ECE-521 Control Systems II
Homework 5

Due Date: Tuesday May 4

Note: For any problem you use Matlab and/or Simulink on, I want you to turn in your Simulink model and the Matlab driver.

1 For the system with model

$$\begin{aligned} \dot{\underline{x}}(t) &= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) &= [0\ 1]\, \underline{x}(t) \end{aligned}$$

and state variable (observer) feedback form

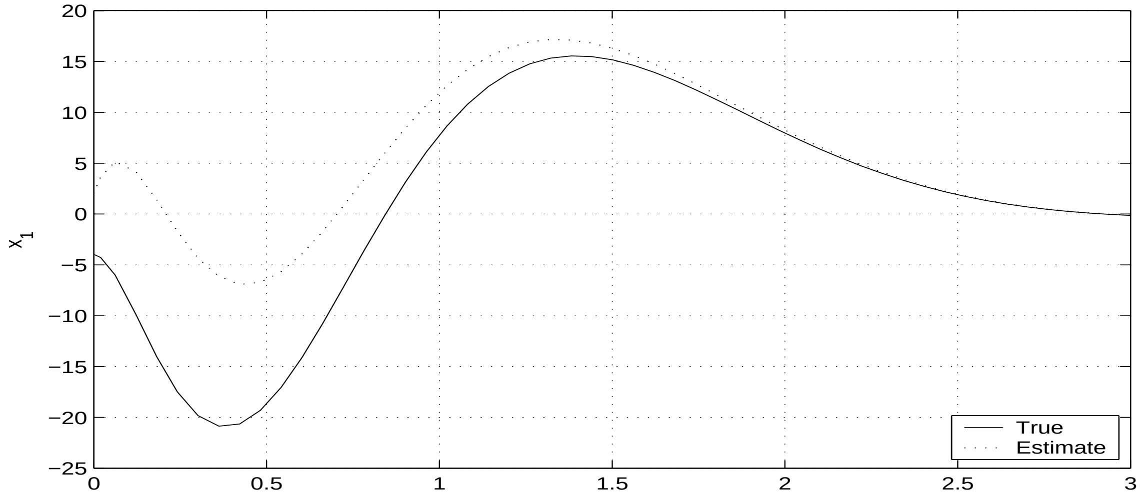
$$u(t) = Fr(t) - K\underline{\tilde{x}}(t)$$

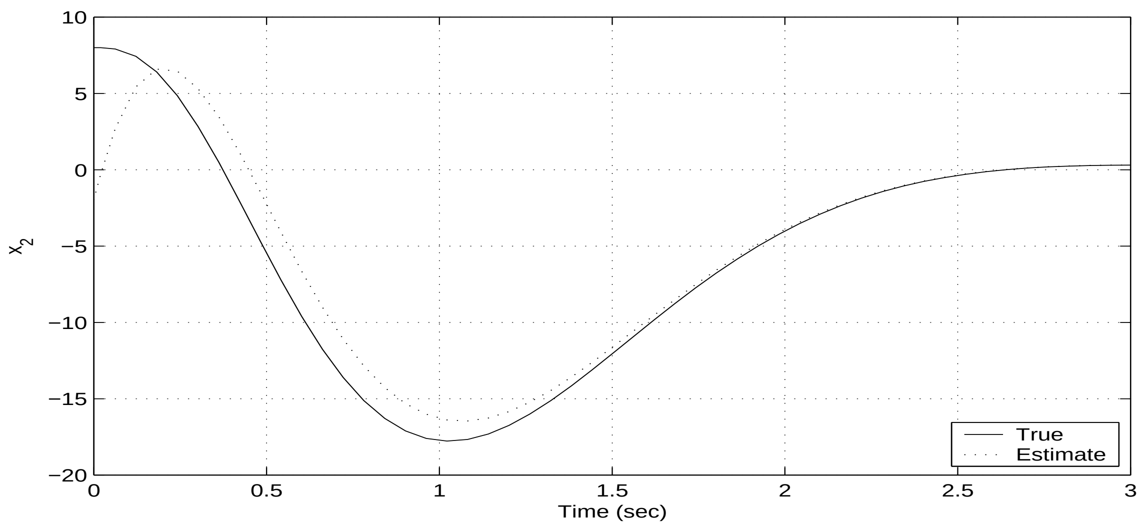
a) Determine the state variable feedback matrix K using Ackermann's formula (by hand) to place the closed loop poles at $-2 \pm 2j$.
b) Determine the observer feedback matrix K_e using Akermann's formula (by hand) to place the observer poles at -3 and -4.
c) Determine the observer-controller transfer function for this system directly (by hand).

