ECE 300
Signals and Systems
Homework 3

Due Date: Tuesday March 25. 2008 *at the beginning of class*

EXAM #1, Thursday March 27

Problems

1. Determine the impulse responses for the following systems:

a) $y(t)=\frac{1}{2}[x(t-1)+x(t+1)]$ b) $y(t)=\int_{-\infty}^{t+1} e^{-(t-\lambda)}x(\lambda+3)d\lambda$

c) $y(t)=\int_{-\infty}^{t+3} e^{-(t-\lambda-2)}x(\lambda-1)d\lambda$ d) $\dot{y}(t)+2y(t)=3x(t-1)$

e) $\dot{y}(t)-3y(t)=x(t+2)$ f) $y(t)=x(t)+\int_{-\infty}^{t} e^{-2(t-\lambda)}x(\lambda)d\lambda$

2. The continuous-time I - interval moving average (MA) filter is given by the input/output relationship

$$y(t)=\frac{1}{I}\int_{t-I}^{t} x(\lambda)d\lambda$$

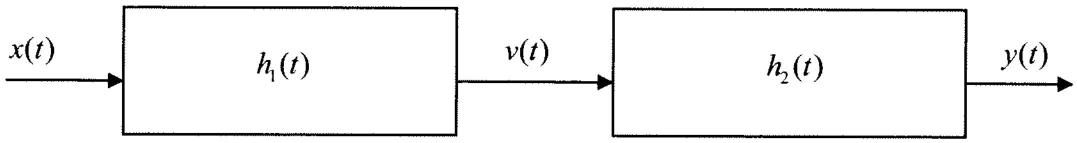
a. Determine the impulse response of the system. Write your answers in terms of unit step functions.

b. Determine the step response of the system, that is, determine the output when the input is a unit step. (*Answer:* $y(t)=\frac{1}{I}[tu(t)-(t-I)u(t-I)]$)

c. Determine the ramp response of the system, that is, determine the output when the input is a unit ramp.

d. Show that for a ramp input, in steady state $(t>I)$ the delay between the input and output is $\frac{I}{2}$ *Hint:* Draw pictures of the integrand and look at what happens as the interval $[t,t-I]$ varies.

3. Consider the following two subsystems, connected together to form a single LTI system.



Determine the impulse response $h(t)$ of the entire system if the impulse responses of the subsystems are given as:

a) $h_1(t) = \delta(t) \quad h_2(t) = 2e^{-t}u(t)$

b) $h_1(t) = e^{-t}u(t) \quad h_2(t) = 2\delta(t-1)$

c) $h_1(t) = e^{-t}u(t) \quad h_2(t) = e^{-t}u(t)$

d) $h_1(t) = 2\delta(t-1) \quad h_2(t) = 3\delta(t-2)$

e) $h_1(t) = 2u(t+2) \quad h_2(t) = u(t-1)$

Simplify your answers as much as possible.

4. Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t+1)}u(t+1)$$

The input to the system is given by

$$x(t) = e^{-(t-1)}u(t-1)$$

Use <u>**both**</u> *graphical convolution* and *analytical convolution* to determine the output $y(t)$ (i.e. find the answer two different ways). Specifically, for the *graphical convolution* you must

a) Flip and slide $h(t)$
b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
c) Determine the range of t for which each part of your solution is valid
d) Set up any necessary integrals and then compute $y(t)$

(Answer: $y(t) = te^{-t}u(t)$)

5. Consider a causal linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$

The input to the system is given by

$$x(t) = u(t) - u(t-1) + u(t-3)$$

Using ***graphical convolution***, determine the output $y(t)$ **for** $2 \le t \le 5$. ***Note the limited range of t we are interested in !***

Specifically, you must

a) Flip and slide $h(t)$
b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
c) Determine the range of t for which each part of your solution is valid
d) Set up any necessary integrals to compute $y(t)$
e) Evaluate the integrals

You should get (in unsimplified form)

$$y(t) = \begin{cases} e^{-(t-1)}[e^1 - 1] & 2 \le t \le 4 \\ e^{-(t-1)}[e^1 - 1] + e^{-(t-1)}[e^{t-1} - e^3] & 4 \le t \le 5 \end{cases}$$

6. Consider a causal linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$

a) Show that the step response of the system (the response to a unit step) is

$$y_s(t) = [1 - e^{-(t-1)}]u(t-1)$$

b) Using linearity and time-invariance, determine the response of the system to the input

$$x(t) = u(t-1) - 2u(t-2)$$

c) Use ***graphical convolution*** to determine the output of the system.

d) Show that your answers to **b** and **c** are the same.

e) Compute the derivative of the step response and show that you indeed obtain the impulse response.

7. Pre-Lab Exercises (to be done by all students. Turn this in with your homework and bring a copy of this with you to lab!)

a) Calculate the impulse response of the RC lowpass filter shown in Figure 1, in terms of unspecified components R and C. Determine the time constant for the circuit.

