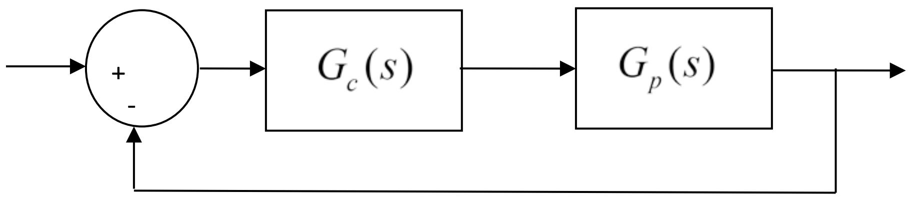
ECE-320: Linear Control Systems
Homework 3

Due: Thursday December 17 at the beginning of class

1) For the following problem, assume we are using the following control system



where the plant is given by

$$G_p(s) = \frac{1}{s^2 + 4s + 29} = \frac{1}{(s+2-5j)(s+2+5j)}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \frac{k}{s}$ (an integral (I) controller)

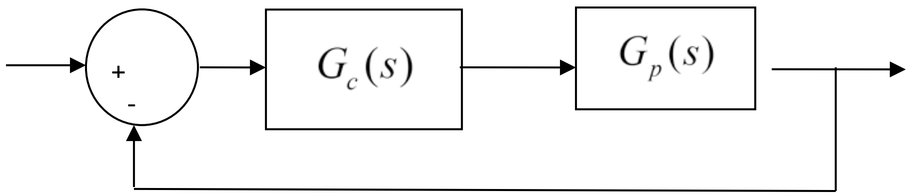
c) $G_c(s) = \frac{k(s+z)}{s}$ (a proportional + integral (PI) controller) Write the centroid σ_c as a function of z. For what values of z will the two asymptotes be in the right half plane? *(For plotting purposes, assume z is equal to 2.)*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller) *(For plotting purposes, assume z is equal to 2.)*

e) $G_c(s) = \frac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) Sketch this for the case where both zeros are real and then when both zeros are complex conjugates.

f) $G_c(s) = \frac{k(s+z)}{(s+p)}$ (a lead controller, $p > z$) Write an expression for σ_c as a function of the distance between the pole and the zero, $l = p - z$. What happens to the asymptotes as l gets larger? *(For plotting purposes, assume p is 5 and z is 1.)*

2) For the following problem, assume we are using the following control system



where the plant is given by

$$G_p(s) = \frac{1}{s+3}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \frac{k}{s}$ (an integral (I) controller)

c) $G_c(s) = \frac{k(s+z)}{s}$ (a proportional + integral (PI) controller) *Sketch this for the case when z is equal to 2 and then assume z is equal to 4; there will be two plots.*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller) *Sketch this for the case where z is equal to 2 and then assume z = 4; there will be two plots.*

e) $G_c(s) = \frac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) *Sketch this for the case where there are zeros are at* $-4 \pm 4j$ *and when they are at -6 and -8; there will be two plots.*