ECE-597: Probability, Random Processes, and Estimation
Homework # 2

Due: Tuesday March 22, 2016

From the book: 5.19, 5.21, 5.24, 5.29, 5.30

Hints and Answers:
5.19, should be easy
5.21, expand out, look at cross terms, and use the uncorrelated property
5.24, there are many possible answers, you should check your C using Matlab
5.29, should be easy, remember $\underline{a}^T\underline{x} = \underline{x}^T\underline{a}$ if $\underline{a}^T\underline{x}$ is a scalar
5.30, use a whitening transform, even if the problem does not say to. Also try the second additional problem below before trying this problem.

Additional Problems

1) Assume

$$\mathbf{Y} = h\mathbf{X} + \mathbf{V}$$

where $\mathbf{X}$ is a random variable with mean μ_x and standard deviation σ_x^2, $\mathbf{V}$ is observation noise which is uncorrelated with $\mathbf{X}$ and has mean $\mu_v = 0$ and variance σ_v^2. We want to estimate $\mathbf{X}$ from observing $\mathbf{Y}$. We showed in class that the optimal linear estimator is of the form

$$\hat{\mathbf{X}} = \mu_x + \frac{Cov(\mathbf{X}, \mathbf{Y})}{\sigma_y^2}(\mathbf{Y} - \mu_y)$$

Show that the optimal linear estimator for this problem is given by

$$\hat{\mathbf{X}}(\mathbf{Y}) = \mu_x + \frac{h\sigma_x^2}{h^2\sigma_x^2 + \sigma_v^2}(\mathbf{Y} - h\mu_x)$$

What happens if σ_v is zero? What happens if h is zero?

2) Consider Gaussian random vector $\mathbf{X}$ with zero mean and covariance matrix

$$K_{XX} = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{bmatrix}$$

a) Determine an expression for $f_{\underline{X}}(\underline{x})$ using equation 5.6-2.
b) Assume we define $\mathbf{Y} = A\mathbf{X}$ where

$$A = \begin{bmatrix} -2 & \sqrt{2} \\ 1 & \sqrt{2} \end{bmatrix}$$

Determine K_{YY}, put this into equation 5.6-2 and determine $f_{Y_1}(y_1)$ and $f_{Y_2}(y_2)$

3) Assume $\underline{\mathbf{X}}$ and $\underline{\mathbf{Y}}$ are random vectors, not necessarily of the same size. Assume also that $K_{\mathbf{XX}}$, $K_{\mathbf{WW}}$ and $K_{\mathbf{WX}}$ are known. Now we make a new random vector

$$\underline{\mathbf{Y}} = A\underline{\mathbf{X}} + B\underline{\mathbf{W}} + \underline{C}$$

where A and B are constant matrices (not necessarily of the same size), and $\underline{C}$ is a constant vector. Determine an expression for $K_{\mathbf{YY}}$ in terms of these known quantities **ONLY**. Do **not** assume the means are zero.

Hint: $(FG)^T = G^T F^T$