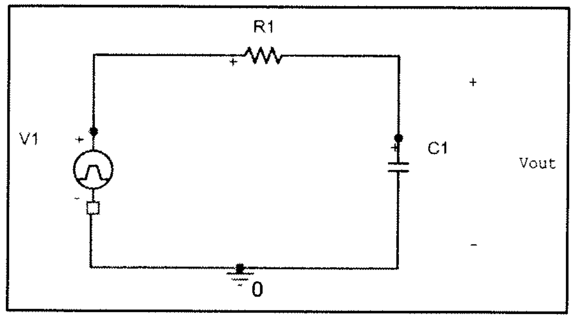


Figure 1. Simple RC lowpass filter circuit.

b) Show that the **step response** of the circuit (the response of the system when the input is a unit step) is $y_s(t) = (1 - e^{-t/\tau})u(t)$, and determine the 10-90% rise time. t_r, as shown below in Figure 2. The rise time is simply the amount of time necessary for the output to rise from 10% to 90% of its final value. Specifically, show that the rise time is given by $t_r = \tau \ln(9)$

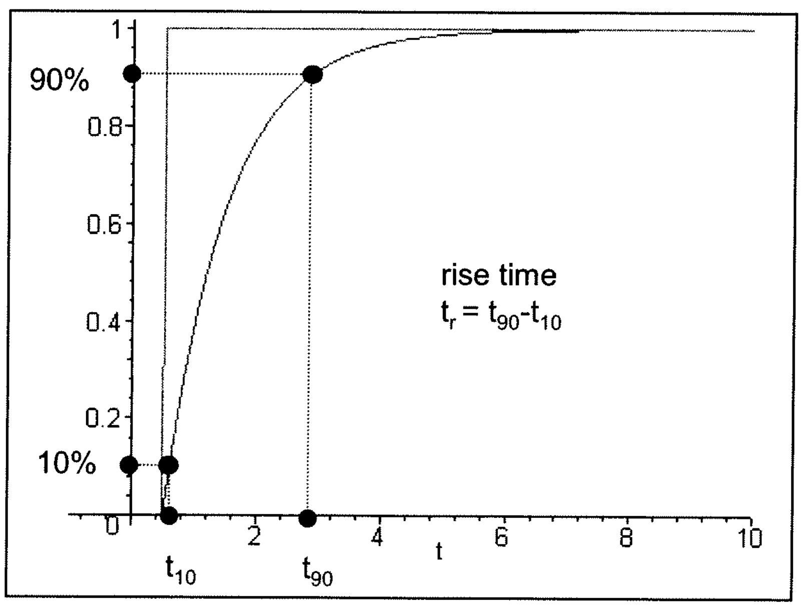


Figure 2. Step response of the RC lowpass filter circuit of Figure 1, showing the definition of the 10-90% risetime.

c) Specify values R and C which will produce a time constant of approximately 1 msec. Be sure to consider the fact that the capacitor will be asked to charge and discharge quickly in these measurements.

d) Using linearity and time-invariance, show that the response of the circuit to a pulse of length *T* and amplitude *A* (, i.e. a pulse of amplitude *A* starting at 0 and ending at *T*) is given by

$$y_{pulse}(t) = A(1-e^{-t/\tau})u(t) - A(1-e^{-(t-T)/\tau})u(t-T)$$

e) Plot the response to a unit (*A=1*) pulse (in Matlab) for $\tau = 0.001$ and $T = 0.003$, 0.001, and 0.0001 from 0 to 0.008 seconds. Note on the plots the times the capacitor is charging and discharging. Use the **subplot** command to make three separate plots, one on top of another (i.e., use subplot(3,1,1), subplot(3,1,2), subplot(3,1,3)).

f) Assume the input is a pulse of amplitude A and width T, and use the results from part **d** to determine an expression for the amplitude of the output at the end of the pulse, $y_{pulse}(T)$. Next, assume that $\frac{T}{\tau} \ll 1$ (the duration for the pulse is much small than the time constant of the circuit) and use Taylor series approximations for the exponentials to show that $y_{pulse}(T) \approx \frac{AT}{\tau}$. This means the amplitude of the output at time T (the end of the pulse) is approximately the area of the pulse divided by the time constant.

(#1)

a) $y(t) = \frac{1}{2}\left[x(t-1) + x(t+1)\right]$ $\boxed{h(t) = \frac{1}{2}\left[\delta(t-1) + \delta(t+1)\right]}$

b) $y(t) = \int_{-\infty}^{t+1} e^{-(t-\lambda)} x(\lambda+3)\,d\lambda$

$h(t) = \int_{-\infty}^{t+1} e^{-(t-\lambda)} \delta(\lambda+3)\,d\lambda = \boxed{e^{-(t+3)} u(t+4) = h(t)}$

need $t+1 > -3$ $t > -4$

c) $y(t) = \int_{-\infty}^{t+3} e^{-(t-\lambda-2)} x(\lambda-1)\,d\lambda$

$h(t) = \int_{-\infty}^{t+3} e^{-(t-\lambda-2)} \delta(\lambda-1)\,d\lambda = \boxed{e^{-(t-3)} u(t+2) = h(t)}$

need $t+3 > 1$

$t > -2$

d) $\dot{y}(t) + 2y(t) = 3x(t-1)$

$\frac{d}{dt}\left[e^{2t} h(t)\right] = e^{2t}\, 3\,\delta(t-1) = e^{2}\, 3\,\delta(t-1)$

$e^{2t} h(t) = \int_{-\infty}^{t} 3e^{2}\delta(\lambda-1)\,d\lambda = 3e^{2} u(t-1)$

