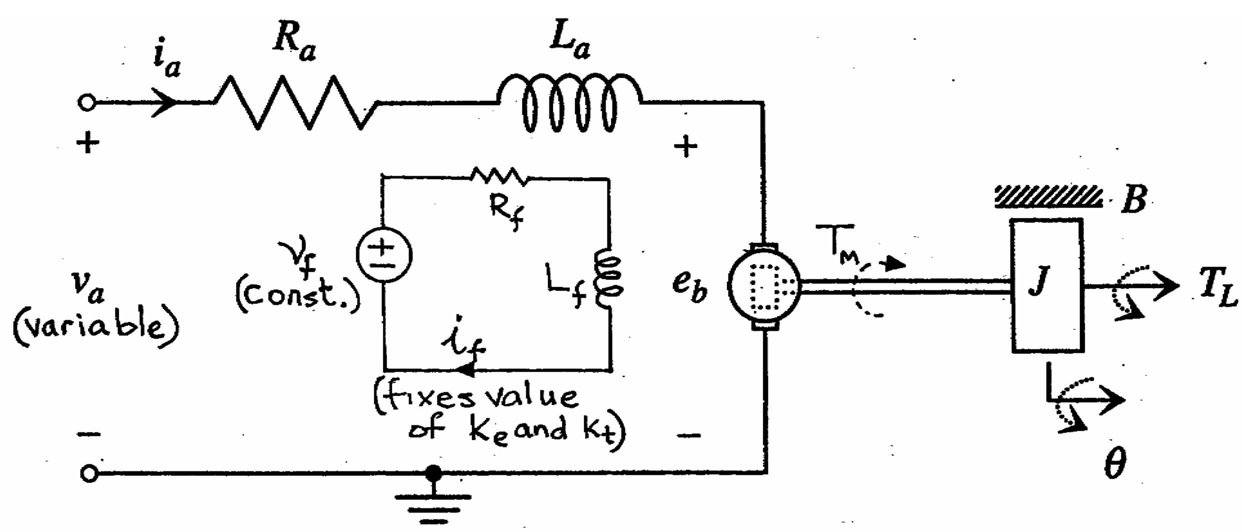
Modeling and Simulation of a Single DC Machine

A. Armature Controlled DC Motor Theory

The components are shown in the figure below. The armature (part that does the work) is located on the rotor, while the field (part that creates the magnetic field) is located on the stator. The field source is constant and hence the strength of the field does not vary; this means that K_e and K_t do not vary.

The armature is fed from a variable DC source, v_a. The resulting current, i_a causes some voltage to be dropped on R_a and L_a, the remaining voltage, e_b is the Aback emf@ which appears across the ideal armature winding. It depends on speed and is given by: $e_b = K_e.\omega$. The armature current also produces the developed torque, which is given by: $T_M = K_t.i_a$.

The product: $e_b.i_a$ is power which is converted into mechanical rotation and is equal to developed torque, T_M times the speed, i.e. $e_b.i_a = T_M.\omega$. This power is fed to the load which is comprised of Inertia J, damping B, and a load torque T_L. The stiffness of the shaft is being neglected for the time being.



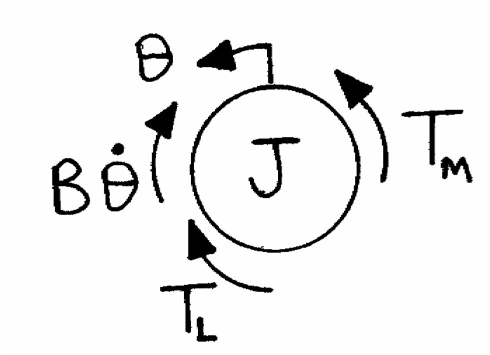
We need to develop an EOM with v_a as the input and ω as the output, for a given value of T_L. This requires writing EOMs for the armature and the load. Begin by taking Laplace Transforms of all quantities and apply KVL to the armature circuit of the motor:

$$\mathbf{V_a(s) = (R_a + L_a s)I_a(s) + K_e W(s)}$$

Re-arranging gives:

$$\mathbf{I_a(s) = \frac{[V_a(s) - K_e W(s)]}{(R_a + L_a s)}} \qquad \mathbf{(1)}$$

Then apply the FBD of the rotor/load:



$$\mathbf{Js^2\ q(s) = T_M - Bs\ q(s) - T_L(s)} \qquad \text{(Note } s\theta(\mathbf{s}) = \Omega(s)\text{)}$$

Which is: $$\mathbf{Js\ W(s) = K_t I_a(s) - BW(s) - T_L(s)}$$

Re-arranging gives: $$\mathbf{W(s) = \frac{[K_t I_a(s) - T_L(s)]}{(B + Js)}} \qquad \mathbf{(2)}$$

Substitute (1) into (2) and multiply out:

$$\mathbf{W(s)(B + Js)(R_a + L_a s) = K_t[V_a(s) - K_e W(s)] - T_L(s)(R_a + L_a s)}$$

The EOM is:

$$\mathbf{W(s)[(B + Js)(R_a + L_a s) + K_t K_e] = K_t V_a(s) - T_L(s)(R_a + L_a s)} \qquad \mathbf{(3)}$$

Development of a Simulink Model

a) When the motor is on no-load the load torque is zero. Apply this condition to equation (3) and write the EOM in the time domain using one of the standard forms.

b) Based on the EOM with the following parameters, predict the static gain, undamped natural frequency, damping ratio, and actual frequency of oscillations.
$K_e = K_t = 2$ Vs/rad, $R_a = 2$ ohm, $L_a = 400$ mH, B = 0.5 N.m.sec, J = 0.4 kg.m^2.

c) Develop a SIMULINK model for the armature controlled DC motor. (Hint: start by working backwards from equation (2) with the $[K_t.I_a(s) - T_L(s)]$ term being the output of a summation block and then feed this into the **transfer function** block (found in the *Continuous* library) to model 1/(B + Js) etc. Continue working backwards to include equation (1) with the $[V_a(s) - K_e\Omega(s)]$ term being the output of a summation block.)

d) Simulate applying a 100 V step at t = 0 to the armature, with the shaft initially on no load and a load of 20 Nm being applied at t = 5 sec. Measure static gain, damping ratio, the frequency of the resulting oscillations, and undamped natural frequency. Compare these measurements with the predictions in part (b).

e) Make plots of Shaft Speed in RPM and Armature Current vs time. Be sure the plots are correctly titled and the axes are labeled.