ECE 300
Signals and Systems
Homework 8

Due Date: Matlab/Prelab, Tuesday April 29 at the beginning of class
Problems 1-6, Friday May 2 at 4 PM

Problems

1. Determine the transfer function $H(\omega)$ that would produce the following input /output relationships. Simplify your answers as much as possible.

a) $y(t) = a\dot{x}(t-b)$

b) $y(t) = ax(t+b) + ax(t-b)$

c) $\dot{y}(t) = x(t) * e^{-t}u(t-b)$

2. Using the ***duality property***, find the corresponding Fourier transform for the following:

a) $g(t) = \mathrm{sinc}^2(Bt)$

b) $g(t) = \mathrm{sinc}(Wt)$

c) $g(t) = \delta(t)$

d) $g(t) = \cos(\omega_0 t)$

Do not just look up the pairs from the table (though you can use any other pairs except the one you are trying to find).

3. Consider a linear time invariant system with transfer function given by

$$H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & else \end{cases}$$

with input $x(t) = \frac{8}{\pi}\mathrm{sinc}^2\left(\frac{2(\text{t-1})}{\pi}\right)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

d) Determine $y(t)$, the output of the system.

Answer $y(t) = \frac{20}{\pi}\mathrm{sinc}\left[\frac{2}{\pi}(t-3)\right] + \frac{10}{\pi}\mathrm{sinc}^2\left[\frac{1}{\pi}(t-3)\right]$

Spring 2008

4. Consider a linear time invariant system with impulse response given by $h(t) = \frac{1}{2\pi}\text{sinc}\left(\frac{t-2}{2\pi}\right)$ with input $x(t) = \frac{4}{\pi}\text{sinc}\left(\frac{2t}{\pi}\right)\cos(t)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Determine the energy in $x(t)$

d) Determine $H(\omega)$.

e) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

f) Determine $y(t)$, the output of the system.

g) Determine the energy in $y(t)$.

5. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t) = 5\,\text{sinc}\left(\frac{t}{0.002}\right) + 5\,\text{sinc}\left(\frac{t}{0.001}\right) \quad \textit{Answer}\ 56\%$$

6. In this problem we'll look at a real world situation when we have to truncate a signal. This actually happens more with digital signal processing, but we can get the basic idea using our continuous time abilities.

a) Find an expression for the Fourier transform of $f(t) = \cos(4t) + \cos(5t)$.

b) Now assume we look at $f(t)$ for a finite time, say T seconds. What we see is actually $y(t) = f(t)rect(t/T)$. Determine an expression for the Fourier transform of $y(t)$, and write your answers in terms of sinc functions.

c) Plot, using **Matlab**, $Y(\omega)$ for ω between 0 and 10 when T=1, T=6, T=10, T=20, and T=40. Can you clearly tell there are two cosines present when you are looking at $Y(\omega)$ for all values of T? What happens as T gets larger (you are looking at more and more data)? Think in terms of the width of the sinc function (the distance between the first nulls). Note: The **sinc** function exists in **Matlab**.

7. (Matlab/Prelab Problem) A Butterworth filter has the property that it is maximally flat in the *passband.* An nth order Butterworth filter has the magnitude squared response

$$|H(\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_p}\right)^{2n}}$$

where ω_p is the *passband frequency.* At this frequency the power has been reduced by one half or 3 dB,

$$|H(\omega_p)|^2 = \frac{1}{1+\left(\frac{\omega_p}{\omega_p}\right)^{2n}} = \frac{1}{2} \quad \text{or} \; 10\log_{10}|H(\omega_p)|^2 = 10\log_{10}\left(\frac{1}{2}\right) = -3dB$$

To determine the required order of a filter we often look at the desired *stopband frequency,* ω_s. Usually we want to indicate the minimum required power difference between the passband and the stopband, Δ. Δ is the *rejection.*Hence we have

$$\Delta = 20\log_{10}|H(0)| - 20\log_{10}|H(\omega_s)|$$

or

$$\Delta = -10\log_{10}\frac{1}{1+\left(\frac{\omega_s}{\omega_p}\right)^{2n}}$$

The ratio $\frac{\omega_s}{\omega_p}$ is called the *transition ratio.*

a) Show that we can write

$$n = \frac{\ln\left(10^{\frac{\Delta}{10}} - 1\right)}{2\ln\left(\frac{\omega_s}{\omega_p}\right)}$$

Note that <u>*n must be an integer*</u>, so we always round up (to the next larger integer).

b) For $\omega_p = 10$ rad/sec, $\omega_s = 20$ rad/sec, and $\Delta = 18$ dB, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using the Table at the end of this problem, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than Δ_{max} .Matlab's

tf command and **Bode** commands will be really useful here. Note that you can click on the curve on the Bode plot to read it more accurately. Turn in your plot.

c) Matlab's command **r = pole(H)**, where H is the transfer function of the Butterworth filter, returns the poles of the transfer function in the array **r**. Using Matlab's commands **abs** and **angle,** relate the magnitude of the poles to ω_p, then plot the pole locations in the complex plane on a circle with radius ω_p. (Note that **angle** returns angles in radians, and you probably want angle in degrees.) Note that the pole locations are all separated by an angle θ. What is this angle?