$h(t) = 3e^{2}e^{-2t}u(t-1) = \boxed{3e^{-2(t-1)}u(t-1) = h(t)}$

e) $\dot{y}(t) - 3y(t) = x(t+2)$

$\frac{d}{dt}\left[e^{-3t} h(t)\right] = e^{-3t}\delta(t+2) = e^{6}\delta(t+2)$

$e^{-3t} h(t) = \int_{-\infty}^{t} e^{6}\delta(\lambda+2)\,d\lambda = e^{6} u(t+2)$

$h(t) = e^{3t}e^{6}u(t+2) = \boxed{e^{3(t+2)}u(t+2) = h(t)}$

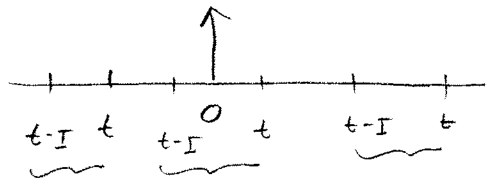
f) $y(t) = x(t) + \int_{-\infty}^{t} e^{-2(t-\lambda)} x(\lambda)\,d\lambda$

$h(t) = \delta(t) + \int_{-\infty}^{t} e^{-2(t-\lambda)} \delta(\lambda)\,d\lambda = \boxed{\delta(t) + e^{-2t}u(t) = h(t)}$

#2 $y(t) = \frac{1}{T}\int_{t-T}^{t} x(\lambda)\,d\lambda$

a) impulse response

$$h(t) = \frac{1}{T}\int_{t-T}^{t} \delta(\lambda)\,d\lambda$$



$$= \frac{1}{T} \text{ for } t>0 \text{ and } t-T<0$$

$$= \frac{1}{T} \text{ for } t>0 \text{ and } t<T$$

$$\boxed{h(t) = \frac{1}{T}\left[u(t) - u(t-T)\right]}$$

b) unit step response

$$y(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\lambda)u(t-\lambda)\,d\lambda$$

$$= \frac{1}{T}\int_{-\infty}^{\infty}\left[u(\lambda) - u(\lambda-T)\right]u(t-\lambda)\,d\lambda = \frac{1}{T}\int_{-\infty}^{\infty} u(\lambda)u(t-\lambda)\,d\lambda - \frac{1}{T}\int_{-\infty}^{\infty} u(\lambda-T)u(t-\lambda)\,d\lambda$$

$u(\lambda) = 1 \quad \lambda > 0$

$u(t-\lambda) = 1 \quad t-\lambda > 0 \quad t > \lambda$

$u(\lambda - T) = 1 \quad \lambda - T > 0 \quad \lambda > T$

need $t>0$ need $t>T$

$$y(t) = \frac{1}{T}\left[\int_0^t d\lambda + \int_T^t d\lambda\right] = \frac{1}{T}\left[t\,u(t) + (t-T)\,u(t-T)\right]$$

$$\boxed{\text{step response} = \frac{1}{T}\left[t\,u(t) - (t-T)\,u(t-T)\right]}$$

#2

c) ramp response

$$y(t) = h(t) * r(t) = \int_{-\infty}^{\infty} r(\lambda)\, h(t-\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} \lambda u(\lambda) \frac{1}{I}\left[u(t-\lambda) - u(t-I-\lambda)\right] d\lambda$$

$$= \frac{1}{I}\int_{-\infty}^{\infty} \lambda u(\lambda) u(t-\lambda)\, d\lambda - \frac{1}{I}\int_{-\infty}^{\infty} \lambda u(\lambda) u(t-I-\lambda)\, d\lambda$$

$$= \frac{1}{I}\int_{0}^{t} \lambda\, d\lambda - \frac{1}{I}\int_{0}^{t-I} \lambda\, d\lambda = \frac{1}{I}\frac{t^2}{2}u(t) - \frac{1}{I}\frac{(t-I)^2}{2}u(t-I)$$

$$\boxed{\text{ramp response} = \frac{1}{I}\left[\frac{t^2}{2}u(t) - \frac{(t-I)^2}{2}u(t-I)\right]}$$

d) for $t > I$ ramp response $= \frac{1}{I}\left[\frac{t^2}{2} - \frac{(t-I)^2}{2}\right]$

$$= \frac{1}{I}\left[\frac{t^2}{2} - \frac{(t^2 - 2tI + I^2)}{2}\right]$$

$$= \frac{1}{2I}\left[2tI - I^2\right] = t - \frac{I}{2}$$

input $= t$ output $= t - I/2$ $\boxed{\text{delay} = I/2}$

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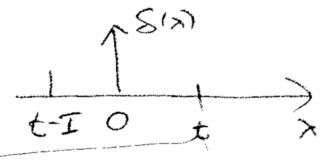
#2 (Alternative Solution)

$$y(t) = \frac{1}{I}\int_{t-I}^{t} x(\lambda)\,d\lambda \quad I>0$$ continuous-time moving average filter

(a) assume $x(t) = \delta(t)$ so $y(t) = \frac{1}{I}\int_{t-I}^{t} \delta(\lambda)\,d\lambda$

draw a picture!

