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ECE-320 Linear Control Systems

Spring 2012, Exam 1

No calculators or computers allowed, except for Problem 8 when you should use Matlab's sisotool.

You must simplify your answers as much as possible, or points will be deducted.

Problem 1 ______/7

Problem 2 ______/7

Problem 3 ______/7

Problem 4 ______/7

Problem 5 ______/7

Problem 6 ______/20

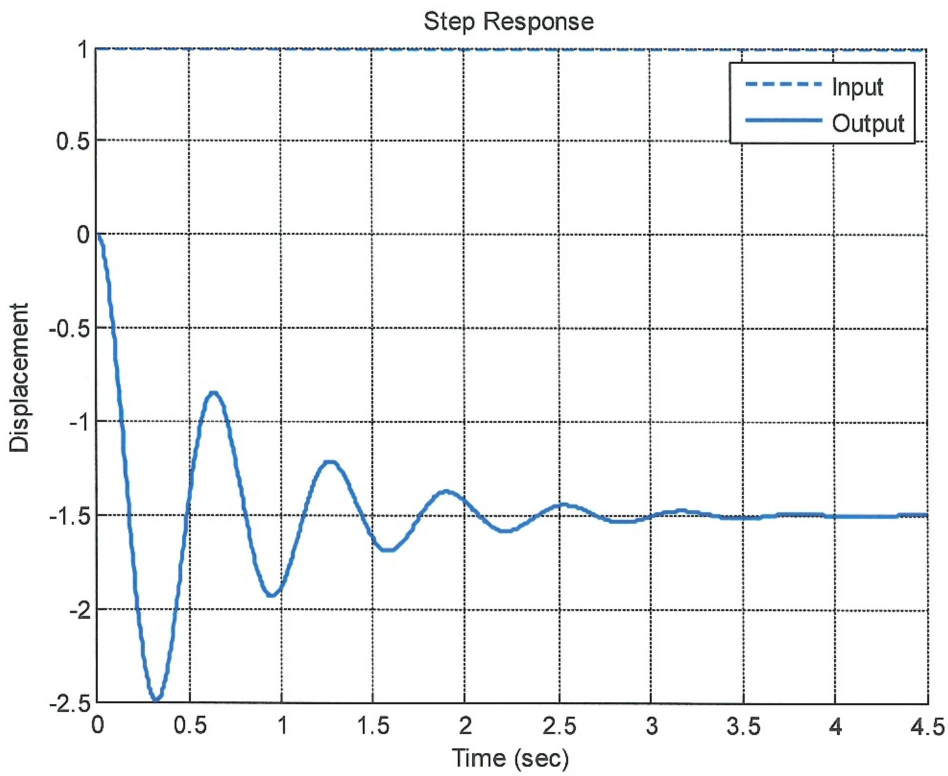
Problem 7 ______/20

Problem 8 ______/25

Total ______/100

Name ______________________________ Mailbox __________

1) (7 points) The following graph shows the **unit step response** for a system.



