

## Computation /Measurement of Average Power in Periodic Signals

### Lab 05

by Bruce A. Black with some tweaking by others

### Objectives

- To predict the average power in different frequency components of a periodic signal using the Fourier series representation of the periodic signal.
- To measure the average power in different frequency components of a periodic signal.
- To compare the predicted average power with the measured average power.
- To become acquainted with the Agilent E4402B Spectrum Analyzer.

### Equipment

Agilent E4402B Spectrum Analyzer,  
AgilentFunction Generator, Oscilloscope

### Background

Recently we learned to represent a periodic signal by using the Fourier series. We have in our lab *spectrum analyzers* that can display the spectrum of a signal in pseudo- real time. The spectrum of a signal is a representation of the average power in each frequency component of the signal. The Agilent E4402B Spectrum Analyzer (SA) can be used to view the *power spectrum* of any signal of frequency up to 3 GHz. The SA displays a “one-sided spectrum” in decibels (dBs) versus frequency. In lab we will observe the spectra of sinusoids, square, and triangle waves, but first we must learn how to convert the Fourier series coefficients that we calculate to the dB values displayed by the spectrum analyzer.

Using a trigonometric Fourier series, we can represent any periodic function  $x(t)$  as

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_o t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_o t)$$

where

$$a_0 = \frac{1}{T_o} \int_{T_o} x(t) dt$$

$$a_k = \frac{2}{T_o} \int_{T_o} x(t) \cos(k\omega_o t) dt$$

$$b_k = \frac{2}{T_o} \int_{T_o} x(t) \sin(k\omega_o t) dt$$

$T_o$  is the fundamental period

$$\omega_o = \frac{2\pi}{T_o} = 2\pi f_o \text{ is the fundamental frequency}$$

A more useful way for us to write the Fourier series is using the complex, or exponential form,

$$x(t) = \sum_{k=-\infty}^{k=\infty} c_k e^{jk\omega_0 t}$$

where

$$c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

If we use Euler's identity to expand this out, we get

$$c_k = \frac{1}{T_0} \int_{T_0} x(t) [\cos(k\omega_0 t) - j\sin(k\omega_0 t)] dt$$

or

$$c_k = \frac{1}{T_0} \int_{T_0} x(t) \cos(k\omega_0 t) dt - j \frac{1}{T_0} \int_{T_0} x(t) \sin(k\omega_0 t) dt$$

Equating terms with the trigonometric Fourier series we get

$$c_0 = a_0$$

$$c_k = \frac{a_k}{2} - j \frac{b_k}{2}$$

Hence, from the trigonometric Fourier series we can also determine the coefficients for the complex (exponential) Fourier series.

Writing out a few terms of the exponential Fourier series gives

$$x(t) = \dots + c_{-2} e^{-j2\omega_0 t} + c_{-1} e^{-j\omega_0 t} + c_0 + c_1 e^{j\omega_0 t} + c_2 e^{j2\omega_0 t} + \dots$$

$$= \dots + |c_2| e^{-j\angle c_2} e^{-j2\omega_0 t} + |c_1| e^{-j\angle c_1} e^{-j\omega_0 t} + c_0 + |c_1| e^{j\angle c_1} e^{j\omega_0 t} + |c_2| e^{j\angle c_2} e^{j2\omega_0 t} + \dots$$

where we have used the fact that  $c_{-k} = c_k^*$  whenever  $x(t)$  is real-valued. Notice that, aside from  $c_0$ , the terms come in pairs (actually, complex conjugate pairs). We can combine each positive-frequency term with its matching negative frequency term to obtain

$$x(t) = c_0 + 2|c_1| \cos(\omega_0 t + \angle c_1) + 2|c_2| \cos(2\omega_0 t + \angle c_2) + \dots$$

From our study of power signals, we know that for the periodic power signal

$$x(t) = A \cos(\omega t + \theta)$$

the average power is  $P_{ave} = \frac{A^2}{2}$

From the complex Fourier series representation, we then know

$$P_1 = \frac{1}{2} [2|c_1|]^2 = 2|c_1|^2 \quad \text{is the average power of } x(t) \text{ at frequency } \omega_0$$

$P_2 = \frac{1}{2}[2|c_2|^2] = 2|c_2|^2$  is the average power of  $x(t)$  at frequency  $2\omega_0$

and in general, for  $k \neq 0$

$P_k = \frac{1}{2}[2|c_k|^2] = 2|c_k|^2$  is the average power of  $x(t)$  at frequency  $k\omega_0$

For  $k = 0$  the average power is  $P_0 = c_0^2$ .