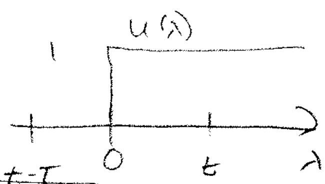


$y(t) = \frac{1}{I}$ if $\delta(\lambda)$ is in the region of integration

$$\boxed{y(t) = \begin{cases} \frac{1}{I} & 0<t<I \\ 0 & \text{else} \end{cases}} = \frac{1}{I}\left[u(t) - u(t-I)\right]$$

(b) assume $x(t) = u(t)$ so $y(t) = \frac{1}{I}\int_{t-I}^{t} u(\lambda)\,d\lambda$

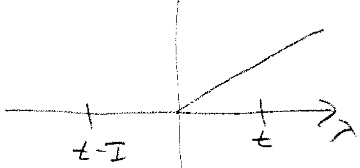
draw a picture!



$$\boxed{y(t) = \begin{cases} \frac{t}{I} & 0<t<I \\ 1 & t \geq I \\ 0 & \text{else} \end{cases}} = \frac{1}{I}\left[t\,u(t) - (t-I)\,u(t-I)\right]$$

(c) assume $x(t) = t\,u(t)$ so $y(t) = \frac{1}{I}\int_{t-I}^{t} \lambda\, u(\lambda)\,d\lambda$

draw a picture!



for $0<t<I$ $\quad y(t) = \frac{1}{I}\int_0^t \lambda^2\,d\lambda = \frac{t^2}{2I}$

for $t>I$ $\quad y(t) = \frac{1}{I}\int_{t-T}^{t} \lambda^2\,d\lambda = \frac{t^2 - (t-I)^2}{2I}$

$$y(t) = \begin{cases} \frac{t^2}{2I} & 0<t\leq I \\ \frac{t^2-(t-I)^2}{2I} & t \geq I \\ 0 & \text{else} \end{cases}$$

#3 $x(t) \longrightarrow \boxed{h_1(t)} \xrightarrow{v(t)} \boxed{h_2(t)} \longrightarrow y(t)$

for the system $v(t) = h_1(t) * x(t)$ $\quad y(t) = h_2(t) * v(t)$

Combining these $y(t) = h_2(t) * [h_1(t) * x(t)] = [h_1(t) * h_2(t)] * x(t)$

so for the system $h(t) = h_1(t) * h_2(t)$

a) $h_1(t) = \delta(t)$ $\quad h_2(t) = 2e^{-t}u(t)$

$$h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)d\lambda = \int_{-\infty}^{\infty} \delta(t-\lambda)2e^{-\lambda}u(\lambda)d\lambda = \boxed{2e^{-t}u(t) = h(t)}$$

b) $h_1(t) = e^{-t}u(t)$ $\quad h_2(t) = 2\delta(t-1)$

$$h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)d\lambda = \int_{-\infty}^{\infty} e^{-(t-\lambda)}u(t-\lambda)\,2\delta(\lambda-1)d\lambda$$

$$= e^{-(t-1)}u(t-1)\,2 = \boxed{2e^{-(t-1)}u(t-1) = h(t)}$$

c) $h_1(t) = e^{-t}u(t)$ $\quad h_2 = e^{-t}u(t)$

$$h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)d\lambda = \int_{-\infty}^{\infty} e^{-(t-\lambda)}u(t-\lambda)e^{-\lambda}u(\lambda)\,d\lambda$$

$$= e^{-t}\int_0^t dt = \boxed{te^{-t}u(t) = h(t)}$$

d) $h_1(t) = 2\delta(t-1)$ $\quad h_2(t) = 3\delta(t-2)$

$$h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)d\lambda = \int_{-\infty}^{\infty} 2\,\delta(t-\lambda-1)\,3\,\delta(\lambda-2)d\lambda$$

$$= \boxed{6\,\delta(t-3) = h(t)}$$

e) $h_1(t) = 2u(t+2)$ $\quad h_2(t) = u(t-1)$

$$h(t) = \int_{-\infty}^{\infty} 2u(t-\lambda+2)\,u(\lambda-1)d\lambda$$

$u(\lambda-1) = 1$ for $\lambda - 1 \geq 0$ $\quad \lambda \geq 1$

$u(t-\lambda+2) = 1$ for $t-\lambda+2 \geq 0$ $\quad t+2 \geq \lambda$

$$h(t) = \int_1^{t+2} 2\,d\lambda = 2[(t+2)-1]u(t+2-1) = \boxed{2(t+1)u(t+1) = h(t)}$$

(#4) $h(t) = e^{-(t+1)}u(t+1) \qquad x(t) = e^{-(t-1)}u(t-1)$

(a) analytical convolution

$$y(t) = \int_{-\infty}^{\infty} h(t-\lambda)x(\lambda)\,d\lambda = \int_{-\infty}^{\infty} e^{-(t-\lambda+1)}u(t-\lambda+1)\,e^{-(\lambda-1)}u(\lambda-1)\,d\lambda$$

$$= \int_{-\infty}^{\infty} e^{-t}e^{\lambda}e^{-1}e^{-\lambda}e^{1}\,u(t-\lambda+1)\,u(\lambda-1)\,d\lambda$$

