Name ____ CM__________

ECE 300
Signals and Systems

Exam 3
13 May, 2008

NAME Solutions

This exam is closed-book in nature. You may use the provided table of Fourier Transform relationships. You may use a calculator for simple calculations, but not for things like integrals. You must show all of your work.
Credit will not be given for work not shown.

Problem 1	________	/ 15
Problem 2	________	/ 40
Problem 3	________	/ 25
Problem 4	________	/ 20

Exam 3 Total Score: _______ / 100

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1. Transfer Functions (15 points)

Assume $x(t)$ and $y(t)$ are related through the equation

$$\dot{y}(t-a) + 2y(t+b) = \dot{x}(t)$$

Determine the transfer function $H(\omega)$ between $X(\omega)$ and $Y(\omega)$.

$$\mathcal{F}\{\dot{y}(t-a)\} = j\omega e^{-j\omega a} Y(\omega)$$

$$\mathcal{F}\{y(t+b)\} = e^{j\omega b} Y(\omega)$$

$$\mathcal{F}\{\dot{x}(t)\} = j\omega X(\omega)$$

$$\left[j\omega e^{-j\omega a} + 2e^{j\omega b}\right] Y(\omega) = \left[j\omega\right] X(\omega)$$

$$\frac{Y(\omega)}{X(\omega)} = \boxed{H(\omega) = \frac{j\omega}{j\omega e^{-j\omega a} + 2e^{j\omega b}}}$$

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2. (40 points) Fourier Analysis of LTI Systems

Assume $x(t) = 2\,\text{sinc}\left[\frac{3}{\pi}(t-3)\right]\cos(2(t-3))$ is the input to an LTI system with transfer function $H(\omega) = \begin{cases} 3e^{-j\omega 2} & |\omega| < 2 \\ 0 & else \end{cases}$

a) Determine the Fourier transform $X(\omega)$ of $x(t)$

b) Accurately sketch the magnitude and phase of $X(\omega)$

c) Determine the energy in $x(t)$

d) Accurately sketch the magnitude and phase of the system output in the frequency domain

e) Determine the system output $y(t)$

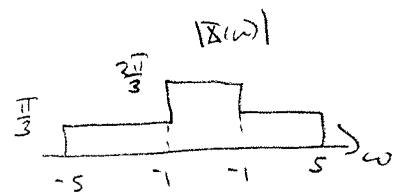
(a) $x_1(t) = 2\,\text{sinc}\left(\frac{3}{\pi}t\right) \longleftrightarrow X_1(\omega) = \frac{2\pi}{3}\,\text{rect}\left(\frac{\omega}{6}\right)$

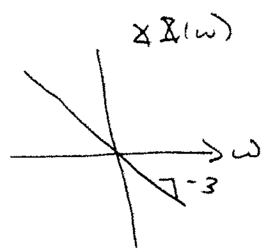
$x_2(t) = x_1(t)\cos(2t) \longleftrightarrow X_2(\omega) = \frac{1}{2}X_1(\omega+2) + \frac{1}{2}X_1(\omega-2)$

$= \frac{\pi}{3}\,\text{rect}\left(\frac{\omega+2}{6}\right) + \frac{\pi}{3}\,\text{rect}\left(\frac{\omega-2}{6}\right)$

$$\boxed{X(\omega) = \left[\frac{\pi}{3}\,\text{rect}\left(\frac{\omega+2}{6}\right) + \frac{\pi}{3}\,\text{rect}\left(\frac{\omega-2}{6}\right)\right]e^{-j3\omega}}$$

(b)

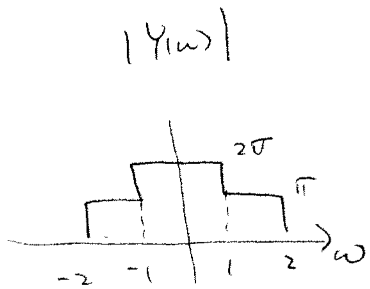


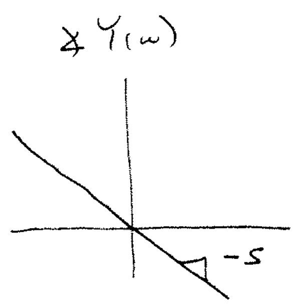


(c) $E_x = \frac{1}{2\pi}\left[2\int_0^1 \left(\frac{2\pi}{3}\right)^2 d\omega + 2\int_1^5 \left(\frac{\pi}{3}\right)^2 d\omega\right]$

$= \boxed{\frac{8\pi}{9} = E_x}$

(d)





(e) $Y(\omega) = \left[\pi\,\text{rect}\left(\frac{\omega}{4}\right) + \pi\,\text{rect}\left(\frac{\omega}{2}\right)\right]e^{-j5\omega}$

