that minimizes J. One way to do this is to use the built-in Excel “Solver” command like you did in Lab 3. You can also use the built-in Matlab command called “fminsearch.”

Repeat this process to determine the discharge coefficient for each tank.

Predicting the response of the two-tank system

After identifying discharge coefficients for the upper and lower tanks, use your Simulink model of the two-tank system to predict the height in the lower tank as a function of time and compare to your experimental results. In particular, we want you to compare how long it takes for the lower tank to drain to a level of 1 inch and also to a level of 3 inches of fluid. How well does your model predict the response of the two tank system? What might explain any differences you observe? Make sure your model initial conditions match the experimental initial conditions.

REPORTING

A memo is required for this lab and there will be a progress check next week. The progress check will take place next week and the memo is due at the beginning of lab during week 8. (Check the course calendar.)

The progress report will consist of showing your professor the following results:

1. Final value of the discharge coefficients for your upper and lower tanks.
2. For each tank, a plot of the experimental data and the theoretical prediction for the initial-condition response.
3. A plot of the experimental data and theoretical results for the two-tank experiment.

The memo shall include all the results in addition to a brief discussion of your experimental procedure, and how you modeled the tanks . The memo must include the following results as a minimum:

- Calibration curve for each tank.
- For each tank you need a plot of the experimental initial-condition response, the theoretical prediction before tuning, and the theoretical prediction after tuning, all on the same graph.
- For the two-tank experiment, compare the predicted value to the experimental value of the time required for the lower tank to reach a fluid level of 1 inch and 3 inches.
- For the two-tank experiment, a plot comparing the theoretical (simulation) to experimental height $h_2(t)$ of the fluid in the lower tank as a function of time.

Figures are to be embedded in the body of the memo and must be thoroughly discussed. Use different line types and legends; plot experimental data as data points and theoretical responses as continuous lines. If the data points make the figures appear cluttered or you cannot tell the difference between the experiment and simulation you can leave off the data points if you mention that you are doing this in the text. If your experimental data and theoretical model match very well it is sometimes nice to include a figure showing the difference between the two, that is $h_{exp} – h_{theory}$, as a function of time. If the model does not match the experiment very well be sure to discuss possible reasons why and how your model could be improved.

Additional comments:

You may include appendices to your memo that include:

- Derivation of the equations of motion
- Copy of the Simulink model, m-files, and Maple worksheets.

You will want to include in the memo the following:

- System schematic
- Model of the single tank and the analytical solution for this model
- Model of the two-tank system
- A table listing system parameters (constants such as the geometry of the tank, the area, C_d, distance from the bottom of the tank to the orifice and so forth) used in the models.

Final comments on modeling:

- In your figures, the height of fluid should be measured from the bottom of the tank.
- In your model, you should model the flow through the orifice as: $Q_0 = A_0 C_d \sqrt{2gh}$ where h is the height of the fluid above the orifice exit, **not** above the bottom of the tank.
- In your two-tank model be sure to use a switch to set the flow out of the upper tank equal to zero when the upper tank is empty.
- When you label your figures your vertical axis should be labeled something like “Height of fluid above the bottom of the tank (inches)”