$u(t-\lambda+1) = 1$ for $t-\lambda+1 \geq 0 \quad t+1 \geq \lambda$

$u(\lambda-1) = 1$ for $\lambda-1 > 0 \quad \lambda > 1$

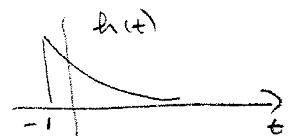
need $t+1 > \lambda > 1$ or $t > 0$

$$= e^{-t}\int_{1}^{t+1} d\lambda = e^{-t}\left[(t+1)-1\right]u(t)$$

$$\boxed{y(t) = te^{-t}u(t)}$$

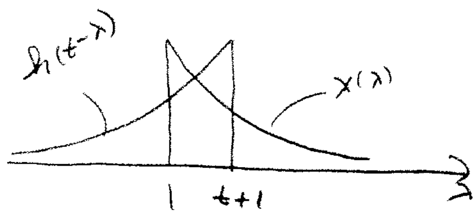
(b) graphical convolution (Flip and slide $h(t)$)

$$y(t) = \int_{-\infty}^{\infty} h(t-\lambda)\,x(\lambda)\,d\lambda$$



$h(-1) = h(t-\lambda)$

$-1 = t-\lambda \quad \lambda = t+1$

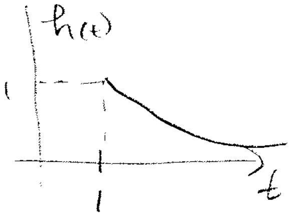


valid for $t \geq 0$

$$y(t) = \int_{1}^{t+1} e^{-(t-\lambda+1)}e^{-(\lambda-1)}\,d\lambda = \int_{1}^{t+1} e^{-t}e^{\lambda}e^{-1}e^{-\lambda}e^{1}\,d\lambda$$

$= e^{-t}\int_{1}^{t+1} d\lambda = te^{-t}$ for $t > 0$ hence $\boxed{y(t) = te^{-t}u(t)}$

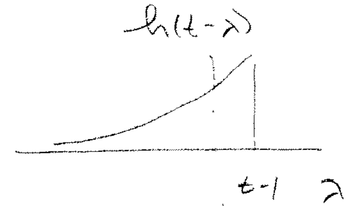
#5 $h(t) = e^{-(t-1)}u(t-1) \quad x(t) = u(t) - u(t-1) + u(t-3)$

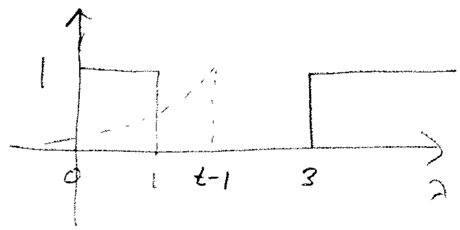


$h(1) = h(t-\lambda)$

$1 = t - \lambda$

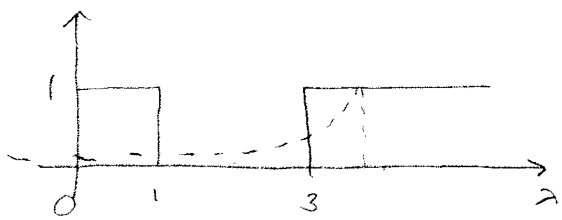
$\lambda = t-1$





$2 \le t \le 4$

$$y(t) = \int_0^1 e^{-(t-\lambda-1)} d\lambda = e^{-(t-1)} \int_0^1 e^{\lambda} d\lambda = e^{-(t-1)}\left[e^1 - 1\right]$$



$t \ge 4$

$$y(t) = \int_0^1 e^{-(t-\lambda-1)} d\lambda + \int_3^{t-1} e^{-(t-\lambda-1)} d\lambda$$

$$= e^{-(t-1)}\left[e^1 - 1\right] + e^{-(t-1)}\left[e^{\lambda}\right]_3^{t-1}$$

$$= e^{-(t-1)}\left[e^1 - 1\right] + e^{-(t-1)}\left[e^{t-1} - e^3\right]$$

$$\boxed{y(t) = \begin{cases} e^{-(t-1)}\left[e^1 - 1\right] & 2 \le t \le 4 \\ e^{-(t-1)}\left[e^1 - 1\right] + e^{-(t-1)}\left[e^{t-1} - e^3\right] & 4 \le t \end{cases}}$$

⑥ $h(t) = e^{-(t-1)} u(t-1)$

a) determine the step response $y_s(t)$

$$y_s(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\lambda) u(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} e^{-(\lambda-1)} u(\lambda-1) u(t-\lambda)\, d\lambda$$

$$= e^{1} \int_{1}^{t} e^{-\lambda} d\lambda = e^{1}\left[-e^{-\lambda}\Big|_{1}^{t}\right] = e^{1}\left[e^{-1} - e^{-t}\right] = 1 - e^{-(t-1)}$$

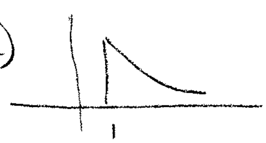
this is only valid for $t \geq \lambda \geq 1$ or $t \geq 1$, so

$$\boxed{y_s(t) = \left[1 - e^{-(t-1)}\right] u(t-1)}$$

b) input $= x(t) = u(t-1) - 2u(t-2)$

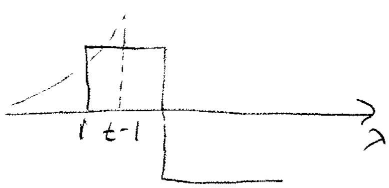
output $= y(t) = y_s(t-1) - 2y_s(t-2)$

$$= \boxed{\left[1 - e^{-(t-2)}\right] u(t-2) - 2\left[1 - e^{-(t-3)}\right] u(t-3) = y(t)}$$

c) 