2 Develop a Simulink model that implements a full order observer, and used the estimated state variable $\tilde{x}$ in the state feedback.

- The system you need to implement should look like the regulator system in Figure 12-12, except we want to include an input, so $u(t) = Fr(t) - K\underline{\tilde{x}}(t)$, where $r(t)$ is the input.
- The Simulink model should have a Matlab driver, and all of the matrices should be loaded from Matlab.
- Your Matlab code should use the **acker** or **place** command to determine the state variable feedback gain K
- Your Matlab code should use the **acker** or **place** command to determine the observer gain K_e.
- Your Matlab code should use subplot to plot the estimated and true states on the same plot, with each state on a different plot.

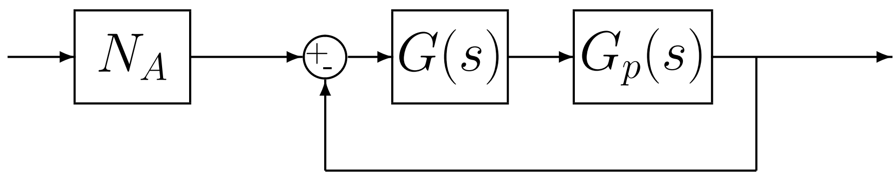
Run the simulation for 3 seconds with the system you analyzed in problem 1 as a regulator system (set the input $r(t) = 0$), assuming $\underline{\tilde{x}}(0) = [2\ \ -2]^T$ and $\underline{x}(0) = [-4\ 8]^T$. Your results should look more or less like the following:





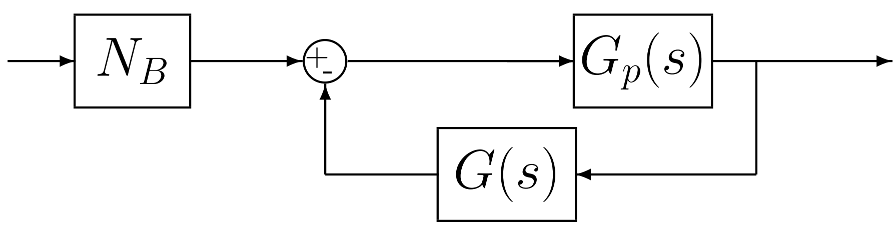
3 Using your Simulink model, look at the effects of the location of the observer poles and the rate of convergence of the estimated states to the true states. Specifically, plot the estimated and true states for observer poles at $[-4\ -5]$, $[-10\ -11]$, and $[-15-16]$. You should note that although the estimates may converge faster as the observer poles move farther away, there may be very large initial errors. Try putting the observer poles at $[-40-41]$ and run your simulations.

$\boxed{4}$ Add a Simulink model for the observer-controller transfer function model to your existing model, as shown below:



Here $G(s)$ is the transfer function of the observer controller, $G_p(s)$ is the plant, and N_A is the prefitler gain. (See Program 12-8 in the text for using Matlab to compute $G(s)$.) Although the plant model has been written in transfer function form, you will need to leave it in state variable form. We will refer to this as configuration A, and at this point we will assume the input is 0 (this is a regulator system) and the value of the prefilter N_A is not important.

