

SYSTEM LINEARITY

Lab 3

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Prelab: No Prelab

Objectives

The purpose of this lab is to gain a better understanding of what LINEARITY means in reference to a real system. You will begin the lab by using a transistor amplifier circuit and measuring the input and output signals as you vary the amplitude of your input signal. You will then compare the results of your experiment to the “expected” outputs by plotting your results in Matlab. You will also explore linearity through the use of sinusoidal analysis of systems by both listening to the input and output sounds, and by looking at the input and output waveforms.

1. Linearity of an amplifier

The common source amplifier, as shown in Figure 1 below, has been fabricated on a printed circuit board for you. The corresponding node names have also been printed on the PCB.

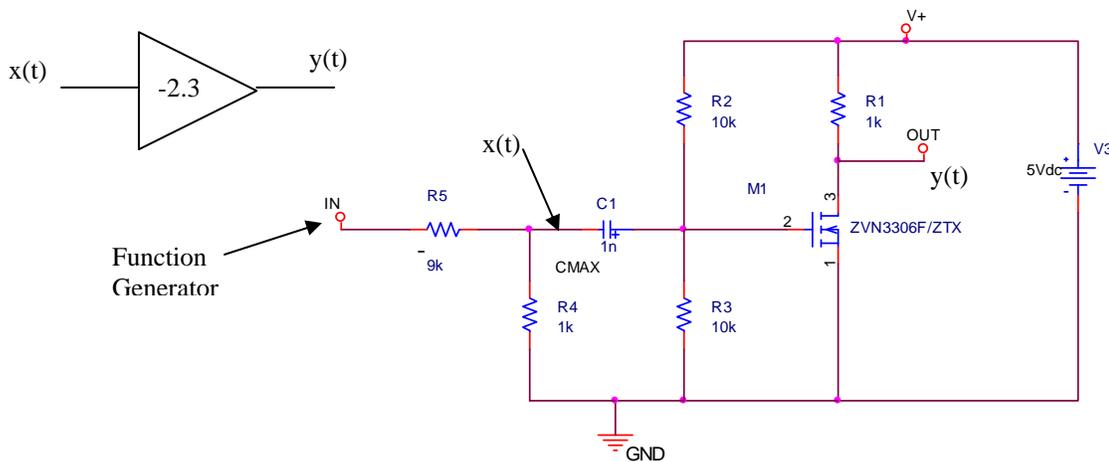


Figure 1: Circuit Diagram of the common-source amplifier. The ideal block diagram representation for this amplifier is shown as the triangle in the upper left.

Ideally, this amplifier should have a gain of -2.3 regardless of the input signal’s amplitude. In other words, in an ideal amplifier, if the input were to go to 100 V then the output should go to -230 V. We know that this is not possible because our power supply only goes to 5 V. Saturation at the energy rails is a problem with any physical device because we cannot physically represent infinity. With transistor amplifiers the problem is even worse, because as the input gets larger, the nonlinearities of the transistors begin to affect the signal. When the input signal to the amplifier is “small”, we can approximate the transistor as a linear system with its small-signal model that is composed of linear elements. As long as the input signal stays small, the amplifier

behaves like a linear system. However, when the input to the amplifier is “large”, the nonlinearities of the amplifier can be observed, and we have to use the nonlinear large-signal model (exponential for BJT, square-law for FETs) of the transistor to understand the operation of the amplifier. “Large” and “small” are fuzzy terms and, as system designers, we would like to have a more quantitative definition for these values. With respect to the amplitude of the input signal, the amplifier’s nonlinearities decrease the gain, called gain compression, and distort the shape of the output signal, called harmonic distortion. One way to quantitatively define “small” versus “large” signal is to measure the magnitude of the input voltage at which the gain has decreased by 1dB, called the 1dB-compression point. Another is to define an acceptable level of distortion and the amplitude threshold at which this distortion level occurs. You won’t understand how to measure the distortion in the signal until the end of this quarter. This lab will focus on measuring the gain compression point while simply observing the distortion that occurs in the output due to the nonlinearities of the amplifier. The goal of the first part of this lab is to:

1. Measure the 1-dB compression point for the amplifier
2. Record the output of the amplifier on the oscilloscope for both a “large” and “small” signal input to observe the distortion.

a) Connect the power supply, function generator, and oscilloscope to the amplifier as shown in the figure above. Of special note when testing the amplifier:

1. Make sure you connect your power supply with the right polarities and that it doesn’t exceed 5 V.
2. The voltage divider connected to the function generator reduces the amplitude of the function generator by a factor of approximately 10. When doing your experiments, make sure you measure the input to the amplifier on the negative node of the capacitor (as indicated by the $x(t)$ signal in the schematic above) not the output of the function generator.
3. Set the coupling of the oscilloscope to AC and the probe gains to 10:1 for each channel. Do not use autoscale because it will change your coupling back to DC.
4. Set the trigger to use the output signal as its source. Use the Mode/Coupling button on the trigger to set HF Reject and Noise Reject for the trigger. **THIS IS VERY IMPORTANT, YOU WILL NOT BE ABLE TO TAKE MEASUREMENTS OTHERWISE!!!**
5. Turn on averaging (with at least 8 samples) in the oscilloscope to reduce the noise of the function generator.
6. Set both channels to have their “gnd” level at the exact center of the scope screen.
7. You can use the quick-measure feature of the scope to measure the pk-pk amplitude of the INPUT, $x(t)$, and divide by 2 to get the peak value. **For the OUTPUT, $y(t)$, use the y-cursors to measure ONLY the positive peak amplitude of the signal.**

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b) Set the function generator to be a sinusoid with frequency of 1 kHz. Start with the peak amplitude of $x(t)=10$ mV as measured on the scope. Measure the positive peak voltage of both $x(t)$ and $y(t)$ as you sweep the amplitude of $x(t)$ as shown on the data recording table at the end of this handout. NOTE: The the peak amplitude of $x(t)$ is different from the function generator because of the voltage divider. You will need to sweep the function generator's amplitude from **approximately** 100mV to 10V, use the scope measurement to ensure the proper amplitude of $x(t)$. Record your data in the table at the end of this handout and then enter the vectors into a Matlab m-file.

c) In the same m-file, plot the gain of the amplifier ($|v_{out}|^2/|v_{in}|^2$) in dB vs. the Power of the input signal into a 1 Ohm resistor in dB for each input voltage amplitude. Your plot should look like the Figure 1 below. Include in your plot a line that indicates the ideal gain of your amplifier. The ideal gain can be found by averaging the measured gain for the first few "small-signal" data points (i.e. those data that lie to the left of the 1-dB compression point).

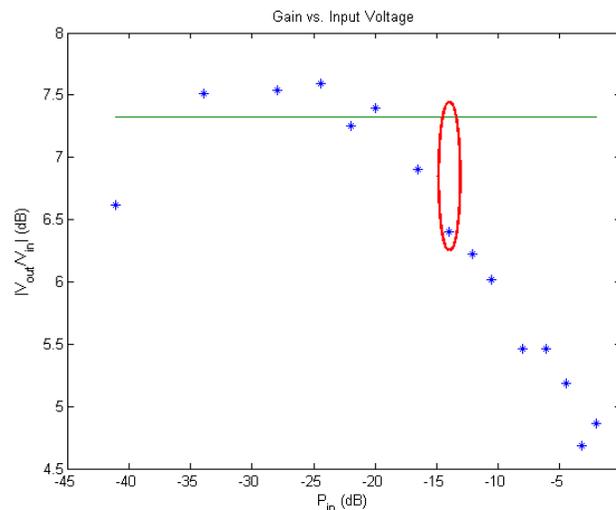


Figure 1: Plot of Voltage Gain (dB) vs. Input Signal Power (dB)

d) You can get a rough estimate of the gain compression point by looking at your plot. This point is defined as the input power at which the gain is reduced by 1dB. You should indicate your gain compression point in your plot similar to the red ellipse shown above. Shapes can be added to your plot using the menu options in the figure window.

e) In the same m-file, use the *figure()* command to assign figure(1) to the plot above and figure(2) for a new plot. The new plot will look like Figure 2 below and will plot the output signal power vs. the input signal power, both in dB relative to a 1 Ohm resistor.

f) To create the new plot you will:

- First use the *polyfit* command in Matlab to fit a line to only the data **that are to the left of** the 1-dB compression point (as these are considered the "small-signal" inputs so that the amplifier is behaving linearly). This line will become the green line shown in Figure 2 below. For the *polyfit* command, use $|v_{in}|$ (in units of Volts) as your x-coordinates and $|v_{out}|$ (in units of Volts) as your y-coordinates. *Polyfit* returns the coefficients of a 1st order polynomial (i.e. a line).

