Newton's Second Law 10/6/03 13

Introduction: The goal of this lab is to find the acceleration of the following system:

- Frictionless tabletop
- \( g = 9.8 - 0.5 \) (LAP)

This will be accomplished by using Logger Pro to analyze position & time data collected by the Motion Detector.

The acceleration values obtained through this procedure for several different masses will be compared to an expected value derived from the following free-body diagram:

\[
\Sigma F_y = N - Mg = Ma = 0 \\
\Sigma F_x = +T = Ma \\
-T + mg = ma_0 \\
\alpha = \left( \frac{m}{m + M} \right) \cdot g
\]

Expected value: I see ok.
<table>
<thead>
<tr>
<th></th>
<th>My measurement</th>
<th>Partner's</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Glider</strong> mass + String &amp; clip</td>
<td>0.21916 Kg</td>
<td>0.21945 Kg</td>
<td>0.219305 ± 0.0001 K</td>
</tr>
</tbody>
</table>

**Table 1**

Given masses on lab handout:

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/16&quot; nut</td>
<td>0.0047 ± 0.0001</td>
</tr>
<tr>
<td>1/2&quot; nut</td>
<td>0.0165 ± 0.0001</td>
</tr>
<tr>
<td>large paper clip</td>
<td>1.2 × 10^-3</td>
</tr>
</tbody>
</table>
Procedure: First each partner massed the glider with 15 paper clips and string attached. Then we performed a leveling test trial with the glider/string/clip to determine whether or not the air-track was level. For this trial we fit a parabola \( y = Ax^2 + Bx + C \) to the \( x-v-t \) data. This fit corresponds to the physics formula \( x = \frac{1}{2}at^2 + vt + x_0 \) and so in the coefficient \( A \) we have a value for half the acceleration. Similarly fitting a line to the \( v-v-t \) data gives us another acceleration value: \( y = mx + b \)

\[ v = \frac{1}{2}at + v_0 \]

As expected, the acceleration for this trial was negligible (more on this later) and so we moved on.

Next, we attached a mass to the paper clip and gathered 5 trials worth of data (which are found on the next page). Rather than run 20 more trials, we gathered data once each for 4 different masses. Below are two example graphs of a trial with a mass on the clip.
<table>
<thead>
<tr>
<th>Masses</th>
<th>X-V-t</th>
<th>V-V-t</th>
<th>Initial</th>
<th>Final</th>
<th>Trial</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.00198 ± 3.06 x 10^-5</td>
<td>m = 0.3716 ± 1.055163</td>
<td>0</td>
<td>0.5</td>
<td>2.85</td>
</tr>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.3668 ± 0.01248</td>
<td>m = 0.7088</td>
<td>0.80</td>
<td>2.85</td>
<td>2</td>
</tr>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.3624 ± 9.614 x 10^-4</td>
<td>m = 0.7122</td>
<td>0.70</td>
<td>2.55</td>
<td>3</td>
</tr>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.3633 ± 1.032 x 10^-3</td>
<td>m = 0.7263</td>
<td>0.95</td>
<td>3.6</td>
<td>4</td>
</tr>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.3659 ± 1.180 x 10^-3</td>
<td>m = 0.7114</td>
<td>0.65</td>
<td>2.85</td>
<td>5</td>
</tr>
<tr>
<td>2-5/16 nuts</td>
<td>A = 0.2203 ± 2.012 x 10^-4</td>
<td>m = 0.4373</td>
<td>0.50</td>
<td>3.15</td>
<td>1</td>
</tr>
<tr>
<td>3-5/16 nuts</td>
<td>A = 0.3109 ± 1.954 x 10^-4</td>
<td>m = 0.6191</td>
<td>0.60</td>
<td>2.65</td>
<td>1</td>
</tr>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.4478 ± 6.854 x 10^-6</td>
<td>m = 0.8927</td>
<td>1.15</td>
<td>3.10</td>
<td>1</td>
</tr>
<tr>
<td>1-1/2 nut</td>
<td>A = 0.530 ± 0.0004812</td>
<td>m = 1.047</td>
<td>0.65</td>
<td>2.35</td>
<td>1</td>
</tr>
</tbody>
</table>

\( m = \text{Mass (kg)} \)
\[ \alpha = \text{(m/s}^2) \]

\( M = 0.222605 \pm 1.45 \times 10^{-4} \text{ Kg} \)

Standard Errors found by using top one as percentage.
Procedure (cont.) On each trial, data was gathered after a force was applied that pushed the glider towards the motion detector. So, the glider was monitored while moving towards the detector and away from it. This was done instead of simply releasing the glider from rest because twice as much data could be gathered. From the data on the opposite page an average value of acceleration was found for each of the masses m attached to the clip. The mass and corresponding acceleration value are shown in the middle of the opposite page. Below are the fits for the unweighted glider:

Position v. Time (Level Run)

![Position v. Time Graph]

Velocity v. Time (Level Run)

![Velocity v. Time Graph]
The average acceleration determined from coefficients of the quadratic & linear fits described in the procedure is: 
\[ a = -0.00358 \pm 0.00038 \text{ m/s}^2 \]

This acceleration is very small and so the track can't be tilted much and the data can't be significantly affected, but for a closer look:

Here is the setup for a tilted track:

\[ \sum F_\perp = N - Mg \cos \theta = Ma \]
\[ \sum F_\parallel = T + Mg \sin \theta = Ma \]
\[ T = M(a - g \sin \theta) \]

**Force Diagram of Glider**

\[ \Sigma F = mg - T = ma \]
\[ mg - M(a - g \sin \theta) = ma \]
\[ mg + Mg \sin \theta = a(M + m) \]
\[ a = \frac{mg}{M + m} + \frac{Mg \sin \theta}{M + m} \]

Here we have an extra term added to the acceleration.

