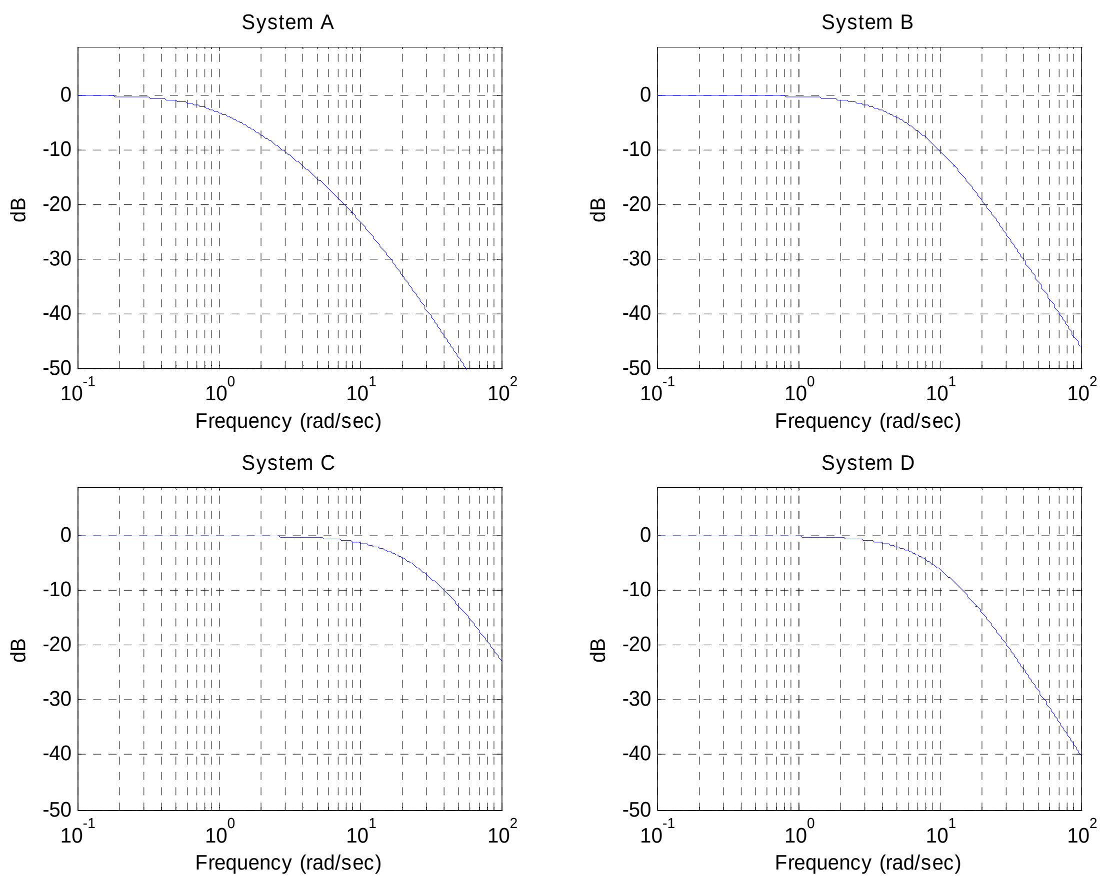
ECE-320: Linear Control Systems
Homework 5

Due: Tuesday April 19 at 10 AM

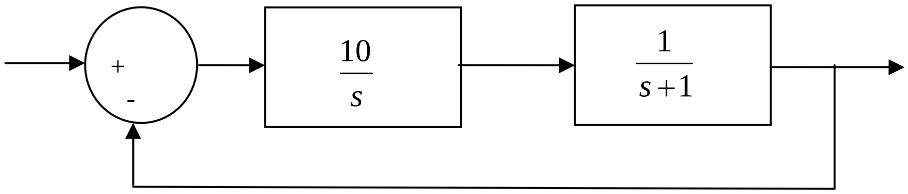
Exam Friday April 15

1) The following four plots display the frequency response (magnitude only) for four different systems with real poles. Estimate the settling time for each system.



Approximate Answers. (Not in the correct order) 0.6, 0.2, 4, 0.8 seconds.