d) For $\omega_p = 15$ rad/sec, $\omega_s = 35$ rad/sec, and $\Delta = 28$ dB, determine the required Butterworth filter order for this filter. (Remember it must be an integer). Using the Table at the end of this problem, plot the Bode plot of your Butterworth filter and verify that all frequencies $\omega > \omega_s$ have magnitude (power) less than $\Delta_{\max}$. Matlab's **tf** command and **Bode** commands will be really useful here. Turn in your plot.

e) Matlab's command **r = pole(H)**, where H is the transfer function of the Butterworth filter, returns the poles of the transfer function in the array **r**. Using Matlab's commands **abs** and **angle,** relate the magnitude of the poles to ω_p, then plot the pole locations in the complex plane on a circle with radius ω_p. (Note that **angle** returns angles in radians, and you probably want angle in degrees.) Note that the pole locations are all separated by an angle θ. What is this angle?

n	denominator(s)
1	$\frac{s}{\omega_p}+1$
2	$\left(\frac{s}{\omega_p}\right)^2+1.414\left(\frac{s}{\omega_p}\right)+1$
3	$\left(\frac{s}{\omega_p}\right)^3+2\left(\frac{s}{\omega_p}\right)^2+2\left(\frac{s}{\omega_p}\right)+1$
4	$\left(\frac{s}{\omega_p}\right)^4+2.6131\left(\frac{s}{\omega_p}\right)^3+3.4142\left(\frac{s}{\omega_p}\right)^2+2.6131\left(\frac{s}{\omega_p}\right)+1$
5	$\left(\frac{s}{\omega_p}\right)^5+3.2361\left(\frac{s}{\omega_p}\right)^4+5.2361\left(\frac{s}{\omega_p}\right)^3+5.2361\left(\frac{s}{\omega_p}\right)^2+3.2361\left(\frac{s}{\omega_p}\right)+1$

Table 1: Denominators of Buterworth filter for filter orders 1-5. The numerator for the Butterworth filter is 1.

#1

(a) $y(t) = a\dot{x}(t-b)$

$Y(\omega) = aj\omega X(\omega) e^{-j\omega b}$ $\quad \boxed{H(\omega) = j\omega a e^{-j\omega b}}$

(b) $y(t) = ax(t+b) + ax(t-b)$

$Y(\omega) = aX(\omega)e^{j\omega b} + aX(\omega)e^{-j\omega b}$

$= a\left[e^{j\omega b} + e^{-j\omega b}\right] X(\omega)$

$= 2a\cos(\omega b)\, X(\omega)$ $\quad \boxed{H(\omega) = 2a\cos(\omega b)}$

(c) $\dot{y}(t) = x(t) * e^{-t}u(t-b)$

$j\omega Y(\omega) = X(\omega)\, \mathcal{F}\{e^{-t}u(t-b)\}$

$$\int_{-\infty}^{\infty} e^{-t}u(t-b)e^{-j\omega t}\,dt = \int_b^{\infty} e^{-(1+j\omega)t}\,dt$$

$$= \left.\frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\right|_b^{\infty} = \frac{e^{-(1+j\omega)b}}{1+j\omega} = \frac{e^{-b}e^{-j\omega b}}{1+j\omega}$$

or $e^{-t}u(t-b) = e^{-(t-b)}e^{-b}u(t-b) = e^{-b}e^{-(t-b)}u(t-b)$

$$\mathcal{F}\{e^{-b}e^{-(t-b)}u(t-b)\} = \frac{e^{-b}e^{-j\omega b}}{1+j\omega}$$

so $\boxed{H(\omega) = \frac{1}{j\omega}\,\frac{e^{-b}e^{-j\omega b}}{1+j\omega}}$

#2

a) $g(t) = \text{sinc}^2(Bt)$

from the table,

$$g_1(t) = \Delta\left(\frac{t}{W}\right) \longleftrightarrow G_1(\omega) = W\,\text{sinc}^2\left(\frac{W}{2\pi}\omega\right)$$

by duality

$$g_2(t) = G_1(t) = W\,\text{sinc}^2\left(\frac{W}{2\pi}t\right) \longleftrightarrow G_2(\omega) = 2\pi g_1(-\omega) = 2\pi\,\Delta\left(\frac{\omega}{W}\right)$$

$$B = \frac{W}{2\pi} \quad \text{so} \quad W = 2\pi B$$

$$g_2(t) = 2\pi B\,\text{sinc}^2(Bt) \longleftrightarrow G_2(\omega) = 2\pi\,\Delta\left(\frac{\omega}{2\pi B}\right)$$

or $$\boxed{\text{sinc}^2(Bt) \longleftrightarrow \frac{1}{B}\Delta\left(\frac{\omega}{2\pi B}\right)}$$

b) $g(t) = \text{sinc}(Wt)$

from the table

$$g_1(t) = \text{rect}\left(\frac{t}{T}\right) \longleftrightarrow G_1(\omega) = T\,\text{sinc}\left(\frac{T}{2\pi}\omega\right)$$