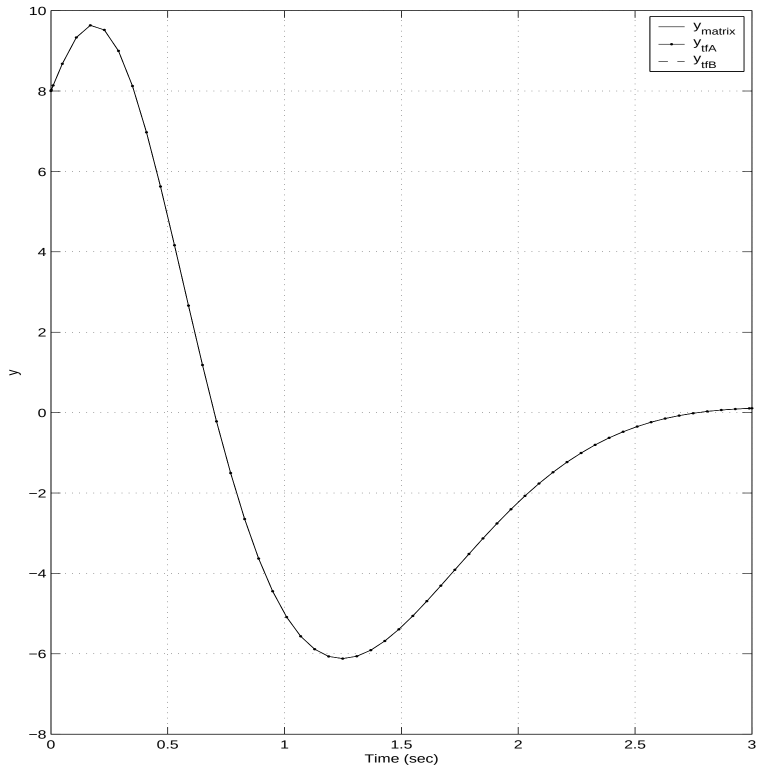
We will also be looking at the following configuration:



which we will refer to as configuration B. Make a Simulink model of this configuration also.

We will want to compare the results for the three different implementations as a check on all three models. You will basically need to determine the observer controller transfer function and you can copy the state variable description of the system. You should compare the Matlab computed observer-controller transfer function with that you computed in problem 1. Your model should produce a plot of the output of the (matrix based) observer controller, and the output of the observer controller (transfer function based) for implementations A and B.

For the system in problem 1, compare the results for the matrix based and transfer function based systems for an initial condition $\underline{x}(0) = [3\ 8]$. (*Note: Since there is no way to set* $\underline{\tilde{x}}(0)$ *in the observer-controller transfer function formulation, we must assume it is zero.*) You should get a plot similar to the following:



5 For our state variable feedback controller with a single input, single output system we have shown that, for the output in steady state to equal the input, the prefilter gain F is given by

$$F = \frac{-1}{C(A-BK)^{-1}B}$$

Assume we use the controller-observer transfer function implementation A. Show that the prefilter gain N_A is given by

$$N_A = 1 + \frac{1}{G_p(0)G(0)}$$

where

$$G_p(0) = -CA^{-1}B$$

if A is invertible.