- You must then use the coefficients from *polyfit* to **create a matrix of data points that represents your best fit line**. This is how the amplifier would behave if it was linear for all amplitudes of the input.
- Note that the line that you just created has units of Volts. You must convert the x- and y- coordinates of this line to units of dB so that it can be displayed with the input and output signal power data that you collected.
- In your *figure(2)* window, plot the measured input and output signal powers relative to 1 Ohm in dB and the line that you generated from the previous step. You should get a figure that looks like the one below. This figure shows how your data compares to a purely linear amplifier. If done correctly, your data should fall 1dB below the ideal line at the compression point. This figure illustrates how the gain compression is typically displayed. Add axes labels and title to this figure and print it out to turn in.

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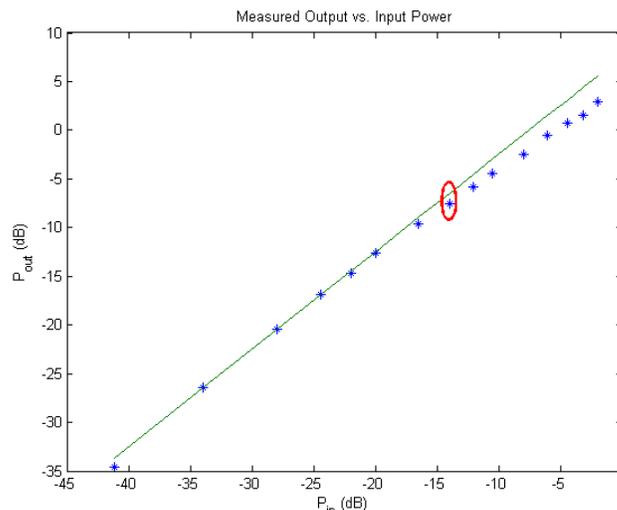


Figure 2: Plot of Output Signal Power (dB) vs. Input Signal Power (dB)

g) Now that you've found the gain compression point, you can quantitatively define "small" and "large" signal. Indicate the peak input voltage and input signal power relative to 1 Ohm in dB on the instructor verification page. Record a screen capture from the oscilloscope showing the traces for the input and output voltages for both a "small-signal" value of $x(t)$ and a "large-signal" value of $x(t)$. Print out the screen captures to hand in with your lab. For the "large-signal" case, highlight on the printout the distortion that is caused by the nonlinearity of the amplifier.

h) Also hand in your table with the recorded data and the final m-file in which you did the data analysis and created the two figures.

Instructor Verification (see last page)

2. Experimentation with Sound, Small Signal Models

The MATLAB program **linear_systems_and_sounds.m** allows you to specify an input signal (lines 11-12) and system (line 24), and listen to how the sound changes as the signal passes through the system. We will use the sounds produced to try to identify the systems as linear or nonlinear. Recall from class that a system is linear if and only if an input signal at frequency f produces an output signal at frequency f . The phase angle and amplitude between the input and output may change, but the frequency cannot. Just think about how transfer functions affected periodic signals when you were using phasors to represent sinusoids- the transfer function could change the magnitude and phase but not the frequency of the input signal. *Note that we can only hear sounds when the speaker is vibrating.*

a) Using the code as is (i.e. with $y=x$), investigate how the frequency of the sinusoidal input signal affects the sound of the signal. Specifically, describe the sound you hear for input signal frequencies f_0 of 200, 1000, and 5000 Hz.. Note that you need to run the program, wait for the input sound (and graph) to appear, then hit enter and you will hear the output sound and see the output plotted in the lower panel.