$\text{rect}\left(\frac{\omega}{4}\right) \longleftrightarrow \frac{2}{\pi}\,\text{sinc}\left(\frac{2}{\pi}t\right) \quad 2\pi W = 4 \quad W = \frac{2}{\pi}$

$\text{rect}\left(\frac{\omega}{2}\right) \longleftrightarrow \frac{1}{\pi}\,\text{sinc}\left(\frac{1}{\pi}t\right) \quad 2\pi W = 2 \quad W = \frac{1}{\pi}$

$$\boxed{y(t) = 2\,\text{sinc}\left(\frac{2}{\pi}(t-5)\right) + \text{sinc}\left(\frac{1}{\pi}(t-5)\right)}$$

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3. (25 points) Fourier Transform Properties

By either direct evaluation of the Fourier transform (or inverse transform) integrals, or by using the Tables and properties, find the corresponding Fourier transform pair for each of the following. *Be sure to simplify your answers as much as possible.*

a) $X(\omega) = \dfrac{j\omega}{2 - j\omega}$, $x(t) = ?$

b) $x(t) = \dfrac{2}{4 + \left[\dfrac{t}{3} + 2\right]^2}$, $X(\omega) = ?$

(a) $x_1(t) = e^{-2t}u(t) \Longleftrightarrow X_1(\omega) = \dfrac{1}{2+j\omega}$

$x_2(t) = x_1(-t) = e^{2t}u(-t) \Longleftrightarrow X_2(\omega) = X_1(-\omega) = \dfrac{1}{2-j\omega}$

$x(t) = \dfrac{d}{dt}x_2(t) = \dfrac{d}{dt}\left[e^{2t}u(-t)\right] = \boxed{2e^{2t}u(-t) - \delta(t)} \Longleftrightarrow X(\omega) = \dfrac{j\omega}{2-j\omega}$

(b) $x_1(t) = \frac{1}{2}e^{-2|t|} \Longleftrightarrow X_1(\omega) = \dfrac{2}{4+\omega^2}$

$x_2(t) = X_1(t) = \dfrac{2}{4+t^2} \Longleftrightarrow X_2(\omega) = 2\pi x_1(-\omega) = \pi e^{-2|\omega|}$

$x_3(t) = x_2(t+2) = \dfrac{2}{4+(t+2)^2} \Longleftrightarrow X_3(\omega) = \pi e^{-2|\omega|}e^{j2\omega}$

$x_4(t) = x_3\left(\frac{t}{3}\right) = \dfrac{2}{4+\left(\frac{t}{3}+2\right)^2} \Longleftrightarrow X_4(\omega) = 3X_3(3\omega) = \boxed{3\pi e^{-6|\omega|}e^{j6\omega}}$

or

$x_3(t) = x_2\left(\frac{t}{3}\right) = \dfrac{2}{4+\left(\frac{t}{3}\right)^2} \Longleftrightarrow X_3(\omega) = 3X_2(3\omega) = 3\pi e^{-6|\omega|}$

$x_4(t) = x_3(t+6) = \dfrac{2}{4+\left(\frac{t+6}{3}\right)^2} \Longleftrightarrow X_4(\omega) = X_3(\omega)e^{j6\omega} = 3\pi e^{-6|\omega|}e^{j6\omega}$

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4. (20 points) Fourier Series

Assume $x(t)$ is a periodic signal with Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{k=\infty} \frac{2}{1+jk} e^{jk4t}$$

Assume $x(t)$ is the input to an LTI system with transfer function

$$H(j\omega) = \begin{cases} 3 & |\omega| < 3 \\ 4e^{-j\frac{\omega}{10}} & 3 < |\omega| < 11 \\ 0 & |\omega| > 11 \end{cases}$$

Determine the steady state output of the system, $y(t)$. Your answer must be written in terms of sines and cosines, not complex exponentials. Your answer must also be in either degrees or radians, but not a mixture.

$\omega_0 = 4$

$$c_0^y = c_0^x H(0) = (2+2)(3) = 12$$

$$c_1^y = c_1^x H(j\omega_0) = \left(\frac{2}{1+j}\right)\left(4e^{-j0.4}\right) = \left(\frac{2}{\sqrt{2}} \angle -\frac{\pi}{4}\right)\left(4 \angle -0.4\right)$$

$$= 5.65 \angle -1.185 \text{ rad}$$

$$c_2^y = c_2^x H(j2\omega_0) = \left(\frac{2}{1+2j}\right)\left(4e^{-j0.8}\right) = \left(\frac{2}{\sqrt{5}} \angle -1.107\right)\left(4 \angle -0.8\right)$$

$$= 3.58 \angle -1.910 \text{ rad}$$

$$\boxed{y(t) = 12 + 11.31\cos(4t - 1.185) + 7.16\cos(8t - 1.910)}$$