ECE 300
Signals and Systems
Homework 2

Due Date: Tuesday September 16, 2008 *at the beginning of class*

Reading: Roberts pages 29-58 and your course notes.

Problems

1) Sketch the following functions:

a) $x(t) = 0.5\text{rect}\left(\frac{t-2}{4}\right) + 0.5\text{rect}\left(\frac{t-3}{5}\right)$ b) $x(t) = \text{rect}(t) + \Lambda(t)$

c) $x(t) = u(t+1) - 2u(t-1) + 3u(t-4) - 2u(t-5)$

d) $x(t) = \text{rect}\left(\frac{t-2}{4}\right) - \Lambda\left(\frac{t-2}{2}\right)$

2) Assume $x(t) = \text{rect}\left(\frac{t+1}{3}\right) + \text{rect}(\text{t})$ and sketch the following:

a) $x_1(t) = x(2t)$ b) $x_2(t) = x(\frac{t}{2})$

c) $x_3(t) = x(1-t)$ d) $x_4(t) = x(1+2t)$

3) Simplify the following as much as possible, giving numerical answers where possible. Use unit step functions as necessary to simplify your answers.

a) $\int_{-\infty}^{\infty} e^{-t}u(t-5)dt$ b) $\int_{-\infty}^{\infty} t^2\left[u(t-6) - u(t-5)\right]dt$

c) $\int_{-\infty}^{\infty} t^2\delta(t-2)dt$ d) $\int_{5}^{\infty} t^2\delta(t-2)dt$

e) $\int_{-\infty}^{\infty} \delta(t-3)\delta(t-4)dt$ f) $\int_{-\infty}^{\infty} u(t-3)\delta(t-4)dt$

g) $\int_{-\infty}^{t} e^{-(t-\lambda-1)}\delta(\lambda-2)\text{d}\lambda$ h) $\int_{-\infty}^{t} e^{-2(t-\lambda)}\delta(\lambda+1)d\lambda$

i) $\int_{-\infty}^{t-1} e^{-3(t-\lambda)}\delta(\lambda-1)d\lambda$ j) $\int_{-t}^{\infty} e^{-(t-\lambda)}\delta(\lambda+2)d\lambda$

4) For each of the following signals, determine if the signal is periodic and, if so, the fundamental period.

a) $x(t) = \sin(2t) + \cos(3t + 30^o)$ b) $x(t) = \cos(2t) + \cos(\pi t)$

c) $x(t) = e^{-t}\cos(t)$ d) $x(t) = 2e^{j2t} + 3e^{j(3t+2)}$

e) $x(t) = e^{j\pi t} - e^{-j(t+3)}$ f) $x(t) = \sin(2t) + e^{j(0.5t+1)}$

5) Use Euler's identity in the form $\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ and $\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$ to prove the following identities:

a) $2\sin(\theta)\cos(\theta) = \sin(2\theta)$ b) $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

c) $\sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$ d) $\frac{d}{d\theta}\cos(\theta) = -\sin(\theta)$ e) $\frac{d}{d\theta}\sin(\theta) = \cos(\theta)$

Matlab Problems

6) Using Matlab, plot each signal from Problem 4 for three *fundamental periods* if the signal is periodic, or three times the longest period in the signal if the signal is not periodic. Be sure there are at least 50 samples per period for each waveform and your graphs are neatly labeled. **Notes:** (1) Matlab works in radians, so all angles must be converted to radians, (2) use **exp** in Matlab to get an exponential, (3) **j** is Matlab's way of indicating the square root of -1, and if you want $x(t) = e^{j2t}$ you should type something like ***x = exp(j*2*t)***, and (4) if the waveform is complex, plot the real and imaginary parts separately. The Matlab commands **real** and **imag** are very useful for this. Turn in your plots.

7) Save the files **unit_step.m, unit_rect.m,** and **unit_triangle.m** from the course website to the directory in which you will be using MATLAB. This directory is called the "working directory" in Matlab. If you do this correctly, you can use theses functions just as you would any other built-in matlab function. To use these supplied Matlab functions to generate the function

