# ECE-320 Lab 3: System Identification and Model Matching Control of a 2 Degree of Freedom System

#### **Overview**

In the first part of this lab you will obtain a system model for one two degree of freedom rectilinear system. In the second part of the lab, you will be using model matching to try and control the system.

It will be easiest to do this if you download the files for this lab and put them in the same folder you used for Labs 1 and 2.

#### **PART A:** System Identification of a Two Degree of Freedom System

<u>Step 0: Set Up the System</u>. Both the first and second carts should move. The third cart should be fixed in place. In addition:

- Either the carts should have an equal amount of weight on them, or the first cart should have more weight than the second cart. You need at least one mass on each cart.
- Either all springs connecting carts should have equal stiffness, or the springs should get less stiff from left to right. You need to use at least two springs.
- If you want to use the active damper, unscrew the screw in the damper.

You will be using this configuration throughout the remainder of the course so be sure you write down all of the information you need to duplicate this configuration.

<u>Step 1) Initial Estimates of</u>  $\omega_1$ ,  $\zeta_1$ ,  $\omega_2$ , and  $\zeta_2$ 

From the equations of motion we have

$$\ddot{x}_1 + 2\zeta_1 \omega_1 \dot{x}_1 + \omega_1^2 x_1 = \frac{k_2}{m_1} x_2 + \frac{1}{m_1} F$$

$$\ddot{x}_2 + 2\zeta_2 \omega_2 \dot{x}_2 + \omega_2^2 x_2 = \frac{k_2}{m_2} x_1$$

a) If there is no applied force (F = 0) and the second cart is fixed in place ( $x_2 = 0$ ), we have

$$s^2 + 2\zeta_1\omega_1 s + \omega_1^2 = 0$$

Use the log decrement method to get our initial estimate of  $\omega_1$  and  $\zeta_1$ .

b) If the second cart is free to move and the first cart is fixed in place ( $x_1 = 0$ ), we have

$$s^2 + 2\zeta_2\omega_2 s + \omega_2^2 = 0$$

Use the log decrement method to get our initial estimate of  $\omega_2$  and  $\zeta_2$ . For the log-decrement analysis you will go throu following steps for <u>each</u> cart:

- Be sure only one cart is free to move
- Reset the system using **ECPDSPresetmdl.mdl.**
- Modify Model210\_Openloop.mdl so the input has zero amplitude.
- Compile Model210\_Openloop.mdl if necessary.
- Connect **Model210\_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Displace whichever cart is free to move and hold it.
- Start (play) Model210\_Openloop.mdl and let the cart go.
- Run the m-file **log\_dec.m.** This should be in the same directory as **Model210\_Openloop.mdl** and **log\_dec.fig**. This routine assumes the position of the first cart is labeled *x1*, the position of the second cart is labeled *x2*, and the time is labeled *time*. (These are the defaults in **Model210\_Openloop.mdl**.)

You will need to include these **two** log-decrement results in your memo.

Step 2) Estimating the Gains  $K_1$  and  $K_2$ 

You will go through the following steps:

- Be sure both carts are free to move
- Reset the system using **ECPDSPresetmdl.mdl.**
- Modify **Model210\_Openloop.mdl** so the input is a step. Set the amplitude to something small, like 0.01 or 0.02 cm.
- Compile Model210\_Openloop.mdl
- Connect Model210\_Openloop.mdl to the ECP system. (The mode should be External.)
- Run **Model210\_Openloop.mdl.** If the carts do not seem to move much, increase the amplitude of the step. If the carts move too much, decrease the amplitude of the step. You may have to recompile.
- Estimate the static gains as

$$K_1 = \frac{x_{1,ss}}{A} \quad K_2 = \frac{x_{2,ss}}{A}$$

where  $x_{ss}$  is the steady state value of the cart position, and A is the input amplitude.

You should do this in Matlab, don't use the X-Y Graph. The variables x1, x2, and *time* should be in your workspace. You need to increase the value of the input amplitude until the first cart is moving about 1.5 cm or so. Use the static gains associated with this input amplitude as your estimates of the static gain. You should try this for three different input values and average the estimated gains.