a) Estimate the **percent overshoot**

$\frac{-2.5-(-1.5)}{-1.5} = \frac{-1}{-1.5} = 2/3$

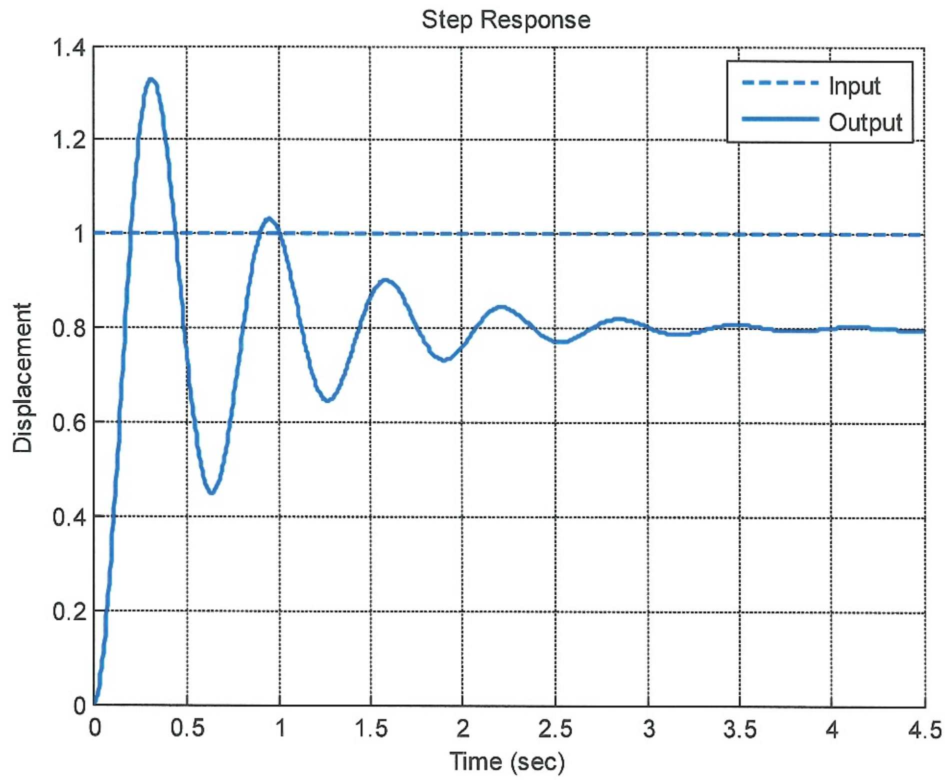
P.O. = 67%

b) Estimate the **static gain**

K(1) = -1.5   K = -1.5

Name ________________________________ Mailbox ____________

2) (7 points) The following graph shows the unit step response of a system.



a) Estimate the **steady state error.**

$e_{ss} = 1 - 0.8 = \boxed{0.2 = e_{ss}}$

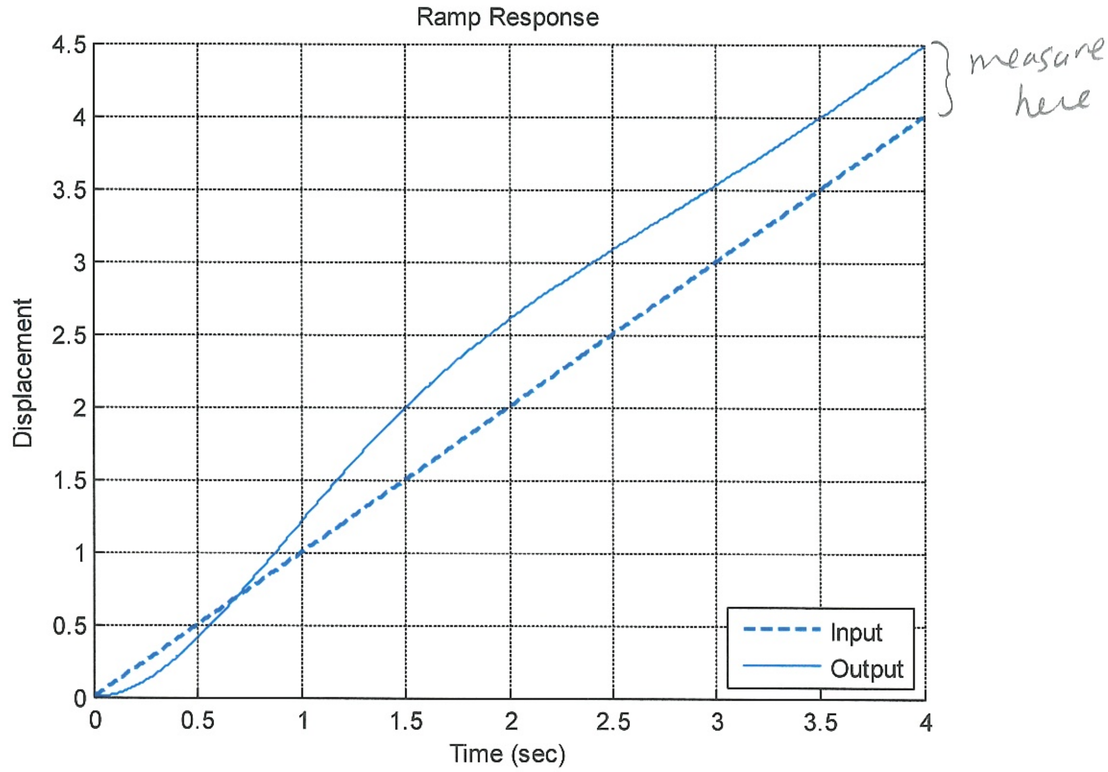
b) Estimate the steady state error for a unit ramp input.

type 0 system

$\boxed{e_{ss} = \infty \text{ for ramp}}$

Name ______________________________ Mailbox __________

3) (7 points) The following figure shows the unit ramp response of a system.



a) Estimate the **steady state error**.