2) Consider the following control system:

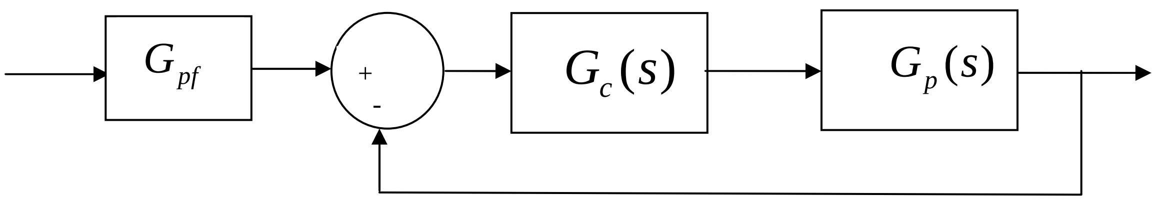


a) If the input to the system is $r(t)=8u(t)$, what is the steady state output?

b) If the input to the system is $r(t)=8\sin(3t)u(t)$, what is the output in steady state? What is the time lag between the input signal and the output signal?
Hint: you can write $\omega t-\theta=\omega(t-t_d)$ *if* θ *is measured in radians.*

Answers: $y(t)=8, y(t)=8\sqrt{10}\sin(3t-71.57^\circ), t_d=0.416\,\text{sec}$

3) Consider the following control system.



We can compute the position error constant K_p as $K_p=G_c(0)G_p(0)$

a) Determine an expression for the closed loop transfer function (from input to output) $G_o(s)$ in terms of G_{pf}, $G_c(s)$, and $G_p(s)$.

b) For zero position error we want $G_o(0)=1$. Use this information to show that we can determine the prefilter gain to be

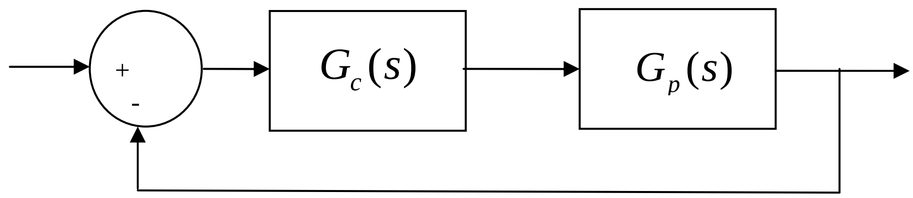
$$G_{pf}=1+\frac{1}{K_p}$$

c) We can find the position error as $e_p=\dfrac{1}{1+K_p}$. Using this, show that we can determine the prefilter gain to be

$$G_{pf}=\frac{1}{1-e_p}$$

Note that if $K_p=\infty$ *(or equivalently* $e_p=0$*), we get* $G_{pf}=1$.

4) For the following problem, assume we are using the following control system



where the plant is given by

$$G_p(s) = \frac{1}{s^2+4s+29} = \frac{1}{(s+2-5j)(s+2+5j)}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \frac{k}{s}$ (an integral (I) controller)

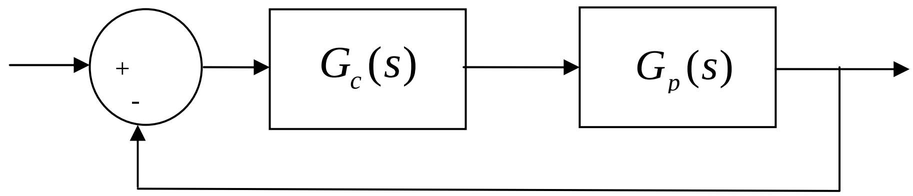
c) $G_c(s) = \frac{k(s+z)}{s}$ (a proportional + integral (PI) controller) Write the centroid σ_c as a function of z. For what values of z will the two asymptotes be in the right half plane? *(For plotting purposes, assume z is equal to 2.)*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller)

e) $G_c(s) = \frac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) Sketch this for the case where both zeros are real and then when both zeros are complex conjugates.

f) $G_c(s) = \frac{k(s+z)}{(s+p)}$ (a lead controller, $p > z$) Write an expression for σ_c as a function of the distance between the pole and the zero, $l = p - z$. What happens to the asymptotes as l gets larger? *(For plotting purposes, assume p is 5 and z is 1.)*

5) For the following problem, assume we are using the following control system



where the plant is given by

$$G_p(s) = \frac{1}{s+3}$$

Want to determine if it is possible to meet the following constraints

- $T_s \leq 1\sec$
- $e_p \leq 0.1$

with P, I, PD, and PID (real zeros and complex conjugate zeros)controllers. For each of these controllers, you need to

- Sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

- State whether it is possible, based on closed loop pole locations, to meet the constraints. If it is necessary to put constraints on k, p, or z to meet the constraints, you must specify them. *(At this point you can only put conditions on k to meet the position error constraints.)*

- Determine if it is possible for the output to oscillate (nonzero ω_d)

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

Preparation for Lab 6.

Nothing this week, don't you have enough to do?