ECE 597: Probability, Random Processes, and Estimation

Homework # 1

Due: Friday March 20, 2015

From the book: 2.17, 2.18, 4.10, 4.20, 4.50, 4.51, 4.74

Hints and Answers:
2.17, (partial answer) P[burns $\geq$ 2 months] = 0.44.
2.18, think in terms of the intersection of events.
4.10, feel free to use Table 4.3.2. This is a very short problem.
4.20, feel free to use what you know about Gaussian random variabls, this is a short problem.
4.50, the answer is

$$R_n(k) \quad = \quad \begin{cases} (1+a^2)\sigma^2 & k = 0 \\ -a\sigma^2 & k = \pm 1 \\ 0 & \text{else} \end{cases}$$

4.51, start with $R_n(0)$, then $R_n(1)$, etc. The answer is $R_n(l) = b^{|l|}K$
4.74, you should get $COV(\epsilon, X) = \alpha\sigma_x^2 - \sigma_x\sigma_y\rho_{xy}$ and if you put in the optimal values the covariance should be zero.

Additional Problems

1) Assume we have random variables $\mathbf{Y} = a\mathbf{X} + b$, where $\mathbf{X}$ is normally distributed (Gaussian) with mean zero and variance one. Determine the parameters a and b so $\mathbf{Y}$ will be normally distributed with a mean of μ and a variance of σ^2.

2) Recall that for a random variable Z with Gaussian pdf, we have

$$F_Z(z) \quad = \quad \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{1}{2\sigma_z^2}(z-\mu_z)^2}$$

and the mean of Z is μ_z and the variance of Z is σ_z^2.

Starting with the joint pdf for two Gaussian random variables X and Y (equation 4.3-27 in the text), show that

$$\begin{aligned} VAR[X|Y = y] &= \sigma_x^2(1 - \rho_{xy}^2) \\ E[X|Y = y] &= \mu_x + \frac{\rho_{xy}\sigma_x}{\sigma_y}(y - \mu_y) \end{aligned}$$

Hint: Write the conditional pdf $f_{X|Y}(x|y) = Ae^{-B}$ and solve for A and B, keeping in mind the general form for a Gaussian pdf. You do not need to do any integration, only algebra. *We will use this result in the next computer project.*