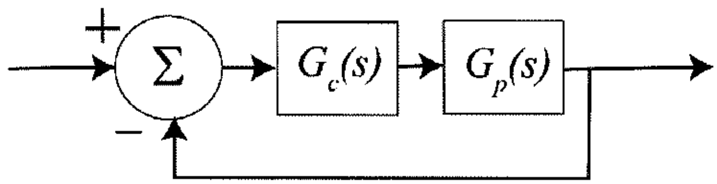
$e_{ss} = 4 - 4.5 = \boxed{-0.5 = e_{ss}}$

b) Estimate the steady state error for a unit step input. Type 1 system

$\boxed{e_{ss} = 0 \text{ for step}}$

Name ______________________________ Mailbox ____________

4) (7 points) For the following systems, assume $G_c(s) = \dfrac{s+2}{s+3}$ and $G_p(s) = \dfrac{s+5}{s+4}$



a) Determine the position error constant K_p

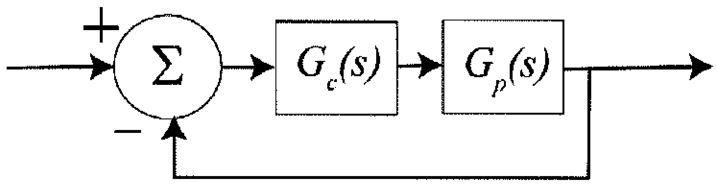
$$K_p = \lim_{s \to 0} G_c(s)\, G_p(s) = \boxed{5/6 = K_p}$$

b) Determine the steady state error for a unit step input.

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5/6} = \boxed{\frac{6}{11} = e_{ss}}$$

Name ______________________________ Mailbox ____________

5) (7 points) For the following systems, assume $G_c(s) = \frac{2}{s}$ and $G_p(s) = \frac{s+5}{s+4}$



a) Determine the velocity error constant K_v.

$K_v = \lim_{s \to 0} s\,G_c(s)\,G_p(s) = \boxed{5/2 = K_v}$

b) Determine the steady state error for a unit ramp input.

$e_{ss} = \frac{1}{K_v} = \boxed{\frac{2}{5} = e_{ss}}$

Name ______________________ Mailbox __________

6) (20 points) For a system with the transfer function $H(s) = \frac{1}{s(s+1)}$

a) Determine the **impulse response** $h(t)$

$$H(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} \qquad A = 1 \quad B = -1$$

$$\boxed{h(t) = (1 - e^{-t})u(t)}$$

b) Determine the **unit step response.**

$$Y(s) = H(s)\frac{1}{s} = \frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$B = 1 \quad C = 1 \qquad \times s$, let $s \to \infty \qquad 0 = A + C \quad A = -C = -1$

$$\boxed{y(t) = (-1 + t + e^{-t})u(t)}$$

Name ______________________ Mailbox ____________

7) (20 points) For the system with closed loop transfer function $G_o(s) = \dfrac{k}{s^2 + 2s + k}$ $= \dfrac{N}{D}$

a) Determine an expression for the sensitivity of G_o to k as a function of s, $S_k^{G_o}$

$$S_K^{G_o} = \frac{K}{N}\frac{\partial N}{\partial K} - \frac{K}{D}\frac{\partial D}{\partial K} = \frac{K}{K}(1) - \frac{K}{s^2+2s+K}(1)$$

$$= 1 - \frac{K}{s^2+2s+K} = \boxed{\frac{s^2+2s}{s^2+2s+K} = S_K^{G_o}}$$

b) Determine and expression for the magnitude of the sensitivity function as a function of frequency, ω. Simplify your answers as much as possible.

$$\left|S_K^{G_o}(j\omega)\right| = \left|\frac{-\omega^2+2j\omega}{-\omega^2+2j\omega+K}\right| = \boxed{\sqrt{\frac{\omega^4+4\omega^2}{(K-\omega^2)^2+4\omega^2}} = \left|S_K^{G_o}(j\omega)\right|}$$

Name ____________________________________ Mailbox ____________

8) (25 points) ***(sisotool problem)***

Consider the plant

$$G_p(s) = \frac{30}{s^2 + 6s + 10}$$

Design a PID controller using *sisotool* with ***real zeros*** so that

$$\begin{aligned} T_s &\leq 1.5\,\text{sec} \\ P.O. &\leq 10\% \end{aligned}$$

In addition, your controller must be designed so that

$$\begin{aligned} k_p &\leq 0.5 \\ k_i &\leq 2 \\ k_d &\leq 0.05 \end{aligned}$$

Write your final values for k_p, k_i, k_d, and the transfer function of the controller in the space below.