ECE 300
Signals and Systems
Homework 1

Due Date: Thursday March 12, 2009 *at the beginning of class*

Reading: Roberts pages 1-28 and your course notes.

Problem:

1) Use Euler's identity in the form $\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2}$ and $\sin(\theta)=\frac{e^{j\theta}-e^{-j\theta}}{2j}$ to prove the following identities:

a) $2\sin(\theta)\cos(\theta)=\sin(2\theta)$ **b)** $\cos^2(\theta)=\frac{1}{2}+\frac{1}{2}\cos(2\theta)$

c) $\sin^2(\theta)=\frac{1}{2}-\frac{1}{2}\cos(2\theta)$ **d)** $\frac{d}{d\theta}\cos(\theta)=-\sin(\theta)$ **e)** $\frac{d}{d\theta}\sin(\theta)=\cos(\theta)$

2) Simplify the following as much as possible. Use unit step functions where necessary instead of inequalities.

a) $y(t)=\delta(t-1)u(t+1)$ **b)** $y(t)=\frac{d}{dt}\left[e^{at}u(-t)\right]$ **c)** $y(t)=\delta(t-1)e^{-t}$

d) $y(t)=\int_{-\infty}^{2}u(\lambda-1)e^{-\lambda}d\lambda$ **e)** $y(t)=\int_{-\infty}^{\infty}u(1-\lambda)u(\lambda-2)e^{-2\lambda}d\lambda$ **f)** $y(t)=\delta(t-1)\delta(t+2)$

g) $y(t)=\int_{-\infty}^{\infty}e^{-2\lambda}u(\lambda)\delta(\lambda-1)d\lambda$ **h)** $y(t)=\int_{-\infty}^{\infty}u(t-\lambda)\delta(\lambda-2)d\lambda$

3) In **ECE-200** you learned (believe it or not) that if the periodic signal $x(t)=A\cos(\omega_0 t+\phi)$ is the input to a transfer function $H(s)$, then the steady state output will be

$$y(t)=|H(j\omega_0)|\,A\cos(\omega_0 t+\phi+\angle H(j\omega_0))$$

This is really just a result of phasor analysis, where we write both the input and the transfer function as phasors, then multiply the magnitudes and add the phases. We will use this relationship a great deal in this course.

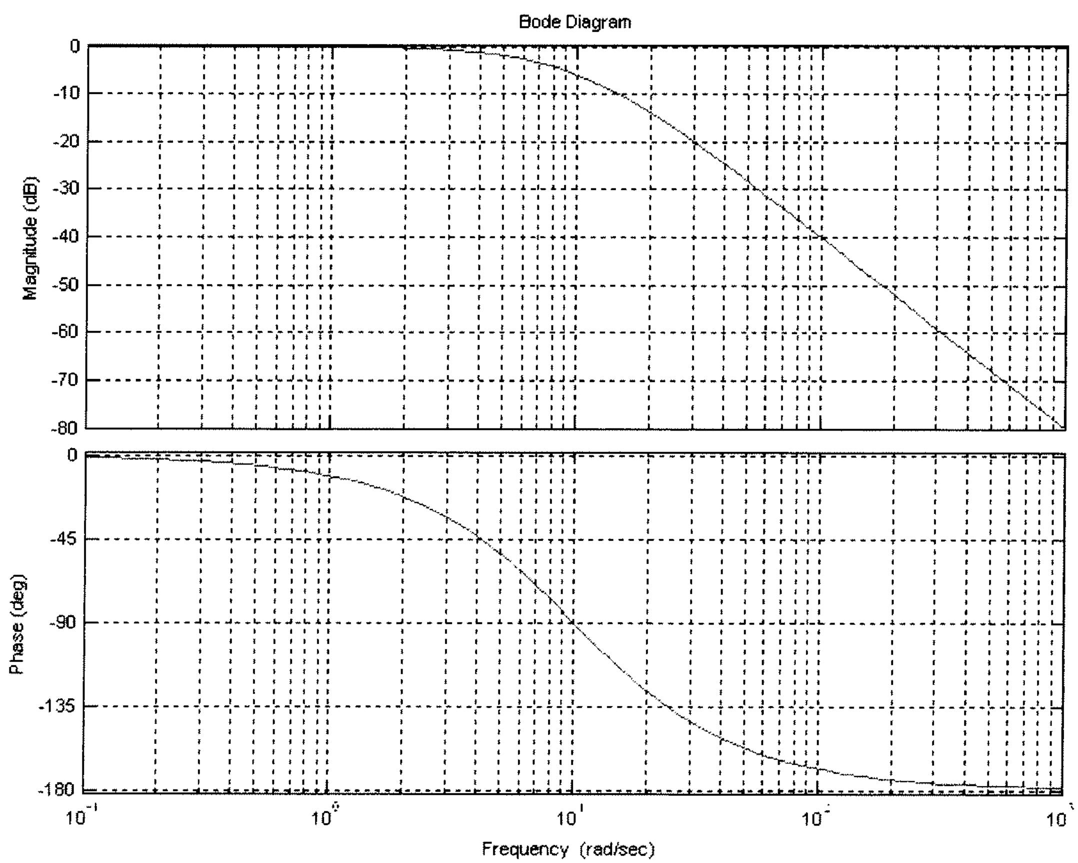
For the following input signals and transfer functions, determine the steady state output of the system

a) $x(t) = 3\cos(2t + 45^o)$ and $H(s) = \dfrac{1}{s+1}$

b) $x(t) = 2\sin(3t + 30^o)$ and $H(s) = \dfrac{s}{s+3}$

4) The Bode plot of a transfer function is just a graphical method of displaying the frequency response of the transfer function. The figure below displays the Bode plot for an unknown system. Use the plot to estimate the steady state output for the following inputs:

a) $x(t) = 100\cos(30t + 30^o)$ **b)** $x(t) = 200\sin(100t - 45^o)$



#1 $\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ $\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

a) $2\cos(\theta)\sin(\theta) = 2\left(\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right)\left(\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right) = 2\left(\dfrac{e^{j2\theta} - e^{-j2\theta}}{4j}\right)$