by duality

$$g_2(t) = G_1(t) = T\,\text{sinc}\left(\frac{T}{2\pi}t\right) \longleftrightarrow G_2(\omega) = 2\pi g_1(-\omega) = 2\pi\,\text{rect}\left(\frac{\omega}{T}\right)$$

$$W = \frac{T}{2\pi} \quad \text{or } T = 2\pi W$$

$$g_2(t) = 2\pi W\,\text{sinc}(Wt) \longleftrightarrow 2\pi\,\text{rect}\left(\frac{\omega}{2\pi W}\right)$$

$$\boxed{\text{sinc}(Wt) \longleftrightarrow \frac{1}{W}\text{rect}\left(\frac{\omega}{2\pi W}\right)}$$

(c) $g(t) = \delta(t)$

from the table

$$g_1(t) = 1 \Leftrightarrow G_1(\omega) = 2\pi\,\delta(\omega)$$

by duality

$$g_2(t) = G_1(t) = 2\pi\,\delta(t) \Leftrightarrow G_2(\omega) = 2\pi g_1(-\omega) = 2\pi$$

$$\boxed{\delta(t) \Leftrightarrow 1}$$

(d) $g(t) = \cos(\omega_0 t)$

for $G_1(\omega) = \cos(T\omega) = \frac{1}{2}e^{j\omega T} + \frac{1}{2}e^{-j\omega T} \Leftrightarrow g_1(t) = \frac{1}{2}\delta(t+T) + \frac{1}{2}\delta(t-T)$

so $g_1(t) = \frac{1}{2}\left[\delta(t+T) + \delta(t-T)\right] \Leftrightarrow G_1(\omega) = \cos(T\omega)$

by duality

$$g_2(t) = G_1(t) = \cos(Tt) \Leftrightarrow G_2(\omega) = 2\pi g_1(-\omega) = \pi\,\delta(-\omega+T) + \pi\,\delta(-\omega-T)$$

$T = \omega_0$

$$\boxed{\cos(\omega_0 t) \Leftrightarrow \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)}$$

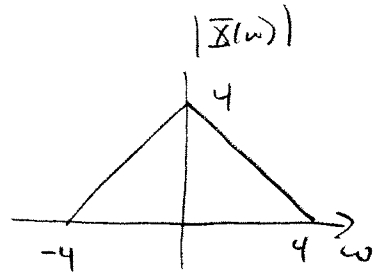
$\delta()$ is an <u>even</u> function

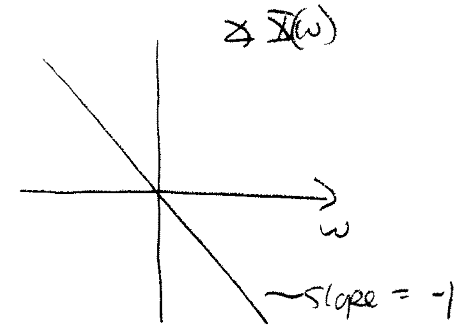
#3 $x(t) = \frac{8}{\pi}\operatorname{sinc}^2\left(\frac{2(t-1)}{\pi}\right)$ $\quad H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & \text{else} \end{cases}$

a) for $x(t) = \operatorname{sinc}^2\left(\frac{2t}{\pi}\right) \longleftrightarrow X(\omega) = \frac{\pi}{2}\Lambda\left(\frac{\omega}{4}\right)$

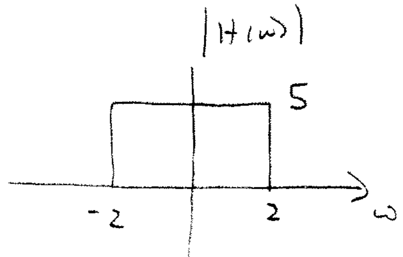
so $\boxed{X(\omega) = 4\Lambda\left(\frac{\omega}{4}\right)e^{-j\omega}}$

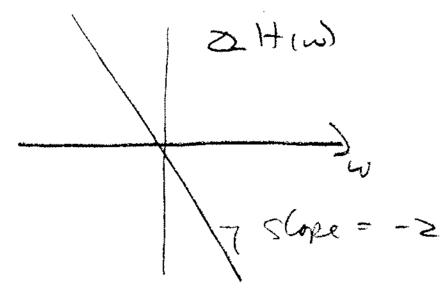
b)



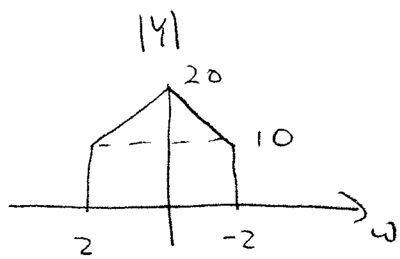


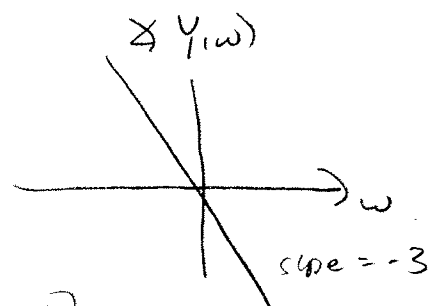
c)





d)





