#1 $\mu_X[n] = (1)\cdot\frac{1}{3} + (0)\cdot\frac{2}{3} = \boxed{\frac{1}{3} = \mu_X[n]}$

$R_{XX}[n,m] = E\{X[n]\,X[m]\}$

$n \neq m \quad E\{X[n]\,X[m]\} = E\{X[n]\}\,E\{X[m]\} = \mu^2 = 1/9$

$n = m \quad E\{X[n]^2\} = (1^2)\cdot\frac{1}{3} + (0^2)\cdot\frac{2}{3} = \frac{1}{3}$

$$\boxed{R_{XX}[n,m] = \frac{1}{9} + \frac{2}{9}\,\delta[n-m]}$$

#2 $\dfrac{f_{X|H_0}}{f_{X|H_1}} \underset{H_1}{\overset{H_0}{\gtrless}} \dfrac{P_1}{P_0}$ $\qquad \dfrac{\frac{2}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(x-1)^2}}{e^{-x}} \underset{H_1}{\overset{H_0}{\gtrless}} 2$

$$e^{-\frac{1}{2}(x-1)^2}\, e^{x} \underset{H_1}{\overset{H_0}{\gtrless}} \sqrt{2\pi}$$

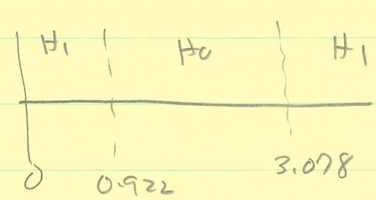
$$-\tfrac{1}{2}(x^2-2x+1) + x \underset{H_1}{\overset{H_0}{\gtrless}} \ln(\sqrt{2\pi})$$

$$x^2-4x+1 \underset{H_0}{\overset{H_1}{\gtrless}} -2\ln(\sqrt{2\pi})$$

$$x^2-4x+1+2\ln\sqrt{2\pi} \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$$(x^2-4x+2.8379) \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$$(x-0.922)(x-3.078) \underset{H_0}{\overset{H_1}{\gtrless}} 0$$



#3 $X[n] = S[n] + V[n]$ $\quad Y[n] = \sum_{k=-\infty}^{\infty} h[k]\, X[n-k]$

$$R_{YX}[m] = E\{Y[n]\, X^*[n-m]\} = \sum_{k=-\infty}^{\infty} h[k]\, E\{X[n-k]\, X^*[n-m]\}$$

$$R_{YX}[m] = \sum_{k=-\infty}^{\infty} h[k]\, R_{XX}[m-k]$$

$$R_{XX}[m] = E\{(S[n] + V[n])(S^*[n-m] + V^*[n-m])\} = R_{SS}[m] + R_{VV}[m]$$

$$\boxed{R_{YX}[m] = \sum_{k=-\infty}^{\infty} h[k]\left(R_{SS}[m-k] + R_{VV}[m-k]\right)}$$

#4 a) $\hat{d}[n] = h_0 X[n] + h_1 X[n-1]$

$$E\{(S[n] - \hat{d}[n])\, X^*[n]\} = E\{S[n] X^*[n] - h_0 X[n] X^*[n] - h_1 X[n-1] X^*[n]\} = 0$$

$$R_{SX}[0] - h_0 R_{XX}[0] - h_1 R_{XX}[-1] = 0$$

$$E\{(S[n] - \hat{d}[n])\, X^*[n-1]\} = E\{S[n] X^*[n-1] - h_0 X[n] X^*[n-1] - h_1 X[n-1] X^*[n-1]\} = 0$$

$$R_{SX}[1] - h_0 R_{XX}[1] - h_1 R_{XX}[0]$$

$$\begin{bmatrix} R_{XX}[0] & R_{XX}[-1] \\ R_{XX}[1] & R_{XX}[0] \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \end{bmatrix} = \begin{bmatrix} R_{SX}[0] \\ R_{SX}[1] \end{bmatrix}$$

$$R_{XX}[m] = E\{(S[n] + V[n])(S^*[n-m] + V^*[n-m])\} = R_{SS}[m] + R_{VV}[m]$$

$$R_{SX}[m] = E\{(S[n])(S^*[n-m] + V^*[n-m])\} = R_{SS}[m]$$

$R_{XX}[0] = 1 + 0.2 = 1.2$

$R_{XX}[1] = R_{XX}[-1] = 0.9$

$R_{SS}[0] = 1 \quad R_{SS}[1] = 0.9$

$$\boxed{\begin{bmatrix} 1.2 & 0.9 \\ 0.9 & 1.2 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.9 \end{bmatrix}}$$

#4 b) $H_O(z) = \frac{1}{K_0}\frac{1}{H_c(z)}\left[\frac{S_{dx}(z)}{H_c(z^{-1})}\right]_+$ $\qquad S_{xx}(z) = \frac{0.4822(1-0.371z^{-1})(1-0.371z)}{(1-0.9z^{-1})(1-0.9z)}$

$R_{dx}[m] = E\{s[n](s^*[n-m] + v^*[n-m])\} = R_{ss}[m]$

$S_{dx}(z) = \frac{1-0.9^2}{(1-0.9z^{-1})(1-0.9z)} = \frac{0.19}{(1-0.9z^{-1})(1-0.9z)}$

$\frac{S_{dx}(z)}{H_c(z^{-1})} = \frac{0.19}{(1-0.9z^{-1})(1-0.9z)} \cdot \frac{1-0.9z}{1-0.371z} = \frac{0.19}{(1-0.9z^{-1})(1-0.371z)}$

$= \frac{A}{1-0.9z^{-1}} + \cancel{\frac{B}{1-0.371z}}$ $\qquad A = \frac{0.19}{1-0.371(0.9)} = 0.285$

$\left[\frac{S_{dx}(z)}{H_c(z^{-1})}\right]_+ = \frac{0.285}{1-0.9z^{-1}}$

$H_O(z) = \frac{1-0.9z^{-1}}{1-0.371z^{-1}} \cdot \frac{1}{0.4822} \cdot \frac{0.285}{1-0.9z^{-1}} = \frac{0.591}{1-0.371z^{-1}}$

$\boxed{h_O[n] = 0.591(0.371)^n u[n]}$

by orthogonality

c) $MSE = E\{(s[n] - \hat{d}[n])^2\} = E\{(s[n] - \hat{d}[n])s[n]\} - E\{(s[n] - \hat{d}[n])\hat{d}[n]\}$ 0

$MSE = R_{ss}[0] - h_0 R_{xs}[0] - h_1 R_{xs}[-1] = \boxed{0.131 = MSE}$