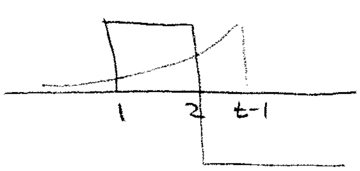
$h(1) = h(t-\lambda)$
$1 = t-\lambda$
$\lambda = t-1$

$$y(t) = \int_{-\infty}^{\infty} h(t-\lambda)\, x(\lambda)\, d\lambda$$



Valid for $t > 2$

$$y(t) = \int_{1}^{t-1} e^{-(t-\lambda-1)} d\lambda = e^{-(t-1)} \int_{1}^{t-1} e^{\lambda} d\lambda = e^{-(t-1)}\left[e^{t-1} - e^{1}\right] = 1 - e^{-(t-2)}$$



valid for $t > 3$

$$y(t) = \int_{1}^{2} e^{-(t-\lambda-1)} d\lambda + \int_{2}^{t-1} e^{-(t-\lambda-1)}(-1)\, d\lambda$$

$$= e^{-(t-1)} \int_{1}^{2} e^{\lambda} d\lambda - e^{-(t-1)} \int_{2}^{t-1} e^{\lambda} d\lambda$$

continued on next page

problem 6 (continued)

$$y(t) = e^{-(t-1)}\left[e^{2} - e^{1}\right] - e^{-(t-1)}\left[e^{t-1} - e^{2}\right]$$

$$= e^{-(t-3)} - e^{-(t-2)} - 1 + e^{-(t-3)}$$

(d) the easiest way to show this is the same as the answer to part b is to add and subtract 1

$$y(t) = e^{-(t-3)} - e^{-(t-2)} - 1 + e^{-(t-3)} \underbrace{+1-1}_{=0}$$

$$= \left(1 - e^{-(t-2)}\right) - 2\left(1 - e^{-(t-3)}\right) \quad \text{for } t \geq 3$$

The other way to do this is to modify the answer from (b)

for $t > 3$

$$y(t) = \left(1 - e^{-(t-2)}\right) - 2\left(1 - e^{-(t-3)}\right)$$

$$= 1 - e^{-(t-2)} - 2 + 2e^{-(t-3)}$$

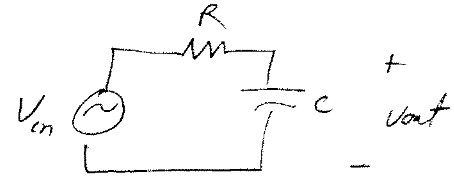
$$= -1 - e^{-(t-2)} + 2e^{-(t-3)}$$ the same result as for part c

(e) $$\frac{d}{dt}\left[y_s(t)\right] = \frac{d}{dt}\left[\left(1 - e^{-(t-1)}\right)u(t-1)\right]$$

$$= -(-1)e^{-(t-1)}u(t-1) + \left(1 - e^{-(t-1)}\right)\delta(t-1)$$

$$= e^{-(t-1)}u(t-1) + 0\,\delta(t-1) = \boxed{e^{-(t-1)}u(t-1) = h(t)}$$

#7



(a)

$$\frac{V_{in} - V_{out}}{R} = C\frac{dV_{out}}{dt} \qquad CR\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

$$\frac{dV_{out}}{dt} + \frac{1}{RC}V_{out} = \frac{1}{RC}V_{in} \qquad \frac{d}{dt}\left(e^{t/RC}V_{out}\right) = \frac{1}{RC}V_{in}\,e^{t/RC}$$

$$\int_{-\infty}^{t}\frac{d}{dt}\left(e^{t/RC}V_{out}(t)\right)dt = e^{t/RC}V_{out} = \int_{-\infty}^{t}\frac{1}{RC}e^{\lambda/RC}V_{in}(\lambda)\,d\lambda$$

for impulse response $V_{in}(t) = \delta(t)$ $V_{out}(t) = h(t)$

$$e^{+t/RC}h(t) = \int_{-\infty}^{t}\frac{1}{RC}e^{\lambda/RC}\delta(\lambda)\,d\lambda = \frac{1}{RC}u(t)$$

$$\boxed{h(t) = \frac{1}{RC}e^{-t/RC}u(t)} \qquad \boxed{\tau = RC}$$

or $$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs+1} = \frac{1}{RC}\frac{1}{s + 1/RC} \Rightarrow h(t) = \frac{1}{RC}e^{-t/RC}u(t)$$

(b) $h(t) = \frac{1}{\tau}e^{-t/\tau}u(t)$

$$y(t) = h(t) * u(t) = \int_{-\infty}^{\infty}h(\lambda)u(t-\lambda) = \int_0^t \frac{1}{\tau}e^{-t/\tau}d\tau = -e^{-t/\tau}\Big|_0^t$$

$$\boxed{y(t) = \left[1 - e^{-t/\tau}\right]u(t)}$$

For rise time $0.9 = 1 - e^{-t_9/\tau}$ or $0.1 = e^{-t_9/\tau}$

$0.1 = 1 - e^{-t_1/\tau}$ or $0.9 = e^{-t_1/\tau}$

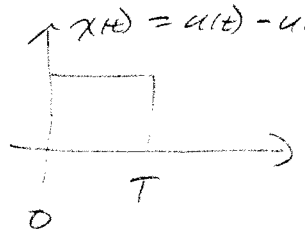
taking ratios $9 = e^{-(t_1 - t_9)/\tau} = e^{(t_9 - t_1)/\tau} = e^{t_r/\tau}$