$Y(\omega) = \left[10\operatorname{rect}\left(\frac{\omega}{4}\right) + 10\Lambda\left(\frac{\omega}{2}\right)\right]e^{-j3\omega}$

$\operatorname{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow W\operatorname{sinc}(Wt) \qquad W = \frac{2}{\pi}$

$\Lambda\left(\frac{\omega}{2\pi B}\right) \longleftrightarrow B\operatorname{sinc}^2(Bt) \qquad B = \frac{1}{\pi}$

$\boxed{y(t) = \frac{20}{\pi}\operatorname{sinc}\left(\frac{2}{\pi}(t-3)\right) + \frac{10}{\pi}\operatorname{sinc}^2\left(\frac{1}{\pi}(t-3)\right)}$

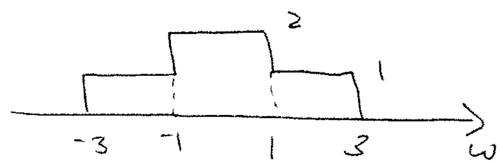
#4 $h(t) = \frac{1}{2\pi}\,\text{sinc}\left(\frac{t-2}{2\pi}\right)$ $\quad x(t) = \frac{4}{\pi}\,\text{sinc}\left(\frac{2t}{\pi}\right)\cos(t)$

a) For $x_1(t) = \text{sinc}\left(\frac{2}{\pi}t\right)$ $\quad X_1(\omega) = \frac{\pi}{2}\,\text{rect}\left(\frac{\omega}{2\pi\cdot\left(\frac{2}{\pi}\right)}\right) = \frac{\pi}{2}\,\text{rect}\left(\frac{\omega}{4}\right)$

For $x_2(t) = \frac{4}{\pi}x_1(t)$ $\quad X_2(\omega) = 2\,\text{rect}\left(\frac{\omega}{4}\right)$

For $x_3(t) = x_2(t)\cos(t)$ $\quad \boxed{X_3(\omega) = \text{rect}\left(\frac{\omega+1}{4}\right) + \text{rect}\left(\frac{\omega-1}{4}\right)}$

b) $|X(\omega)|$ $\qquad \measuredangle X(\omega) = 0$



c) $E_x = \frac{1}{2\pi}\left[\int_{-3}^{-1} 1^2\,d\omega + \int_{-1}^{1} 2^2\,d\omega + \int_{1}^{3} 1^2\,d\omega\right] = \frac{1}{2\pi}\left[2 + 4\cdot 2 + 2\right] = \boxed{\frac{6}{\pi} = E_x}$

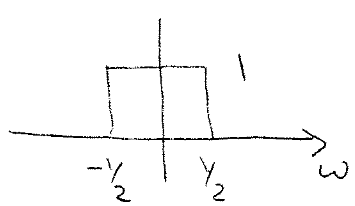
d) $h(t) = \frac{1}{2\pi}\,\text{sinc}\left(\frac{t-2}{2\pi}\right)$

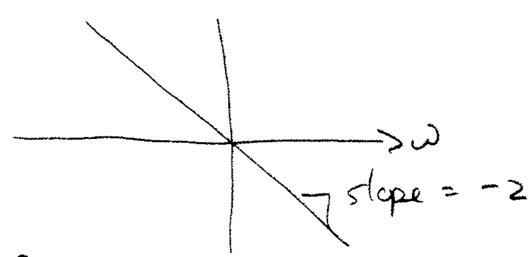
For $h_1(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$ $\quad H_1(\omega) = 2\pi\,\text{rect}\left(\frac{\omega}{2\pi\cdot\left(\frac{1}{2\pi}\right)}\right) = 2\pi\,\text{rect}(\omega)$

For $h_2(t) = \frac{1}{2\pi}h_1(t)$ $\quad H_2(\omega) = \text{rect}(\omega)$

For $h_3(t) = h_2(t-2)$ $\quad H_3(\omega) = e^{-j2\omega}H_2(\omega) = \boxed{\text{rect}(\omega)e^{-j2\omega} = H(\omega)}$

e) $|H(\omega)|$ $\qquad \measuredangle H(\omega)$





f) $Y(\omega) = 2\,\text{rect}(\omega)e^{-j2\omega}$

$$\boxed{y(t) = \frac{1}{\pi}\,\text{sinc}\left(\frac{t-2}{2\pi}\right)}$$

g) $E_y = \frac{1}{2\pi}\int_{-1/2}^{1/2} 2^2\,d\omega = \frac{4}{2\pi} = \boxed{\frac{2}{\pi} = E_y}$

#5

$x(t) = 5\,\mathrm{sinc}\left(\frac{t}{0.002}\right) + 5\,\mathrm{sinc}\left(\frac{t}{0.001}\right)$

