ECE300 Hw4 Sol W0809

Tuesday, January 06, 2009
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Problem 1: Determine if the following systems are BIBO stable.

(a) $y(t) = \int_{-\infty}^{t} (x(\lambda) - 5)\, d\lambda$

$h(t) = \int_{-\infty}^{t} \delta(\lambda) - 5 \; d\lambda = u(t) - 5\,[t + \infty]$

The system is obviously not stable because even an impulse produces an unbounded output

NOT BIBO stable

(b) $y(t) = \cos\left(\frac{1}{x(t)}\right)$

yes it is BIBO stable. The cosine function is bounded between ± 1 for all values of $x(t)$

(c) $y(t) = e^{-|x(t)|}$

$e^{-|x|}$ is bounded to be less than 1 for all values of x. yes it is BIBO stable

(d) $y(t) = x(t) + y(t)\,x(t)$

$y(t) = \dfrac{x(t)}{1 - x(t)}$ if $x(t) = 1$ $y(t) = \infty$

NOT BIBO stable

Problem 2

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For systems w/ following Impulse response, determine if the system is stable

(a) $h(t) = e^{-t}u(t)$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^{\infty} e^{-t}\,dt = 1$$ so BIBO stable

(b) $h(t) = u(t)$

$$\int_{-\infty}^{\infty} u(t)\,dt = \int_0^{\infty} dt = \infty$$ Not BIBO stable

(c) $h(t) = u(t) - u(t-10)$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^{10} dt = 10$$ BIBO stable

(d) $h(t) = \delta(t-1)$

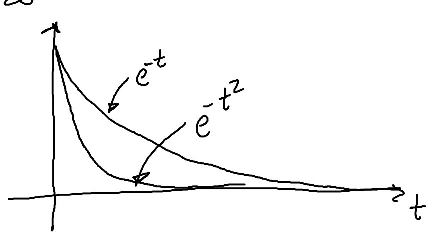
$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} \delta(t-1)\,dt = 1$$ BIBO stable

(e) $h(t) = \sin(t)\,u(t)$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^{\infty} |\sin(t)|\,dt = \infty$$ Not BIBO stable

(f) $h(t) = e^{-t^2}u(t)$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^{\infty} e^{-t^2}\,dt \le \int_0^{\infty} e^{-t}\,dt$$ because e^{-t^2} decays to 0 faster than e^{-t} which is part (a)



So BIBO stable

Problem 3

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(a) Use Eulers Identity to show that

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\beta-\alpha)\right]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2j}\left[e^{j\alpha} - e^{-j\alpha}\right]\frac{1}{2}\left[e^{j\beta} + e^{-j\beta}\right]$$

$$= \frac{1}{4j}\left[e^{j(\alpha+\beta)} + e^{j(\alpha-\beta)} - e^{j(\beta-\alpha)} - e^{-j(\alpha+\beta)}\right]$$

$$= \frac{1}{2}\left[\frac{e^{j(\alpha+\beta)} - e^{-j(\alpha+\beta)}}{2j} - \frac{e^{j(\beta-\alpha)} - e^{-j(\beta-\alpha)}}{2j}\right]$$

$$= \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\beta-\alpha)\right]$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\sin(\alpha)\sin(\beta) = \frac{-1}{4}\left[e^{j(\alpha)} - e^{-j\alpha}\right]\left[e^{j\beta} - e^{-j\beta}\right]$$

$$= \frac{-1}{4}\left[e^{j(\alpha+\beta)} - e^{j(\alpha-\beta)} - e^{j(\beta-\alpha)} + e^{-j(\alpha+\beta)}\right]$$

$$= \frac{-1}{2}\left[\frac{\left[e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}\right]}{2} - \frac{\left[e^{j(\alpha-\beta)} + e^{-j(\alpha-\beta)}\right]}{2}\right]$$

$$= \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

(b) show that for $x(t) = V_m \sin\left(\frac{\omega_o t}{2}\right)$ $0<t<T_o$ the trig Fourier Series are given by

$$a_o = \frac{2V_m}{\pi}$$

$$a_k = \frac{V_m}{\pi}\frac{1}{0.25-k^2}$$

$$b_k = 0$$

$$a_k = \frac{2}{T_o}\int_0^{T_o} V_m \sin\left(\frac{\omega_o t}{2}\right)\cos(k\omega_o t)\,dt$$