We can make a plot of the average power in each frequency, which is called a **single sided (or one sided) power spectrum**. The spectrum analyzer displays the one-sided spectrum of a signal, but instead of showing the value of  $2|c_k|^2$  at each frequency, the spectrum analyzer shows average power in decibels with respect to a one millivolt RMS reference. For the sinusoid at frequency  $k\omega_0$ , the average power in decibels is given by

$$P_k|_{dB} = 10 \log_{10} \frac{P_k}{P_{ref}},$$

where the power  $P_k$  represents the power spectrum coefficient  $2|c_k|^2$ , and the power  $P_{ref}$  is the average power delivered to a one-ohm resistor by a one millivolt RMS sinusoid. We have

$$P_k|_{dBmV} = 10 \log_{10} \frac{2|c_k|^2}{(0.001)^2} \text{ dBmV}.$$

The units “dBmV” indicate that the reference for the decibels is a one millivolt RMS sinusoid.\*

**Note:** The spectrum analyzer will not display the DC term  $|c_0|^2$  even when one is present in the signal. Instead it displays a large spike at zero frequency allowing for easy location of DC on the display. Also, because it is showing a power spectrum, the spectrum analyzer does not measure or display the phase angles.  $\angle c_k$

\* Further information on working with dBs is available in the document called “Guide to dBs” available on the class webpage. It is suggested you read this before lab.

## Procedure

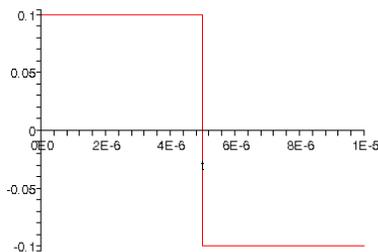
1. We have shown that, if we have computed the  $a_k$  and  $b_k$  from the trigonometric Fourier series, we can determine the  $c_k$  as

$$c_0 = a_0$$

$$c_k = \frac{a_k}{2} - j \frac{b_k}{2}$$

Now assume instead we have computed the  $c_k$  and want to compute the  $a_k$  and  $b_k$ . Show how we can do this. *Hint:* one simple way to do this is to use  $c_k$  and its complex conjugate,  $c_k^*$ . Write your answers on the answer sheet at the end of the lab.

2a. Use your trigonometric Fourier Series program from the homework to determine the trigonometric Fourier series representation for a square wave of period  $10 \mu\text{s}$  and peak-to-peak amplitude  $0.2 \text{ V}$ . The waveform we want to use is displayed below



**Note:** The function we use for numerical integration, `quadl`, has a default tolerance. This tolerance level is used so the function can determine when its estimate of the integral is “good enough”. When dealing with either small valued functions, or very short intervals, you may need to change this tolerance. For the signal in this lab, you will need to change the tolerance to get the proper results (use help `quadl`).

2b. Plot the trigonometric Fourier series representation and the single-sided power spectrum in dBmV. Use 9 terms. To plot the single sided power spectrum, we just plot the average power terms versus the corresponding frequency  $0 \quad \omega_0 \quad 2\omega_0 \quad \dots \quad N\omega_0$ . The average power for each harmonic is given by  $|c_k|^2/2$ . Since the fundamental frequency  $\omega_0$  is common to all of the frequency terms, we often just plot the Fourier indices, or *harmonics*,  $0 \quad 1 \quad 2 \quad \dots \quad N$ . To get the powers in terms of dB, you should use the **log10** command to compute the base 10 logarithm. You should also use the **subplot** command so you can plot both a signal and its Fourier series representation in one graph, and its single-sided power spectrum in a subsequent graph within the same window. Utilize the **stem** command in Matlab to do the plotting. For the power spectrum plot, plot the power versus the harmonic. The y-axis should be labeled *Power(dBmV)*, the x-axis labeled *Harmonic*. Your plots for this function should look like those in Figure 1. (Note the axis on the top graph is limited using the command **axis('tight')**, and is limited on the

second plot using the command `axis([0 N 0 40]);` ) Be sure to plot both the Fourier series representation and the single sided power spectrum on one page, print it out and turn it in.

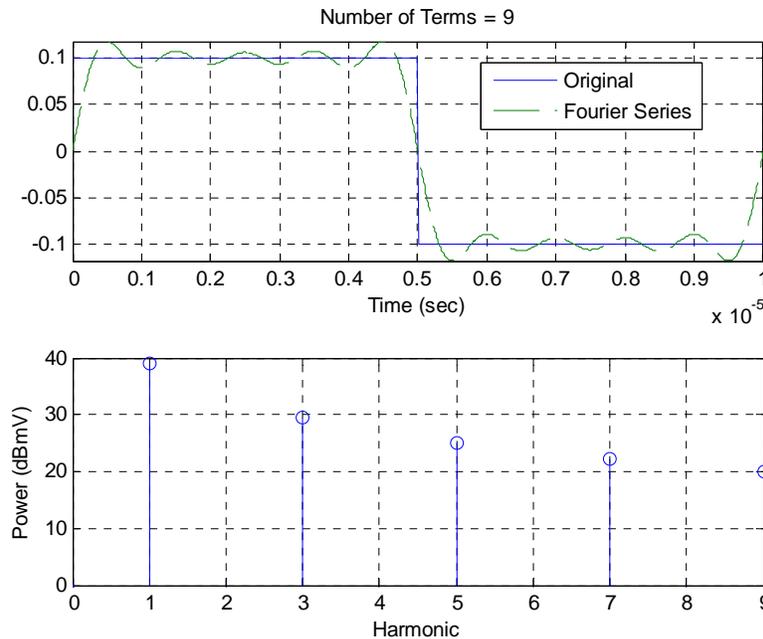


Figure 1: Results for problem 2b.