$\ln 9 = t_r/\tau$ $\boxed{t_r = \tau \ln 9}$

#7 (continued)

(c) up to you, but c should be small

(d) $x(t) = u(t) - u(t-T)$



for $x(t) = u(t)$ $\quad y(t) = \left[1 - e^{-t/\tau}\right] u(t)$

since LTI for $x(t) = u(t) - u(t-T)$, $\boxed{y(t) = \left[1 - e^{-t/\tau}\right] u(t) - \left[1 - e^{-(t-T)/\tau}\right] u(t-T)}$

(e) see plots

(f) For $x(t) = A\left[u(t) - u(t-T)\right]$ the output is

$$y(t) = A\left[1 - e^{-t/\tau}\right] u(t) - A\left[1 - e^{-(t-T)/\tau}\right] u(t-T)$$

$$y(T) = A\left[1 - e^{-T/\tau}\right]$$

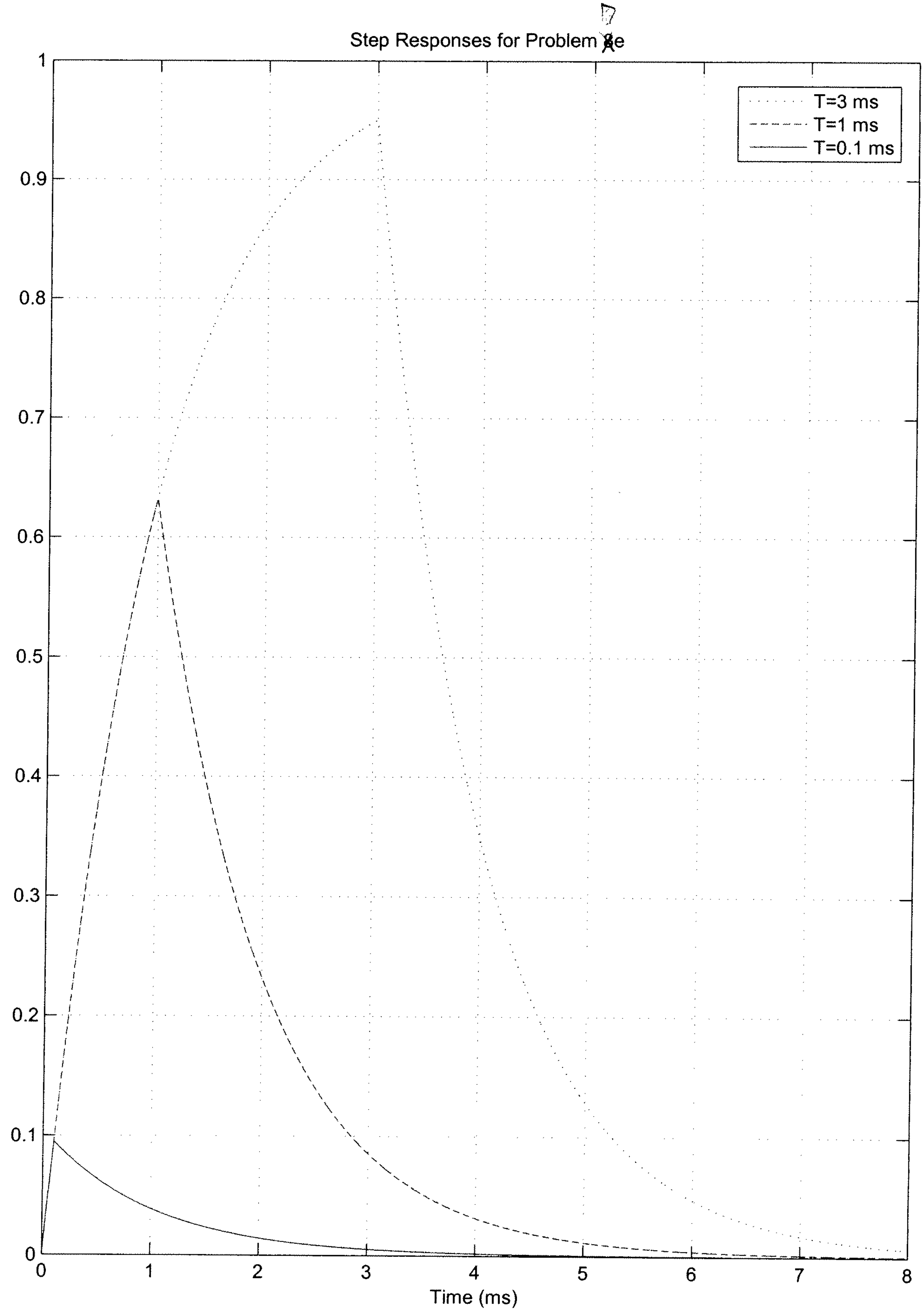
For $T/\tau \ll 1$ $\quad e^{-T/\tau} \approx 1 - T/\tau$

