Name Solutions CM

ECE 300
Signals and Systems

Exam 1
1 October, 2009

This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam. ***You must show your work to receive credit for a problem.***

Problem 1	________	/ 10
Problem 2	________	/5
Problem 3	________	/ 30
Problem 4	________	/ 30
Problem 5	________	/ 25

Exam 1 Total Score: ________ / 100

50-59 4
60-69 7
70-79 10
80-89 12
90-99 7

median = 79
average = 78

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1. (10 points) For the following signal, determine if the signal is periodic and if it is perioidic determine its fundamental period.

$$x(t) = \cos\left(\frac{\pi}{2}t\right) + e^{j(\frac{\pi}{5}t+\sqrt{2})}$$

$$x(t+T_0) = \cos\left(\frac{\pi}{2}t + \frac{\pi}{2}T_0\right) + e^{j(\frac{\pi}{5}t + \frac{\pi}{5}T_0 + \sqrt{2})}$$

$$\frac{\pi}{2}T_0 = g(2\pi) \qquad \frac{\pi}{5}T_0 = r(2\pi)$$

$$T_0 = g(4) = r(10) \qquad g = 5, r = 2$$

$\boxed{\text{Periodic with period } T_0 = 20 \text{ seconds}}$

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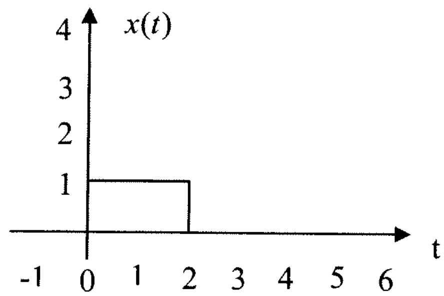
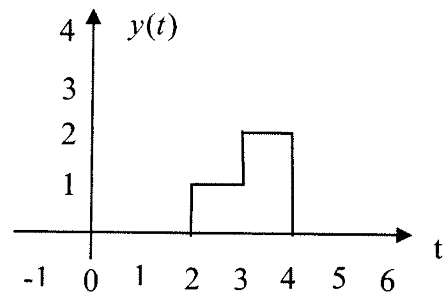
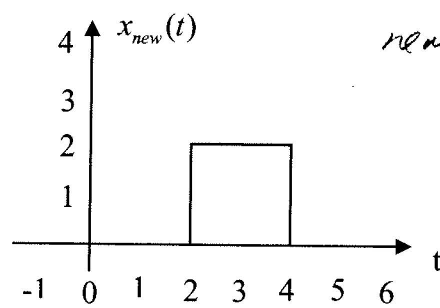
2. (5 points) Assume we know a system is a linear time invariant (LTI) system. We also know the following input $x(t)$ – output $y(t)$ pair:





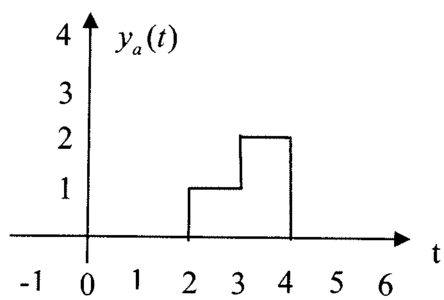
If the input to the system is now $x_{new}(t)$

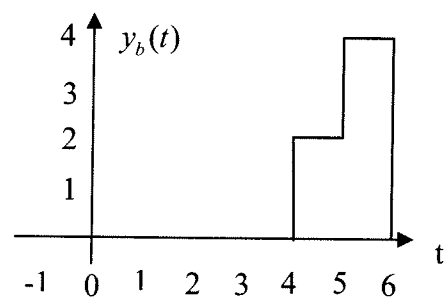


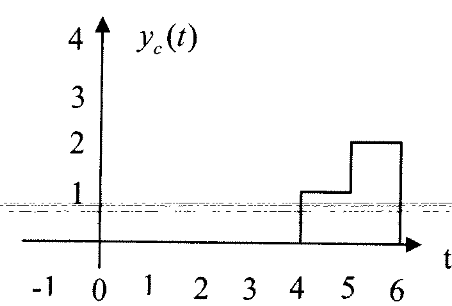
new input delayed by 2, scaled by 2

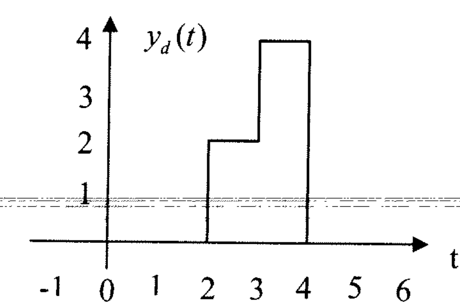
Which of the following best represents the output of the system?