**BIG NOTE:** You will need to turn in your code for this lab. It must be neat and not contain superfluous stuff, only what is needed for this lab! (You can comment out each function....)

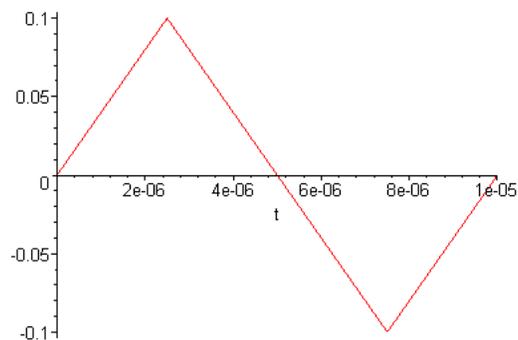
3. Read the documents "SA\_hints\_E4402B" and "Reading\_SA\_Display\_E4402B", both available on the class webpage.
4. Calibrate the spectrum analyzer as described in the document **SA\_hints\_E4402B**.
5. Use the function generator to generate a sinusoid of frequency 100 kHz and amplitude 0.1 V. Use the oscilloscope to verify that the amplitude is the correct value for the spectrum analyzer (meaning that the SA sees an amplitude of 0.1V). Now observe the signal power spectrum on the spectrum analyzer. Measure the power level and frequency. Record your measurements in the table on the final page of this lab.

\*\* **Note** concerning Agilent FG amplitude readings.

The Agilent function generators have an interesting feature built into their displays. The FG is a 50  $\Omega$  output impedance device, designed to deliver maximum power to a 50  $\Omega$  load. The FG by default will display the voltage amplitude delivered to a 50  $\Omega$  load, independent of what is actually connected. By changing the display setting to "High-Z", the FG display will display the voltage delivered to a high impedance load, assuming a high impedance load is connected.

However, the FG remains a  $50\ \Omega$  output impedance device. Since the scope and spectrum analyzer have different input impedances, caution must be used in reading the FG display.

6. Informally vary the frequency and the amplitude of the sinusoid and observe how the spectrum analyzer display changes.
7. Use the function generator to generate a square wave of period  $10\ \mu\text{s}$  and peak-to-peak amplitude  $0.2\ \text{V}$ , as seen by the SA. Use the oscilloscope to verify the amplitude\*\*. Now observe the signal power spectrum on the spectrum analyzer. Measure the power level and frequency. Record your measurements in the table on the final page of this lab.
8. Informally vary the frequency and the amplitude of the square wave and observe how the spectrum analyzer display changes.
9. Generate the trigonometric Fourier series for a triangle wave of period  $10\ \mu\text{s}$  and peak-to-peak amplitude  $0.2\ \text{V}$ . The waveform we want to use is displayed below



10. Plot the single sided power spectrum (average power versus harmonic) for this signal. Fill in the average powers in the table for waveform 3 at the end of this lab. Be sure to determine the average power values more accurately than by reading the graph. *Be sure to plot both the Fourier series representation and the single sided power spectrum on one page, print it out and turn it in.*
11. Use the function generator to generate a triangle wave of frequency  $100\ \text{kHz}$  and peak-peak amplitude of  $0.2\ \text{V}$  as seen by the SA. Use the oscilloscope to verify the amplitude\*\*. Now observe the signal power spectrum on the spectrum analyzer. Measure the power level and frequency. Record your measurements in the table on the final page of this lab and compare the measured average powers with the predicted average powers.
12. Informally vary the frequency and the amplitude of the triangle wave and observe how the spectrum analyzer display changes.

**Measurement of Fourier Coefficients-Instructor Verification Sheet**

Name \_\_\_\_\_ Date of Lab: \_\_\_\_\_

Record your procedure for using the spectrum analyzer in the space below. How should the output of the signal generator be set? Is the signal measured on the oscilloscope the size predicted by the signal generator?

Answers for problem 1, (how to get the  $a_k$  and  $b_k$  from the  $c_k$ ) Show all your work! Maple is not allowed as a method!

## Summary of Results for Waveform 1 (Part 5)

Harmonic	Predicted Power (dBmV)	Measured Power (dBmV)	Error (dBmV)	Frequency (kHz)
1	37.0			

## Summary of Results for Waveform 2 (Part 7)

Harmonic	Predicted Power (dBmV)	Measured Power (dBmV)	Error (dBmV)	Frequency (kHz)
1	39.1			
2	---			
3	29.5			
4	---			
5	25.1			
6	---			
7	22.2			
8	---			
9	20.0			

## Summary of Results for Waveform (Parts 10 and 11)

Harmonic	Predicted Power (dBmV)	Measured Power (dBmV)	Error (dBmV)	Frequency (kHz)
1				
2				
3				
4				
5				
6				
7				
8				
9				

**Be sure to attach your plots for parts 2b and 10**

**Attach your code for this lab to this sheet**