$$= \frac{V_m}{T_o}\int_0^{T_o}\left[\sin\left(\frac{\omega_o t}{2} + k\omega_o t\right) - \sin\left(k\omega_o t - \frac{\omega_o t}{2}\right)\right]dt$$

$$= \frac{V_m}{T_o}\left[\frac{-1}{\omega_o(k+\frac{1}{2})}\cos\left(\omega_o t\left(k+\tfrac{1}{2}\right)\right) + \frac{1}{\omega_o(k-\frac{1}{2})}\cos\left(\omega_o t\left(k-\tfrac{1}{2}\right)\right)\right]\Bigg|_0^{T_o}$$

$$= \frac{V_m}{T_o}\left[\frac{-1}{\omega_o(k+\frac{1}{2})}\cos\left(2\pi\left(k+\tfrac{1}{2}\right)\right) + \frac{1}{\omega_o(k-\frac{1}{2})}\cos\left(2\pi\left(k-\tfrac{1}{2}\right)\right) + \frac{1}{\omega(k+\frac{1}{2})} - \frac{1}{\omega_o(k-\frac{1}{2}}\right]$$

$$\cos\left(2\pi\left(k+\tfrac{1}{2}\right)\right) = \cos\left(2\pi\left(k-\tfrac{1}{2}\right)\right) = -1 \quad \forall\, k = 1,2,3,\ldots$$

$$a_k = \frac{V_m}{T_o}\left[\frac{2}{\omega_o(k+\frac{1}{2})} - \frac{2}{\omega_o(k-\frac{1}{2})}\right] = \frac{2V_m}{T_o}\left[\frac{k-\frac{1}{2}-k-\frac{1}{2}}{\omega_o\left(k^2-\frac{1}{4}\right)}\right]$$

$$= \frac{2V_m}{T_o}\,\frac{1}{\omega_o\left(\frac{1}{4}-k^2\right)} = \boxed{\frac{V_m}{\pi\left(\frac{1}{4}-k^2\right)}}$$

$$a_o = \frac{1}{T_o}\int_0^{T_o} V_m \sin\left(\frac{\omega_o t}{2}\right)dt = -\frac{2V_m}{\omega_o T_o}\cos\left(\frac{\omega_o t}{2}\right)\Big|_0^{T_o} = -\frac{V_m}{\pi}\left[\cos(\pi) - \cos(0)\right]$$

$$= \frac{2V_m}{\pi}$$

(b) cont'd.

$$b_k = \frac{2}{T_0}\int_0^{T_0} \sin\left(\frac{\omega_0 t}{2}\right)\sin(k\omega_0 t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T}\left[\cos\left(\frac{\omega_0 t}{2} - k\omega_0 t\right) - \cos\left(\frac{\omega_0 t}{2} + k\omega_0 t\right)\right]dt$$

$$= \frac{1}{T_0}\left[\frac{1}{\omega_0\left(\frac{1}{2}-k\right)}\sin\left(\frac{\omega_0 t}{2} - k\omega_0 t\right) - \frac{1}{\omega_0\left(\frac{1}{2}+k\right)}\sin\left(\frac{\omega_0 t}{2} + k\omega_0 t\right)\right]\Bigg|_0^{T_0}$$

$$= \frac{1}{T_0}\left[\frac{1}{\omega_0\left(\frac{1}{2}-k\right)}\sin\left(2\pi\left(\frac{1}{2}-k\right)\right) - \frac{1}{\omega_0\left(\frac{1}{2}+k\right)}\sin\left(2\pi\left(\frac{1}{2}+k\right)\right) - 0 + 0\right]$$

$$\sin\left(2\pi\left(\frac{1}{2}-k\right)\right) = \sin\left(2\pi\left(\frac{1}{2}+k\right)\right) = 0 \quad \forall\ k = 1, 2, 3, \dots$$