a) $y_a(t)$ b) $y_b(t)$ c) $y_c(t)$ d) $y_d(t)$









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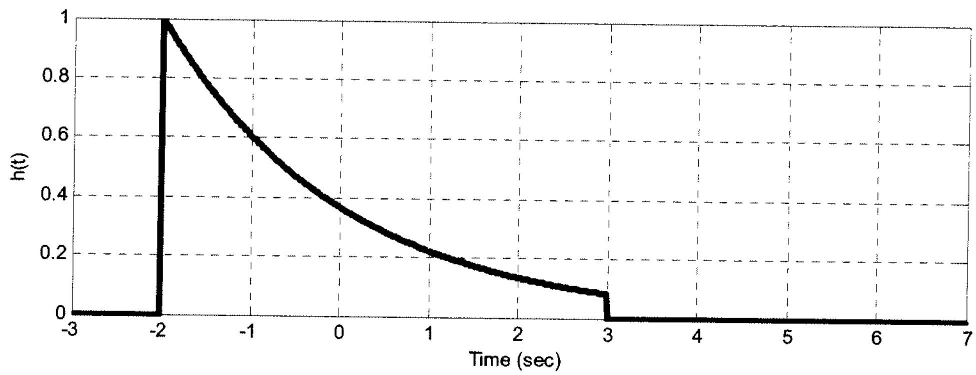
3. Graphical Convolution (30 points)

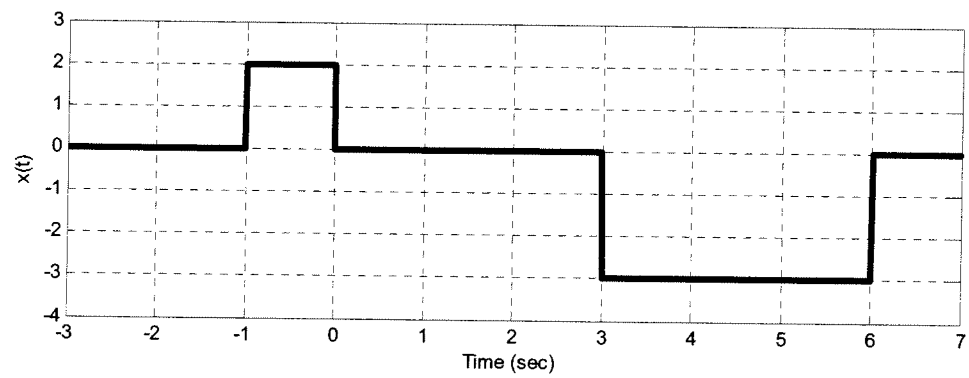
Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = e^{-0.5(t+2)}[u(t+2) - u(t-3)]$$

The input to the system is given by

$$x(t) = 2[u(t+1) - u(t)] - 3[u(t-3) - u(t-6)]$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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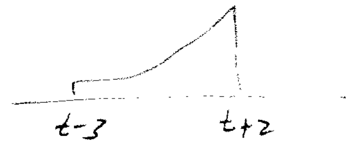
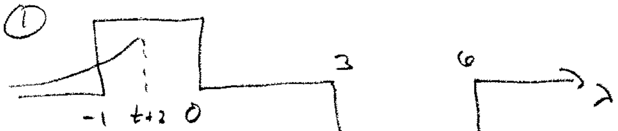
$h(-2) = h(t-\lambda) \quad -2 = t-\lambda \quad \lambda = t+2$