In the case of the unweighted glider, \( m = 0 \) and the extra term reduces to \( g \sin \theta \).

**Unweighted**
\[ a = g \sin \theta \]
\[ \theta = \sin^{-1} \left( \frac{a}{g} \right) \]
\[ \theta = \sin^{-1} \left( \frac{-3.58 \times 10^{-3}}{9.8} \right) \]
\[ \theta = -0.021^\circ \]

Our track was actually tilted backwards at an angle of \(-0.021^\circ\) with the horizontal.

To correct the acceleration values, \( \frac{Mg}{M + m} \sin(-0.021^\circ) \)
will be subtracted from each a \( a \) value.

2. The Standard Error should be used as a percentage for other accelerations b/c with different size accelerations you would expect different size standard errors.

4. Using Equation \( a = \left( \frac{m}{M+m} \right) g \) from problem lab we can obtain value for \( g \) by graphing \( a \) versus \( \frac{m}{M+m} \). First, acceleration values need to be modifying as mentioned @ beginning of Analysis.

\[
\begin{align*}
M &= \text{mass of cart, } \text{railed track, } \text{and } \text{cart on track} \approx 0.1303 \text{ kg} \\
M &= 0.222605 \pm 1.45 \times 10^{-4} \text{ kg} \\
M &= \text{mass of cart, } \text{payload, } \text{and } \text{railed track} \approx 0.1303 \text{ kg} \\
M &= 0.72444 \\
0.0177 &= \text{acceleration, } a \text{ of cart, } \text{payload, } \text{and } \text{railed track} \approx 0.72444 \\
0.0106 &= \text{acceleration, } a \text{ of cart, } \text{payload, } \text{and } \text{railed track} \approx 0.43899 \\
0.0153 &= \text{acceleration, } a \text{ of cart, } \text{payload, } \text{and } \text{railed track} \approx 0.6205 \\
0.0224 &= \text{acceleration, } a \text{ of cart, } \text{payload, } \text{and } \text{railed track} \approx 0.89423 \\
0.0271 &= \text{acceleration, } a \text{ of cart, } \text{payload, } \text{and } \text{railed track} \approx 1.0513
\end{align*}
\]

The Graph on p. 20 shows this fit in which \( g \) is the slope.

By Tinkering with the linear fit options in Logger Pro, I found that the root mean squared error, or standard deviation was \( 0.00690 \). Dividing this by \( \sqrt{N} \) or \( \sqrt{5} \) in this case gives an uncertainty of \( g \) of \( \pm 0.003 \).
$g = 9.752 \pm 0.003 \text{ N/kg}$

Curiously, the unmodified acceleration data gives a closer estimate of $g$. 

Regardless, the angle $\theta$ did not have a tremendous effect.
Some Possible Sources of error:

1) As in Lab 1, velocity data was obtained from discrete position/time data and so any errors would be propagated.
2) The string connected to the glider was not exactly horizontal. The setup was more like the following:

\[
\begin{align*}
\Sigma F_y &= N - Mg + T \sin \theta = 0 = M a_y \implies 0 = M a_y \\
\Sigma F_x &= T \cos \theta = M a_x \\
T &= \frac{M a_x}{\cos \theta} \\
\end{align*}
\]

\[
\begin{align*}
\Sigma F &= M g - T = ma \\
m g - \frac{M a_x}{\cos \theta} &= ma \\
T &= m g - M a_x \\
a &= \frac{m g - M a_x}{(M + m) \cos \theta}
\end{align*}
\]

If anything, this should have increased the estimate for \(g\).

\(\theta\) is constantly changing, however, and it remains below a maximum angle.

Just to get a feel for the impact of this error...

Suppose \(\theta\) is constant @ 10°. This yields

\(g = 9.88\) using the modified acceleration value.

This error may have played a significant part in this experiment. (Note: final conclusion on next page)
Conclusion

In this lab, a motion detector was used to obtain x-t, v-t, and a-t data for an accelerating system. Using force diagrams and Newton's second law, a value for g was obtained. 

\[
g = 9.752 \pm 0.003 \text{ N/kg}
\]

3 main sources of error include:

1) tilted air track - although we noticed the unweighted glider accelerating, the angle and therefore effect was negligible
2) tilted string - may or may not have played huge role in experiment, hard to say
3) velocity taken to be average at midpoint of time intervals - definitely negligible effect

served to increase approximation of g
served to decrease approximation of g

An AMAZING RESULT - Must thank Mr. Brown

Well done. :3