#### Step 3) Fitting the Estimated Frequency Response to the Measured Frequency Response

We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine the parameters we need. For each frequency  $\omega = 2\pi f$  we have as input  $u(t) = A\cos(\omega t)$  where, for out systems, A is measured in centimeters.

After a transition period, the steady state output will be  $x_1(t) = B_1 \cos(\omega t + \theta_1)$  for the first cart and

 $x_2(t) = B_2 \cos(\omega t + \theta_2)$  for the second cart, where both  $B_1$  and  $B_2$  are also measured in cm. Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles  $\theta_1$  and  $\theta_2$ .

You will go through the following steps

For frequencies f = 0.5, 1, 1.5...7.5 Hz

- Make sure both carts are free to move, and the third cart is fixed.
- Modify Model210\_Openloop.mdl so the input is a sinusoid. You may have to set the mode to Normal.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 0.01 cm. Generally this amplitude should be as large as you can make it without the system hitting a limit. This amplitude will probably vary with each frequency.
- Compile **Model210\_Openloop.mdl**, if necessary. (Assume it is not necessary. The system will let you know if it is necessary.)
- Connect **Model210\_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210\_Openloop.mdl.** If the carts do not seem to move much, increase the amplitude of the input sinusoid. If the carts move too much, decrease the amplitude of the input sinusoid.
- Record the input frequency ( f ), the amplitude of the input ( A ), and the amplitude of the output ( B<sub>1</sub> and B<sub>2</sub> ) when the system is in steady state. Modify the program **get\_Amp2.m** to help record these amplitudes accurately. Be sure the plot from **get\_Amp2** shows the system in steady state. Be sure to look at the graph and understand what the code is doing before you use it!!!

Enter the values of f, A,  $B_1$  and  $B_2$  into the program **process\_data\_2dof.m** (you need to edit the file)

At the Matlab prompt, type **data = process\_data\_2dof**;

Run the program **model\_2dof.m.** There are ten input arguments to this program:

- data, the measured data as determined by process\_data\_2dof.m
- the estimated value of  $K_2$
- $\omega_a$ , the estimated frequency of the first resonance, when both carts are moving, in radians/sec
- $\zeta_a$ , the estimated first damping ratio when both carts are moving. Assume  $\zeta_a = 0.1$ .
- $\omega_b$ , the estimated frequency of the second resonance, when both carts are moving, in radians/sec
- $\zeta_b$ , the estimated second damping ratio when both carts are moving. Assume  $\zeta_b = 0.1$ .
- $\omega_1$  the estimated natural frequency of the first cart when it is the only cart moving (from the log decrement analysis)
- $\zeta_1$  the estimated damping ratio of the first cart when it is the only cart moving (from the log decrement analysis)
- $\omega_2$  the estimated natural frequency of the second cart when it is the only cart moving (from the log decrement analysis)
- $\zeta_2$  the estimated damping ratio of the second cart when it is the only cart moving (from the log decrement analysis)

The program **model\_2dof.**m will produce the following:

- A graph indicating the fit of the identified transfer function to the measured data for the first cart (You need to include the *final* graph of this fit in your memo.)
- A graph indicating the fit of the identified transfer function to the measured data for the second cart (You need to include the *final* graph of this fit in your memo.)
- The optimal estimates of all parameters (written at the top of the graphs)
- A file **state\_model\_2dof.mat** in your directory. This file contains the A, B, C, and D matrices for the state variable model of the system. If you subsequently type **load state\_model\_2dof** you will load these matrices into your workspace.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of  $\zeta$  (which correspond to very sharp peaks) At this point you probably should go back and add a few points near both the resonant peaks and nulls.