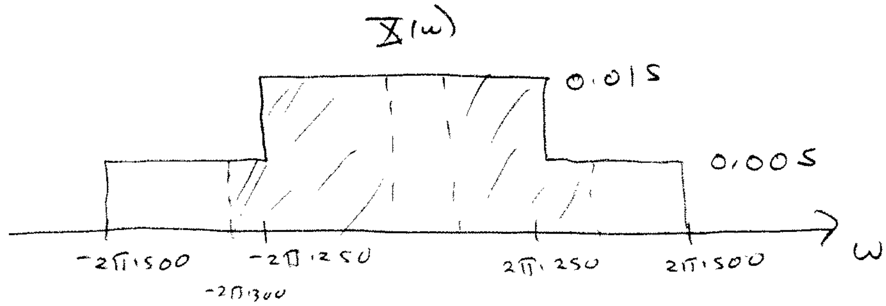
compute the % of energy between 100 and 300 Hz

for $\mathrm{sinc}(Wt) \Longleftrightarrow \frac{1}{W}\,\mathrm{rect}\left(\frac{\omega}{2\pi W}\right)$

here $W = \frac{1}{0.002} = 500$ or $W = \frac{1}{0.001} = 1000$

$$X(\omega) = \frac{5}{500}\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 500}\right) + \frac{5}{1000}\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 1000}\right)$$

$$= 0.01\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 500}\right) + 0.005\,\mathrm{rect}\left(\frac{\omega}{2\pi\cdot 1000}\right)$$



$$E_{total} = \frac{1}{2\pi}\int_{-2\pi\cdot 500}^{2\pi\cdot 500} |X(\omega)|^2 d\omega = \frac{1}{2\pi}\left[2\cdot\int_{-2\pi\cdot 500}^{-2\pi\cdot 250} (0.005)^2 d\omega + 2\int_{-2\pi\cdot 250}^{0} (0.015)^2 d\omega\right]$$

$$= \frac{1}{\cancel{2\pi}}\left[2\cdot(\cancel{2\pi}\cdot 250)(0.005)^2 + 2\cdot(\cancel{2\pi}\cdot 250)(0.015)^2\right]$$

$$= 2\cdot 250\cdot(0.005)^2 + 2\cdot 250\cdot(0.015)^2 = \boxed{0.125 = E_{total}}$$

$$E_{Band} = \frac{1}{2\pi}\left[2\cdot\int_{-2\pi\cdot 300}^{-2\pi\cdot 250} (0.005)^2 d\omega + 2\int_{-2\pi\cdot 250}^{-2\pi\cdot 100} (0.015)^2 d\omega\right]$$

$$= \frac{1}{\cancel{2\pi}}\left[2\cdot(\cancel{2\pi}\cdot 50)(0.005)^2 + 2(\cancel{2\pi}\cdot 150)(0.015)^2\right]$$

$$= 2\cdot(50)(0.005)^2 + 2(150)(0.015)^2 = \boxed{0.070 = E_{Band}}$$

$$\text{ratio} = \frac{0.070}{0.125} = 0.560 \qquad 56\%$$

#6

a) $f(t) = \cos(4t) + \cos(5t)$

$$\boxed{F(\omega) = \pi\left[\delta(\omega+4) + \delta(\omega-4)\right] + \pi\left[\delta(\omega+5) + \delta(\omega-5)\right]}$$