$h(3) = h(t-\lambda) \quad 3 = t-\lambda \quad \lambda = t-3$

$h(t-\lambda) = e^{-0.5(t-\lambda+2)}$

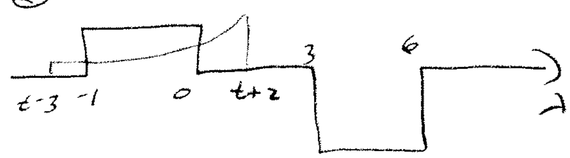


①



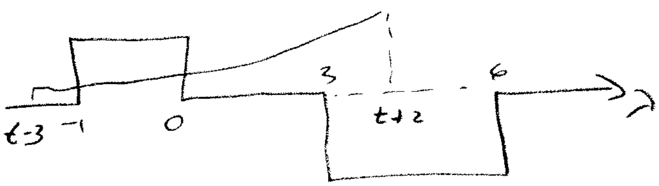
$-3 \le t \le -2 \quad y(t) = \int_{-1}^{t+2} e^{-0.5(t-\lambda+2)}(2)\,d\lambda$

②



$-2 \le t \le 1 \quad y(t) = \int_{-1}^{0} e^{-0.5(t-\lambda+2)}(2)\,d\lambda$

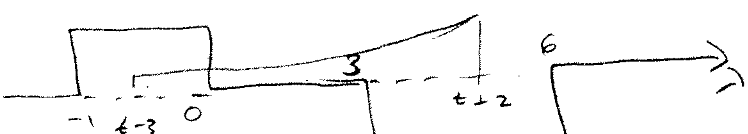
③



$1 \le t \le 2$

$y(t) = \int_{-1}^{0} e^{-0.5(t-\lambda+2)}(2)\,d\lambda + \int_{3}^{t+2} e^{-0.5(t-\lambda+2)}(-3)\,d\lambda$

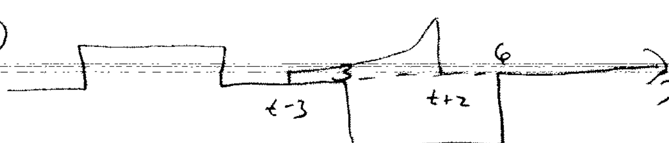
④



$2 \le t \le 3$

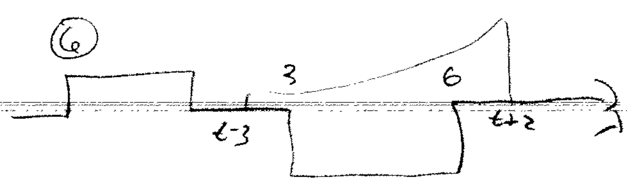
$y(t) = \int_{t-3}^{0} e^{-0.5(t-\lambda+2)}(2)\,d\lambda + \int_{3}^{t+2} e^{-0.5(t-\lambda+2)}(-3)\,d\lambda$

⑤



$3 \le t \le 4 \quad y(t) = \int_{3}^{t+2} e^{-0.5(t-\lambda+2)}(-3)\,d\lambda$

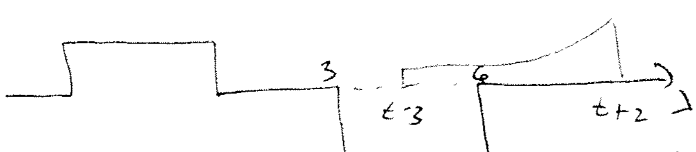
⑥



$4 \le t \le 6$

$y(t) = \int_{3}^{6} e^{-0.5(t-\lambda+2)}(-3)\,d\lambda$

⑦



$6 \le t \le 9 \quad y(t) = \int_{t-3}^{6} e^{-0.5(t-\lambda+2)}(-3)\,d\lambda$

$y(t) = 0 \quad t \le -3$

$y(t) = 0 \quad t \ge 9$

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4. Impulse Response (30 points)
For each of the following systems, determine the impulse response $h(t)$ between the input $x(t)$ and output $y(t)$. *Be sure to include any necessary unit step functions. For full credit, simplify your answers as much as possible.*

a) $y(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)} x(\lambda - 2) d\lambda + e^{-t} x(t)$

$h(t) = \int_{-\infty}^{t-2} e^{-(t-\lambda)} \delta(\lambda-2) d\lambda + e^{-t}\delta(t)$