#### Part B: Model Matching Control of a One degree of Freedom System

We would like our final design to meet the following design requirements:

- Settling time less than 1.0 seconds.
- Absolute value of the steady state error less than 0.1 cm for a 1 cm step, and less than 0.05 cm for a 0.5 cm step
- Percent Overshoot less than 10%

You should start with the lower step amplitudes, and work your way up to the higher step amplitudes. Not all systems or controllers will work for a 1 cm step. Your real systems may oscillate a bit. If this happens try to reduce the input level, since this limits the allowed control effort. *It may not be possible to eliminate all of the oscillations, since these types of controllers depend on canceling the plant dynamics,* and if your model is not accurate enough the controller will not cancel the real plant well enough. We will be trying only to control the position of the second cart!

<u>Step 1:</u> Modify **closedloop\_driver.m** to read in the correct model file.

Step 2: Modify **closedloop driver.m** to use the correct saturation level for the system you are using.

<u>Step 3:</u> Modify **closedloop\_driver.m** to use  $C = [0\ 0\ 1\ 0]$  since you will be trying to control the position of the second cart.

<u>Step 4:</u> Fourth Order ITAE Model Matching Control. (Be sure to record the value of  $\omega_a$  you use.)

- Using a fourth order ITAE system, vary  $\omega_o$  until you meet the design specs with the *simulation* (*closedloop\_driver.m*). The larger the value of  $\omega_o$ , the faster your system will respond and the closer your model will predict the response of the real system. Be sure you do not reach the limiter on the control effort, unless you really like restarting your computer. At this point all of the variables you should need are in your current Matlab workspace. You should run the simulation until the system is clearly at a steady state value, but at least one second.
- Open **Model210\_Closedloop.mdl** and be sure the *Base Address* is correct for your system.

- Modify Model210\_Closedloop.mdl so the feedback is x2 since you are trying to control the position of the second cart
- Compile **Model210\_Closedloop.mdl**. The parameters this file needs are in the workspace. The yellow block is what makes the ECP system work, and replaces your model of the system with the real system.
- Reset the system by running ECPDSPReset.mdl.
- Connect Model210\_Closedloop.mdl to the system and then run it.
- Use the **compare1.m** file (or a modification of it) to plot the results of both the simulation and the real system on one nice, neatly labeled graph. You may need to modify this file to get the plot you want. *You need to include this graph in your memo*.

<u>Step 5:</u> Fourth Order Deadbeat Model MatchingControl. (Be sure to record the value of  $\omega_o$  you use.)

- Using a fourth order deadbeat system, vary  $\omega_o$  until you meet the design specs with the *simulation* (*closedloop driver.m*). You should run the simulation until the system is clearly at a steady state value, but at least one second.
- Compile the correct closed loop ECP Simulink driver, connect to the system, and run the simulation.
- Use the **compare1.m** file (or a modification of it) to plot the results of both the simulation and the real system on one nice, neatly labeled graph. *You need to include this graph in your memo*.

Your memo should compare the difference between the predicted response (from the model) and the real response (from the real system) for each of the systems. There should be two figures per page, and they should be large enough I can read them.

# Be sure in include the number of the ECP system you are using. These sometimes get moved around and you need to be able to find the exact same one again. They are not interchangeable.

Your memo should also include a description of your two degree of freedom system (so you can set them it again), a table comparing the estimated values of the static gains, the natural frequencies, and the damping ratios using the two different methods (time domain and optimized frequency domain), and a brief comparison of the values. The damping ratios are often quite different, so that's OK. The other values should be close. You should include as attachments 6 additional graphs (2 log-decrement and 2 frequency response graphs, and two step response graphs using model matching), each with a figure number and caption. You should also include the data used for estimating the static gain.

### state\_model\_2dof.mat is now named :

process\_data\_2dof.m is now named:

ECP System Number:

## 2 dof rectilinear systems (model 210)

Damper:	Yes No	
Left Spring:	stiff/light/none	Spring Number =
Number of Large Masse on First Cart:	2 3 4	
Number of Small Masses on First Cart:	1 2 3 4	
Middle Spring:	stiff/light	Spring Number =
Number of Large Masse on Second Cart:	2 3 4	
Number of Small Masses on Second Cart:	1 2 3 4	
Right Spring:	stiff/light	Spring Number =