$$b_k = 0$$

Problem 4

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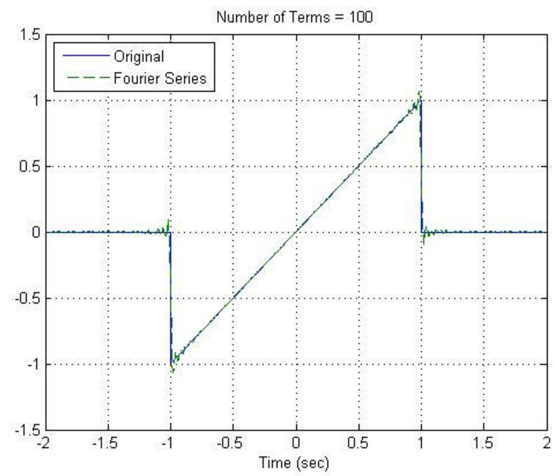
Matlab

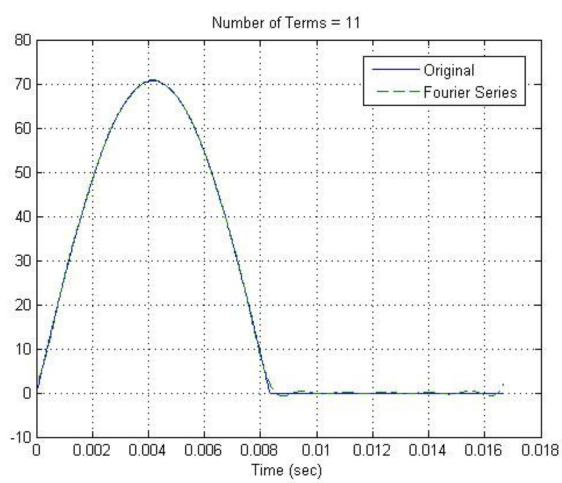
(a) The result of a N=100 sine series is:

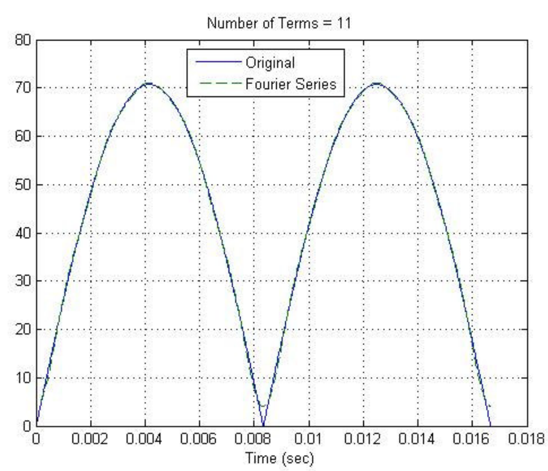


(b) • What variable in the code represents the Fourier Series Approx?
est

• Why does a_0 not need to be in "for loop"?
Because it is the DC offset of the estimate it can be used as the initial condition before the for loop.

d)





```
%
% Thsir routine implements a trigonometric Fourier  Series
%
% Inputs: N is the number of terms to be used in the series
%
function Trig_Fourier_series(N)
%
% one period of the function goes from low to high
%
 low = 0;
 high = 1/60;
%
% the difference between low and high is one period
%
 T = high-low;
 w0 = 2*pi/T;
%
% the periodic function
%
 x  = @(t) 100/sqrt(2)*abs(sin(2*pi*60*t)).*(t<=T);
%
%  find b(1) to b(N)
%
 for k = 1:N
    barg = @(t) x(t).*sin(k*w0*t);
    aarg = @(t) x(t).*cos(k*w0*t);
    a(k) = (2/T)*quadl(aarg,low,high);
    b(k) = (2/T)*quadl(barg,low,high);
 end;
%
% determine a time vector over one period
%
 t = linspace(low,high,1000);
%
% Find the Fourier series representation
%
 a0arg = @(t) x(t);
 a0 = (1/T)*quadl(a0arg,low,high);
 est = a0;
 for k = 1:N
    est = est + b(k)*sin(k*w0*t) + a(k)*cos(k*w0*t);
 end;
%
% plot the results
%
 plot(t,x(t),'-',t,est,'--'); grid; xlabel('Time (sec)');
 legend('Original','Fourier Series','Location','NorthWest');
 title(['Number of Terms = ', num2str(N)]);
%
```