$-\infty$ 2 $t+2$ $t+2 > 2$ $t > 4$

$\boxed{h(t) = e^{-(t-2)} u(t-4) + \delta(t)}$

b) $2\dot{y}(t) + y(t) = x(t-1)$

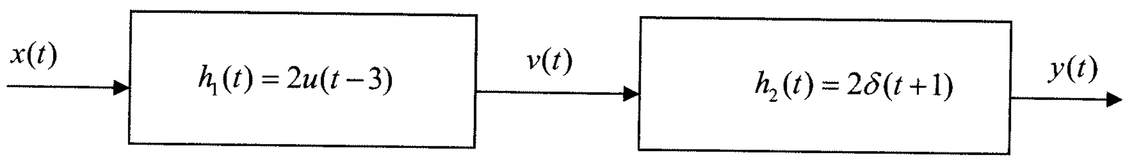
$\dot{h} + \frac{1}{2} h(t) = \frac{1}{2}\delta(t-1)$

$\frac{d}{dt}(h e^{t/2}) = \frac{1}{2} e^{t/2} \delta(t-1) = \frac{1}{2} e^{1/2} \delta(t-1)$

$h(t) e^{t/2} = \int_{-\infty}^{t} \frac{1}{2} e^{\frac{1}{2}} \delta(\lambda-1) d\lambda = \frac{1}{2} e^{\frac{1}{2}} u(t-1)$

$\boxed{h(t) = \frac{1}{2} e^{-t/2} e^{1/2} u(t-1)}$

c) Determine the impulse response for the following system



$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda) h_2(\lambda) d\lambda = \int_{-\infty}^{\infty} 2u(t-\lambda-3)\, 2\delta(\lambda+1) d\lambda$

$= 4u(t-\lambda-3)\Big|_{\lambda=-1} = \boxed{4u(t-2) = h(t)}$

d) If the response of a system to a step of amplitude A is given by

$$s(t) = A[1 + e^{-t/\tau}]u(t)$$

determine the **unit** impulse response of the system. (*Do not just guess the answer, you will probably be wrong, and besides, you need to show your work!*)

For a unit step $A = 1$, to get unit impulse take derivative

$h(t) = \frac{d}{dt}\left\{[1 + e^{-t/\tau}]u(t)\right\} = -\frac{1}{\tau} e^{-t/\tau} u(t) + [1 + e^{-t/\tau}]\delta(t)$

$\boxed{h(t) = -\frac{1}{\tau} e^{-t/\tau} u(t) + 2\delta(t)}$

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5. System Properties (25 points)

a) Fill in the following table with a Y (Yes) or N (No). Only your responses in the table will be graded, not any work. Assume $x(t)$ is the system input and $y(t)$ is the system output. Also assume we are looking at all times (positive and negative times).

System	Linear ?	Time-Invariant?	Memoryless?	Causal?
$\dot{y}(t)+y(t)=e^{(t+1)}x(t)$	Y	N	N	Y
$y(t)=x\left(-\frac{t}{2}\right)$	Y	N	N	N
$y(t)=x(t-1)-1$	N	Y	N	Y
$y(t)=x^2(t)$	N	Y	Y	Y

b) For the system described below, use a formal technique such as we used in class (and on the homework) to determine if the system is time invatriant. *You will be graded more on your method of arriving at an answer than the answer itself!*

$$y(t)=\int_{-\infty}^{t} e^{-(t+\lambda)}x(\lambda)d\lambda$$

$$z_1 = H\{x(t-t_0)\} = \int_{-\infty}^{t} e^{-(t+\lambda)} x(\lambda-t_0)\,d\lambda$$

$$z_2 = H\{x(t)\}\Big|_{t=t-t_0} = \int_{-\infty}^{t-t_0} e^{-(t-t_0+\lambda)} x(\lambda)\,d\lambda$$

in z_1, let $\sigma = \lambda - t_0$ $d\sigma = d\lambda$, $\sigma + t_0 = \lambda$

$$z_1 = \int_{-\infty}^{t-t_0} e^{(t+t_0+\sigma)} x(\sigma)\,d\sigma \neq z_2$$ not TI