$= \dfrac{e^{j2\theta} - e^{-j2\theta}}{2j} = \boxed{\sin(2\theta)}$

b) $\cos^2(\theta) = \left(\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right)\left(\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right) = \dfrac{e^{j2\theta} + e^{-j2\theta} + 2}{4}$

$= \left(\dfrac{e^{j2\theta} + e^{-j2\theta}}{2}\right)\dfrac{1}{2} + \dfrac{1}{2} = \boxed{\dfrac{1}{2} + \dfrac{1}{2}\cos(2\theta)}$

c) $\sin^2(\theta) = \left(\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right)\left(\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right) = \dfrac{e^{j2\theta} + e^{-j2\theta} - 2}{-4}$

$= \dfrac{1}{2} - \dfrac{1}{2}\left(\dfrac{e^{j2\theta} + e^{-j2\theta}}{2}\right) = \boxed{\dfrac{1}{2} - \dfrac{1}{2}\cos(2\theta)}$

d) $\dfrac{d}{d\theta}\left[\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right] = \dfrac{je^{j\theta} - je^{-j\theta}}{2} = \dfrac{-e^{j\theta} + e^{-j\theta}}{2j} = -\left[\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right]$

$= \boxed{-\sin(\theta)}$

e) $\dfrac{d}{d\theta}\left[\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right] = \dfrac{je^{j\theta} + je^{-j\theta}}{2j} = \dfrac{e^{j\theta} + e^{-j\theta}}{2} = \boxed{\cos(\theta)}$

#2

(a) $y(t) = \delta(t-1)u(t+1) = \delta(t-1)u(2) = \boxed{\delta(t-1)}$

(b) $y(t) = \frac{d}{dt}\left[e^{at}u(-t)\right] = \left[\frac{d}{dt}e^{at}\right]u(-t) + e^{at}\frac{d}{dt}\left[u(-t)\right]$

$= ae^{at}u(-t) + e^{at}\delta(-t)(-1) = \boxed{ae^{at}u(-t) - \delta(t) = y(t)}$

$= ae^{at}u(-t) - \delta(-t)$

(c) $y(t) = \delta(t-1)e^{-t} = \boxed{e^{-1}\delta(t-1)}$

(d) $y(t) = \int_{-\infty}^{2} u(\lambda-1)e^{-\lambda}d\lambda$ $\quad u(\lambda-1) = 1$ for $\lambda-1 \geq 0 \quad \lambda \geq 1$

$= \int_{1}^{2} e^{-\lambda}d\lambda = -e^{-\lambda}\Big|_{1}^{2} = e^{-1} - e^{-2} = \boxed{0.2325}$

(e) $y(t) = \int_{-\infty}^{\infty} u(1-\lambda)u(\lambda-2)e^{-2\lambda}d\lambda$ $\quad u(1-\lambda) = 1$ for $1-\lambda \geq 0 \quad 1 \geq \lambda$

$u(\lambda-2) = 1$ for $\lambda-2 \geq 0 \quad \lambda \geq 2$

$= \boxed{0}$

(f) $y(t) = \delta(t-1)\delta(t+2) = \boxed{0}$

(g) $y(t) = \int_{-\infty}^{\infty} e^{-2\lambda}u(\lambda)\delta(\lambda-1)d\lambda = e^{-2}u(1) = e^{-2} = \boxed{0.135}$

(h) $y(t) = \int_{-\infty}^{\infty} u(t-\lambda)\delta(\lambda-2)d\lambda = \boxed{u(t-2)}$

#3

(a) $x(t) = 3\cos(2t + 45^\circ)$ $\quad H(s) = \dfrac{1}{s+1}$

$\omega_0 = 2 \quad H(j\omega_0) = \dfrac{1}{j2+1} = 0.447 \measuredangle -63.43$

$y(t) = (3)(0.447)\cos(2t + 45^\circ - 63.43^\circ) = \boxed{1.34\cos(2t - 18.43^\circ) = y(t)}$

(b) $x(t) = 2\sin(3t + 30^\circ)$ $\quad H(s) = \dfrac{s}{s+3}$

$\omega_0 = 3 \quad H(j\omega_0) = \dfrac{j3}{j3+3} = 0.707 \measuredangle 45^\circ$

$y(t) = (2)(0.707)\sin(2t + 30.0 + 45^\circ) = \boxed{1.41\sin(2t + 75.0) = y(t)}$

#4

(a) $x(t) = 100\cos(30t + 30^\circ)$

from graph $\measuredangle H(j30) \approx -140^\circ$

$|H(j30)| \approx -20\,dB$

$-20 = 20\log_{10}|H(j30)| \qquad |H(j30)| = 0.1$

$y(t) \approx (100)(0.1)\cos(30t + 30^\circ - 140^\circ) \approx \boxed{10\cos(30t - 110^\circ) \approx y(t)}$

(b) $x(t) = 200\sin(100t - 45^\circ)$

from graph $\measuredangle H(j100) \approx -170^\circ$

$|H(j100)| \approx -40\,dB$

$-40 = 20\log_{10}|H(j100)|$

$10^{-2} = |H(j100)|$

$y(t) \approx (200)(10^{-2})\sin(100t - 45^\circ - 170^\circ) \approx \boxed{2\sin(100t - 215^\circ) \approx y(t)}$

$\approx 2\cos(100t - 215^\circ + 90^\circ)$

$\approx 2\cos(100t - 125^\circ)$