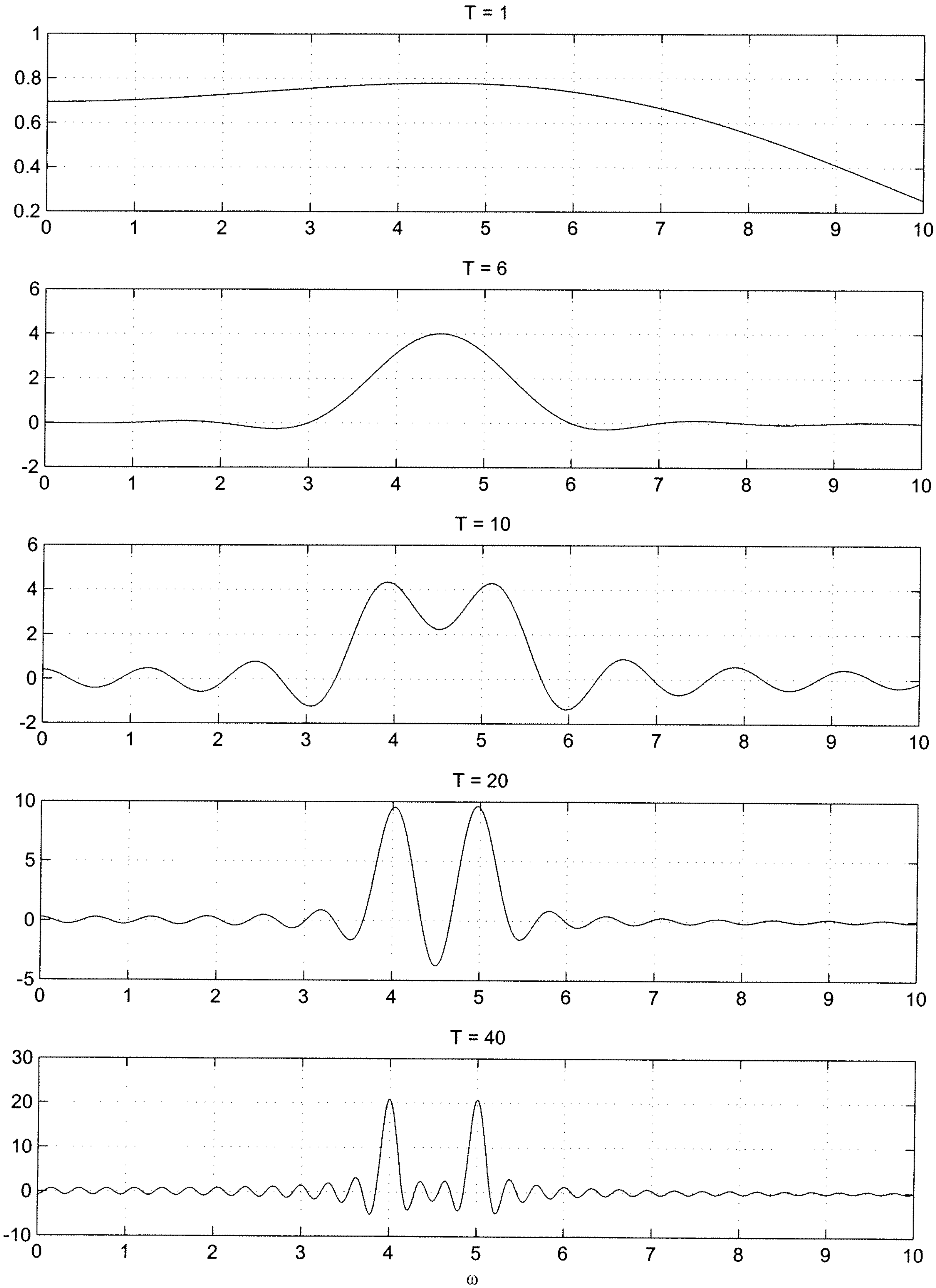
b) $y(t) = f(t)\,\mathrm{rect}\left(\frac{t}{T}\right)$

$$Y(\omega) = \frac{1}{2\pi} F(\omega) * \mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{T}\right)\right\}$$

$$= \frac{1}{2\pi} F(\omega) * T\,\mathrm{sinc}\left(\frac{T}{2\pi}\omega\right)$$

$$\boxed{Y(\omega) = \frac{T}{2}\left\{\mathrm{sinc}\left(\frac{T}{2\pi}(\omega+4)\right) + \mathrm{sinc}\left(\frac{T}{2\pi}(\omega-4)\right) + \mathrm{sinc}\left(\frac{T}{2\pi}(\omega+5)\right) + \mathrm{sinc}\left(\frac{T}{2\pi}(\omega-5)\right)\right.}$$

c) See Attached Plots


T = 1
T = 6
T = 10
T = 20
T = 40
ω

```
%
% plot for problem 5 of homework 5
%
w = linspace(0,10,1000);

T = 1;
a = T/(2*pi);
Y1 = @(w) (T/2)*(sinc(a*(w+4))+sinc(a*(w-4))+sinc(a*(w+5))+sinc(a*(w-5)))

T = 6;
a = T/(2*pi);
Y6 = @(w) (T/2)*(sinc(a*(w+4))+sinc(a*(w-4))+sinc(a*(w+5))+sinc(a*(w-5)))

T = 10;
a = T/(2*pi);
Y10 = @(w) (T/2)*(sinc(a*(w+4))+sinc(a*(w-4))+sinc(a*(w+5))+sinc(a*(w-5)))

T = 20;
a = T/(2*pi);
Y20 = @(w) (T/2)*(sinc(a*(w+4))+sinc(a*(w-4))+sinc(a*(w+5))+sinc(a*(w-5)))

T = 40;
a = T/(2*pi);
Y40 = @(w) (T/2)*(sinc(a*(w+4))+sinc(a*(w-4))+sinc(a*(w+5))+sinc(a*(w-5)))

orient tall
subplot(5,1,1); plot(w,Y1(w)); grid; title('T = 1');
subplot(5,1,2); plot(w,Y6(w)); grid; title('T = 6');
subplot(5,1,3); plot(w,Y10(w)); grid; title('T = 10');
subplot(5,1,4); plot(w,Y20(w)); grid; title('T = 20');
subplot(5,1,5); plot(w,Y40(w)); grid; title('T = 40');
xlabel('\omega ');
```

#17

(a) $\Delta = -10 \log_{10} \dfrac{1}{1+\left(\frac{\omega_s}{\omega_p}\right)^{2n}}$

$$10^{-\Delta/10} = \frac{1}{1+\left(\frac{\omega_s}{\omega_p}\right)^{2n}}$$

$$1+\left(\frac{\omega_s}{\omega_p}\right)^{2n} = 10^{+\Delta/10}$$

$$\left(\frac{\omega_s}{\omega_p}\right)^{2n} = 10^{+\Delta/10} - 1$$

$$2n \ln\left(\frac{\omega_s}{\omega_p}\right) = \ln\left[10^{+\Delta/10} - 1\right]$$

$$\boxed{n = \frac{\ln\left[10^{+\Delta/10} - 1\right]}{2 \ln\left(\frac{\omega_s}{\omega_p}\right)}}$$