If we determine the observer-controller in Matlab as suggested on page 869 (Program 12-8), then we will have a piece of code like

```
[numG,denG] = ss2tf(AA,BB,CC,DD)
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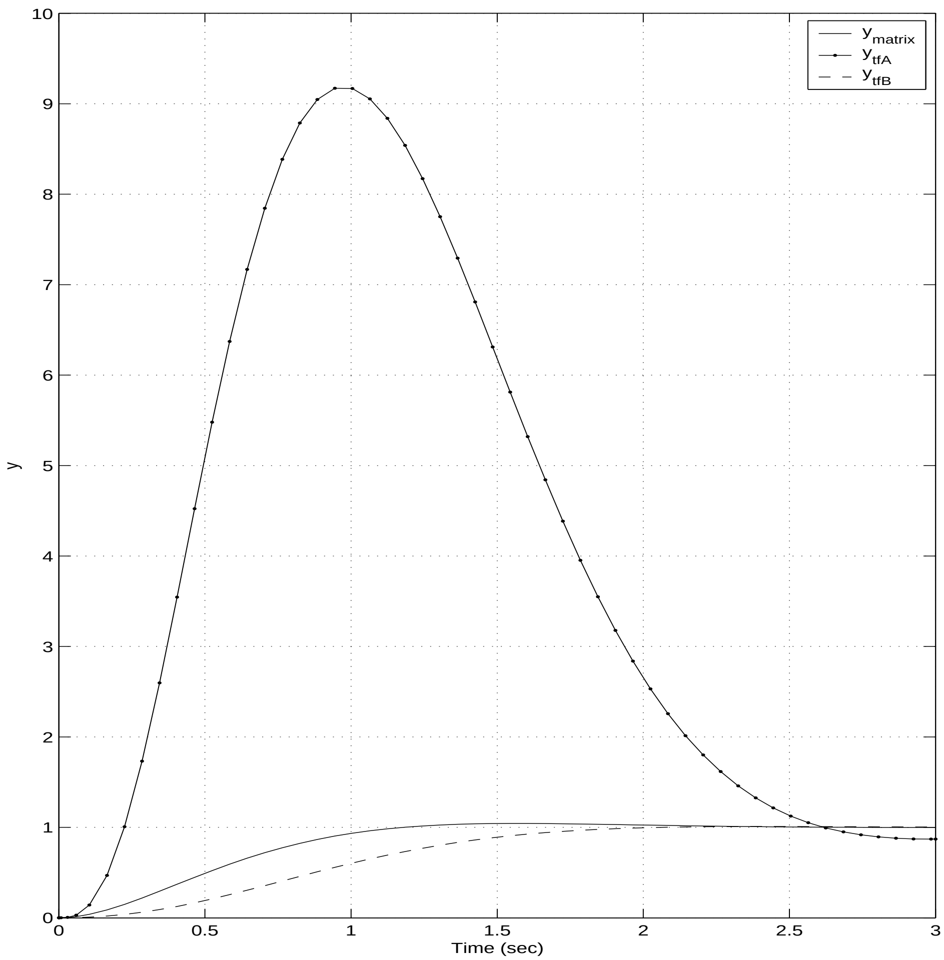
To compute $G(0)$ in Matlab we can type

```
G0 = numG(end)/denG(end);
```

6 Assume we use the controller-observer transfer function implementation B. Show that the prefilter gain N_B is given by

$$N_B = G(0) + \frac{1}{G_p(0)}$$

7 Set all initial conditions to zero, and look at the step response for both the transfer function observer controllers and the matrix based observer controller for our system with state variable feedback poles at $-2 \pm 2j$ and observer poles at $[-3 \ \ -4]$. All systems should end at a steady state value of 1, but they will not look very much alike. This is because the observer controller transfer function was designed assuming there was no input (it was designed as a regulator). Sometimes this transfer function is effective, but sometimes it is not. Your results should look something like the following:



$\boxed{8}$ Modify your systems to be used with the model

$$\begin{aligned} \underline{\dot{x}}(t) &= \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [1\ 0]\, \underline{x}(t) \end{aligned}$$

with closed loop poles at -1.8 ± 2.4 and observer poles both at -8 (Note that you may need to use **acker** in Matlab to get both poles at -8). Look at the unit step response for both the matrix based and the transfer function based observer controllers. In this case they should be very similar.

$\boxed{9}$ If A is singular (has a zero eigenvalue), then we cannot compute N_A and N_B in the way we did above. In this case, we use the above formulas assuming $G_p(0) \rightarrow \infty$. For the system

$$\begin{aligned} \underline{\dot{x}}(t) &= \begin{bmatrix} 0 & 0 \\ 20.6 & 0 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [1\ 1]\, \underline{x}(t) \end{aligned}$$

with closed loop poles at -1.8 ± 2.4 and observer poles both at -8, compare the step response of both types of observer-controllers and look at the effects of changing N.