$$y(T) \approx A\left[1 - (1 - T/\tau)\right] = \boxed{\frac{AT}{\tau} = y(T)}$$

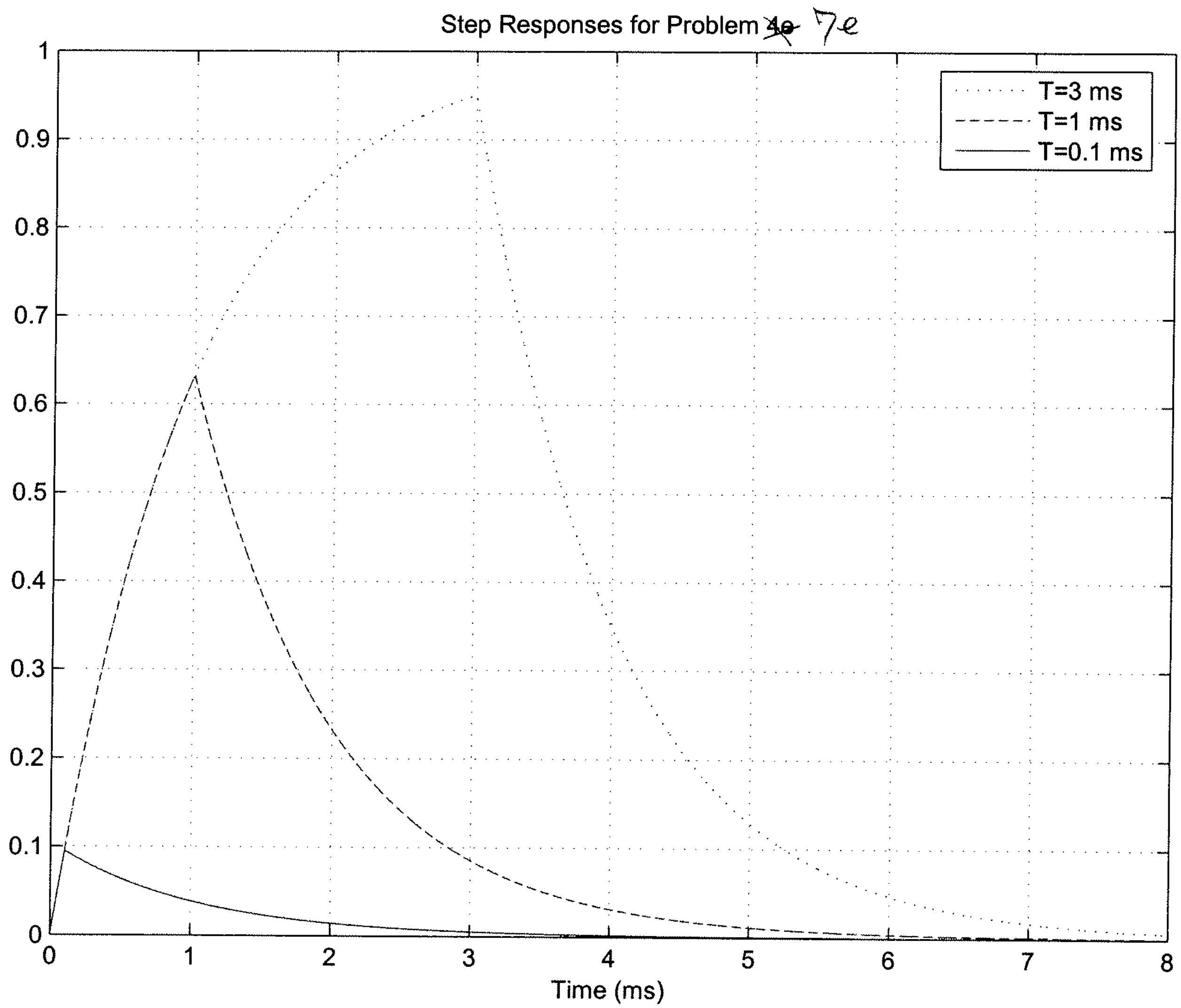
```
%
%   step response plot for homework 3 (prelab for impulse response lab)
%
t = linspace(0,0.008,10000);
tau = 0.001;
%
  T = 0.003;
  y1 = (1-exp(-t/tau)).*unit_step(t,0)-(1-exp(-(t-T)/tau)).*unit_step(t,T);
  T = 0.001;
  y2 = (1-exp(-t/tau)).*unit_step(t,0)-(1-exp(-(t-T)/tau)).*unit_step(t,T);
  T = 0.0001;
  y3 = (1-exp(-t/tau)).*unit_step(t,0)-(1-exp(-(t-T)/tau)).*unit_step(t,T);
  orient tall

  plot(t*1000,y1,':',t*1000,y2,'--',t*1000,y3,'-'); grid;
  legend('T=3 ms','T=1 ms','T=0.1 ms');
  xlabel('Time (ms)'); title('Step Responses for Problem 8e');

  figure;
  orient tall
  subplot(3,1,1); plot(t*1000,y1); grid; ylabel('T = 3 ms'); xlabel('Time (ms)');
  title('Results for Problem 8e');
  subplot(3,1,2); plot(t*1000,y2); grid; ylabel('T = 1 ms'); xlabel('Time (ms)');
  subplot(3,1,3); plot(t*1000,y3); grid; ylabel('T = 0.1 ms'); xlabel('Time (ms)');
```



Step Responses for Problem ~~8~~e 7
T=3 ms
T=1 ms
T=0.1 ms
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
Time (ms)


Step Responses for Problem ~~4e~~ 7e
T=3 ms
T=1 ms
T=0.1 ms
Time (ms)

Results for Problem ~~8~~7e