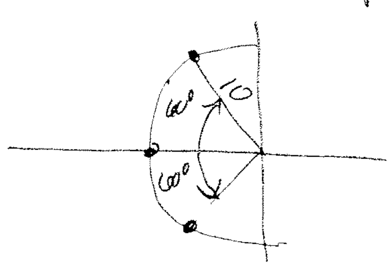
(b) $\omega_p = 10$ rad/sec $\omega_s = 20$ rad/sec $\Delta = +18$ dB

$n = \dfrac{\ln\left[10^{+1.8} - 1\right]}{2 \ln\left(\frac{20}{10}\right)} = 2.978$ need $n = 3$ see plot

(c) For this filter the poles are at -10, $-5 \pm j8.660$

The magnitudes for each of these is 10 rad/sec $= \omega_p$

The phases are $-180, 120, -120$ poles separated by $60°$



$\Theta = 60°$

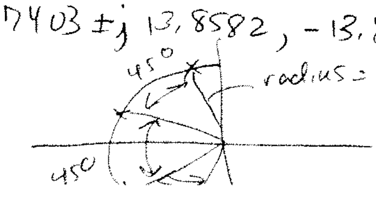
(d) $\omega_p = 15$ rad/sec $\omega_s = 35$ rad/sec $\Delta = +28$ dB

$n = \dfrac{\ln\left[10^{2.8} - 1\right]}{2 \ln\left(\frac{35}{15}\right)} = 3.8$ need $n = 4$ see plot

(e) For this filter the poles are at $-5.7403 \pm j\,13.8582$, $-13.8580 \pm j\,5.7408$

The magnitudes are all 15 rad/sec $= \omega_p$

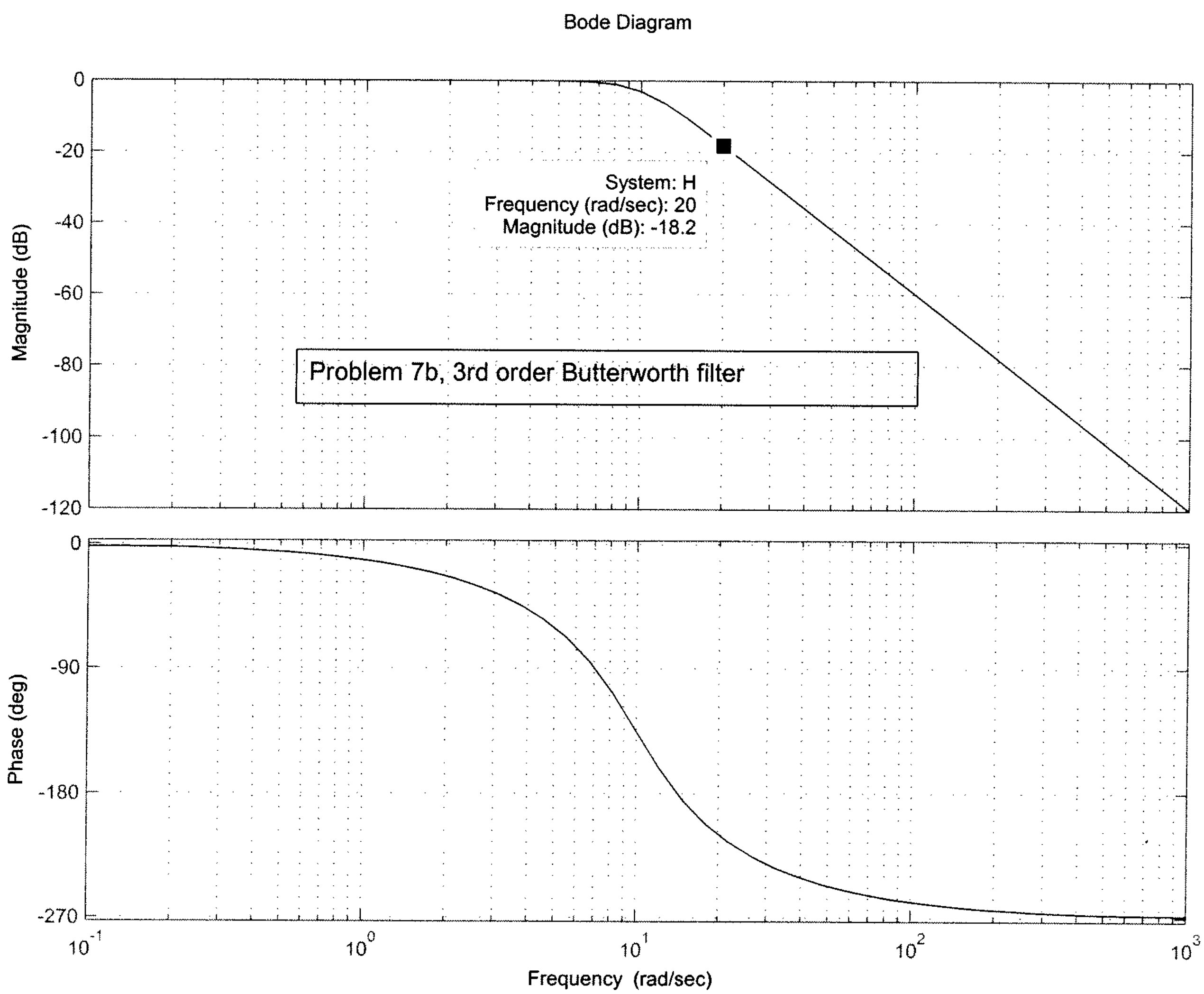
The phases are $\pm 112.5°$, $\pm 157.5°$