b) Now change the system so the output is zero, i.e., $y = 0*x$ (this will make sure the y vector is the correct length). What do you hear for the system output, if anything?

c) Now change the system so the output is $y=0*x+10$; What do you hear? How fast (at what frequency) is the speaker vibrating?

d) For each of the mathematical models of systems in the following table, assume x is a periodic signal with frequency 200 Hz. For each of these models, determine if the mathematical model of the system is linear or not, and if you would classify the system models as linear or not based on both listening to and looking at the input and output. Discrete-time solutions for model (i) is included in the code, and is commented out. Uncomment the correct lines when you need to run these models.

e) For many system, such as transistors, we are really interested in *small signal models*. For these systems we only care about the behavior of the system about a set or bias point for an input with a small amplitude. For $x(t)$ small, we can use a Taylor series approximation of $y(x)$ around $x(t) = 0$ of the form

$$y(x) \approx y_0 + \nu x(t)$$

where y_0 and ν are determined by the Taylor series expansion of $y(x)$ about the point $x = 0$. If we subtract the bias point (y_0) from both sides of the equation and rename variables we have

$$y(x) - y_0 \equiv \Delta y(x) = \nu x(t)$$

This is now a description of a linear system between input $x(t)$ and output $\Delta y(x)$. This relationship is only approximately linear if the input $x(t)$ is sufficiently small that the Taylor series approximation is valid.

For the systems that are mathematically nonlinear, but appeared/sounded linear, determine the small signal linear approximation, that is, determine y_0 and ν .

Note that our input amplitude is 1, and hence something like $z = x/10$ would be considered small. The following formula's for Taylor series about the point $z = 0$ may be helpful:

Taylor series (general form) $f(z) \approx f(0) + z \left. \frac{df}{dz} \right|_{z=0}$

$$y(z) = [a + bz]^c \approx a^c + ca^{c-1}bz \quad y(z) = \sin(az) \approx az$$

$$y(z) = \log(a + bz) = \log(a) + \frac{bz}{a+b} \quad y(z) = e^{az} \approx 1 + az$$

(Note that log here means natural log, typing log in Matlab takes the natural log of the function.)

As a check, for those systems where the small signal model is valid for this particular input signal, if you look at the plots from *linear_systems_and_sounds.m*, the output signal should be centered at y_0 and should oscillate between $y_0 + \nu$ and $y_0 - \nu$. **Show your work in deriving the small signal model, do not just look at the system output!**

Don't forget that if you want to process element by element, you may need to use `.`, `./`, or `.^` in Matlab*

System	Mathematical Model	Mathematical Model Linear? (Y/N)	Sounds/Looks Linear?(Y/N)	Small Signal Model (if valid)
a	$y(t) = \frac{1}{2 + x(t)}$			
b	$y(t) = \frac{1}{2 + \frac{x(t)}{10}}$			
c	$y(t) = \sin(2x(t))$			
d	$y(t) = \sin\left(\frac{x(t)}{10}\right)$			
e	$y(t) = x(t) $			
f	$y(t) = e^{x(t)}$			
g	$y(t) = \log(1.1 + x(t))$			
h	$y(t) = \left(10.1 + \frac{x(t)}{10}\right)^{0.2}$			
i	$\dot{y}(t) + ay(t) = bx^2(t)$			

Instructor Verification (see last page)

Report

For this lab, turn in all of your Matlab code and plots in addition to this verification sheet.

Lab 03 Introduction to MATLAB
Instructor Verification Sheet

Name _____ Date of Lab: _____

Part 1(a) Demonstrate that your circuit works appropriately.

Verified: _____ Date/Time: _____

Part 1(f) Present your plots.

Verified: _____ Date/Time: _____

Part 1(h) Present your plots.

Verified: _____ Date/Time: _____

1-dB Compression Point:

In dB relative to 1 Ohm _____ In peak voltage _____

Part 2(e) Present your table.

Verified: _____ Date/Time: _____

$x(t)$ (mVp)	$y(t)$ (mVp)
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
200	
300	
400	
500	
600	
700	
800	