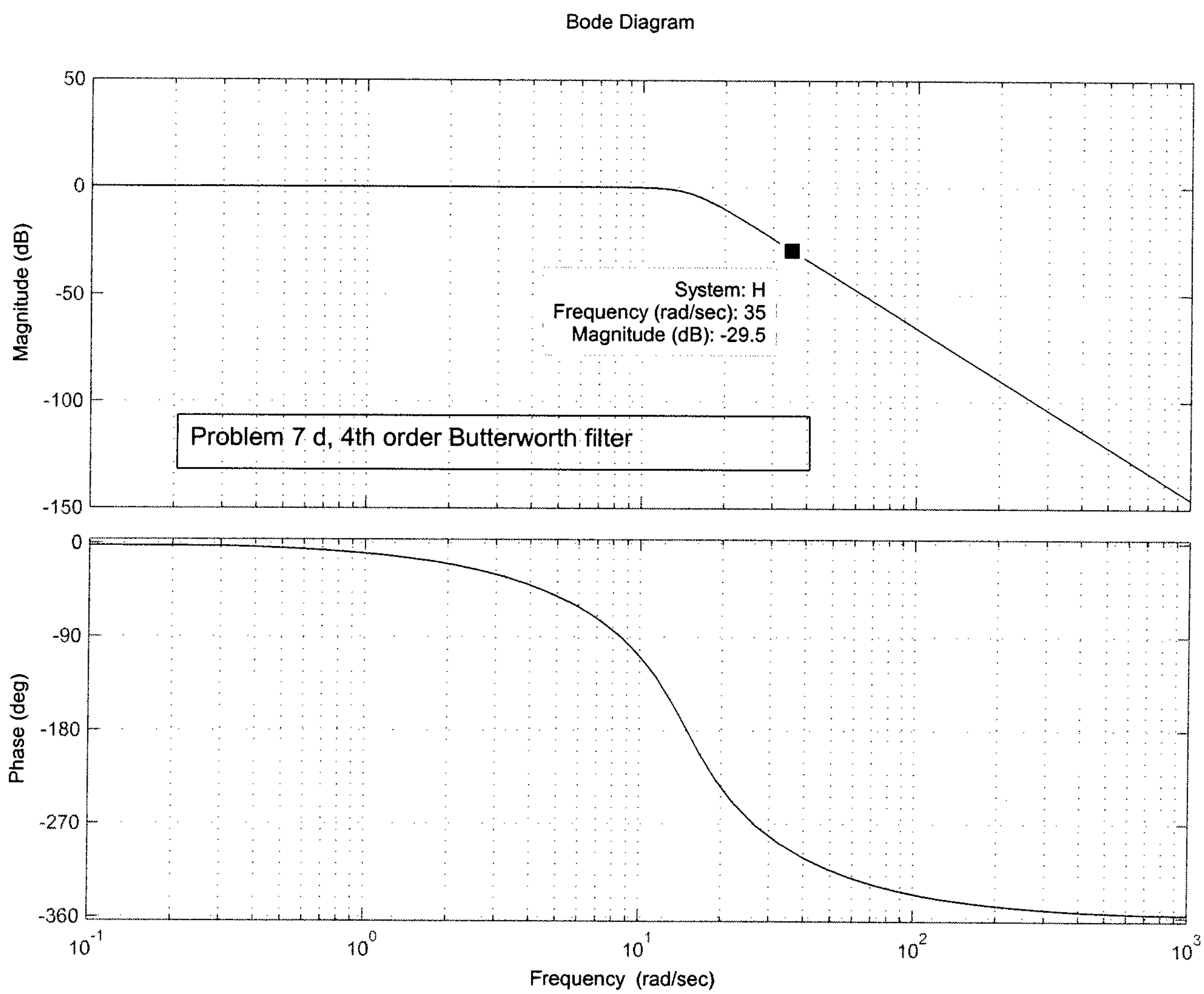


radius = 15°

separated by 45°

$\Theta = 45°$


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
-100
-120
System: H
Frequency (rad/sec): 20
Magnitude (dB): -18.2
Problem 7b, 3rd order Butterworth filter
Phase (deg)
0
-90
-180
-270
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)


Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
System: H
Frequency (rad/sec): 35
Magnitude (dB): -29.5
Problem 7 d, 4th order Butterworth filter
Phase (deg)
0
-90
-180
-270
-360
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)