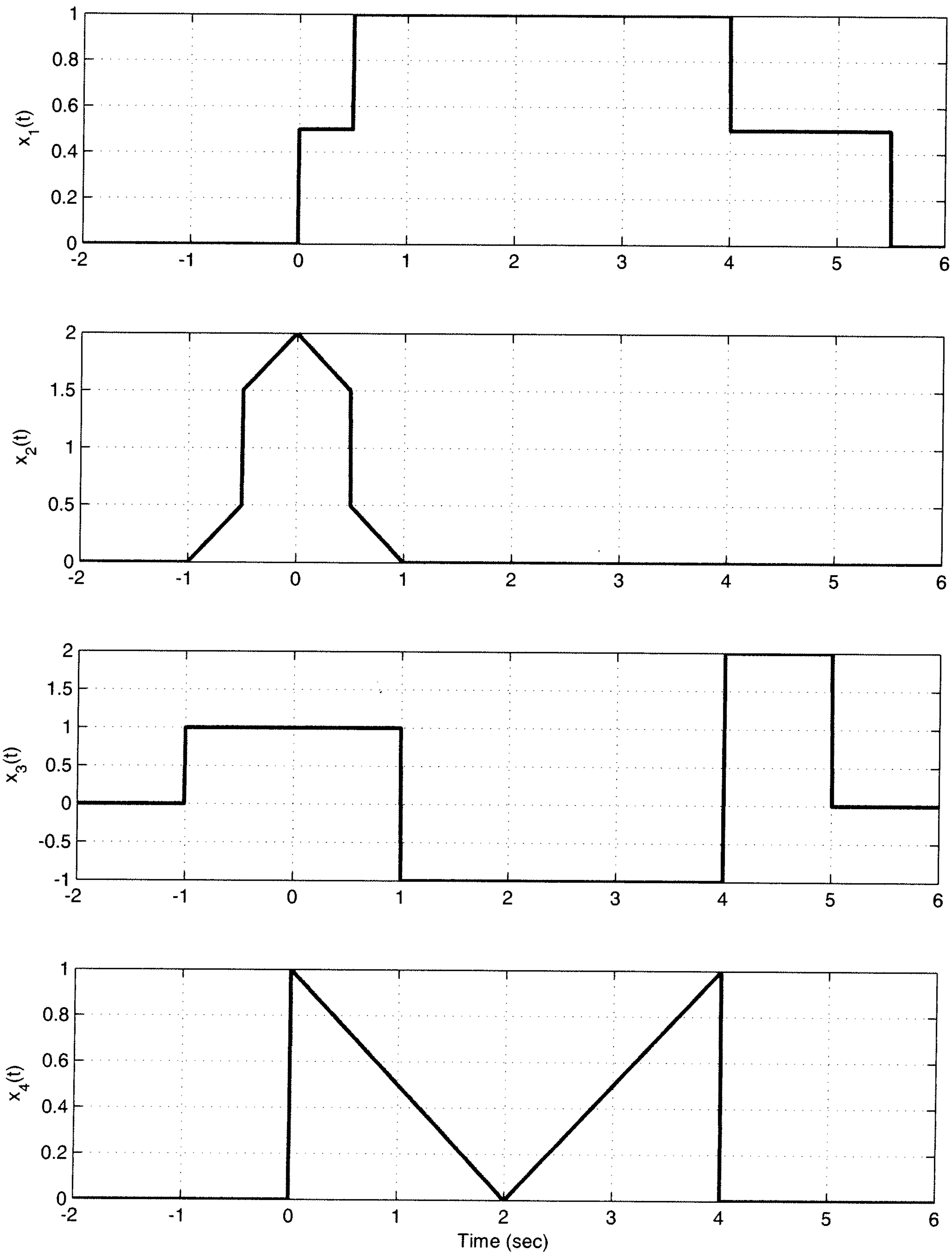
$$x(t) = 3u(t-2) + 4\text{rect}\left(\frac{t-4}{5}\right) - 3\Lambda\left(\frac{t+1}{4}\right)$$

from t = -10 to 10, you might type the following in Matlab

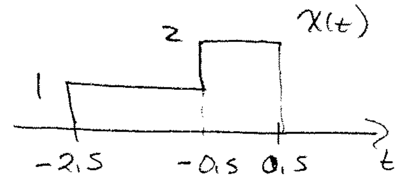
```
t = linspace(-10,10,1000);
x = 3*unit_step(t-2)+4*unit_rect((t-4),5)-3*unit_triangle((t+1),4);
```

Use these functions to plot the functions from problem 1. Plot all of the functions from t = -2 to 6 on one page using the **subplot** command.

#1 + #7



#2 $x(t) = \underbrace{rect\left(\frac{t+1}{3}\right)}_{\substack{\text{width}=3 \\ \text{centered at } -1}} + \underbrace{rect(t)}_{\substack{\text{width}=1 \\ \text{centered at } 0}}$



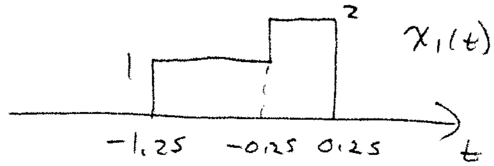
the endpoints are at $t=-2.5$ and $t=0.5$

(a) $x_1(t) = x(2t)$

$2t = 0.5 \quad t = 0.25 \quad x_1(0.25) = x(0.5)$

$2t = -2.5 \quad t = -1.25 \quad x_1(-1.25) = x(-2.5)$

$2t = -0.5 \quad t = -0.25 \quad x(-0.25) = x(-0.5)$



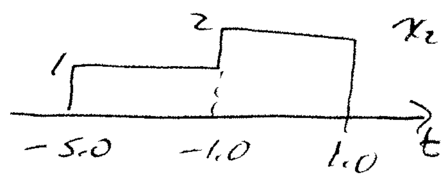
$x_1(t)$ compressed

(b) $x_2(t) = x\left(\frac{t}{2}\right)$

$\frac{t}{2} = 0.5 \quad t = 1.0 \quad x_2(1.0) = x(0.5)$

$\frac{t}{2} = -2.5 \quad t = -5.0 \quad x_2(-5.0) = x(-2.5)$

$\frac{t}{2} = -0.5 \quad t = -1 \quad x_2(-1.0) = x(-0.5)$



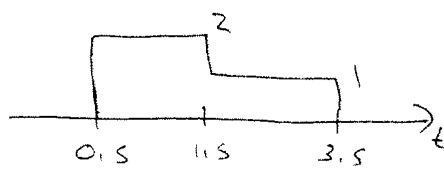
$x_2(t)$ expanded

(c) $x_3(t) = x(1-t)$

$1-t = 0.5 \quad t = 0.5 \quad x_3(0.5) = x(0.5)$

$1-t = -2.5 \quad t = 3.5 \quad x_3(3.5) = x(-2.5)$

$1-t = -0.5 \quad t = 1.5 \quad x_3(1.5) = x(-0.5)$



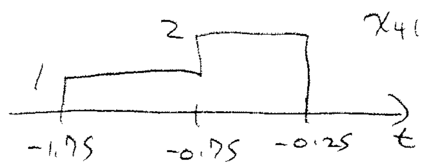
$x_3(t)$

(d) $x_4(t) = x(1+2t)$

$1+2t = 0.5 \quad t = -0.25 \quad x_4(-0.25) = x(0.5)$

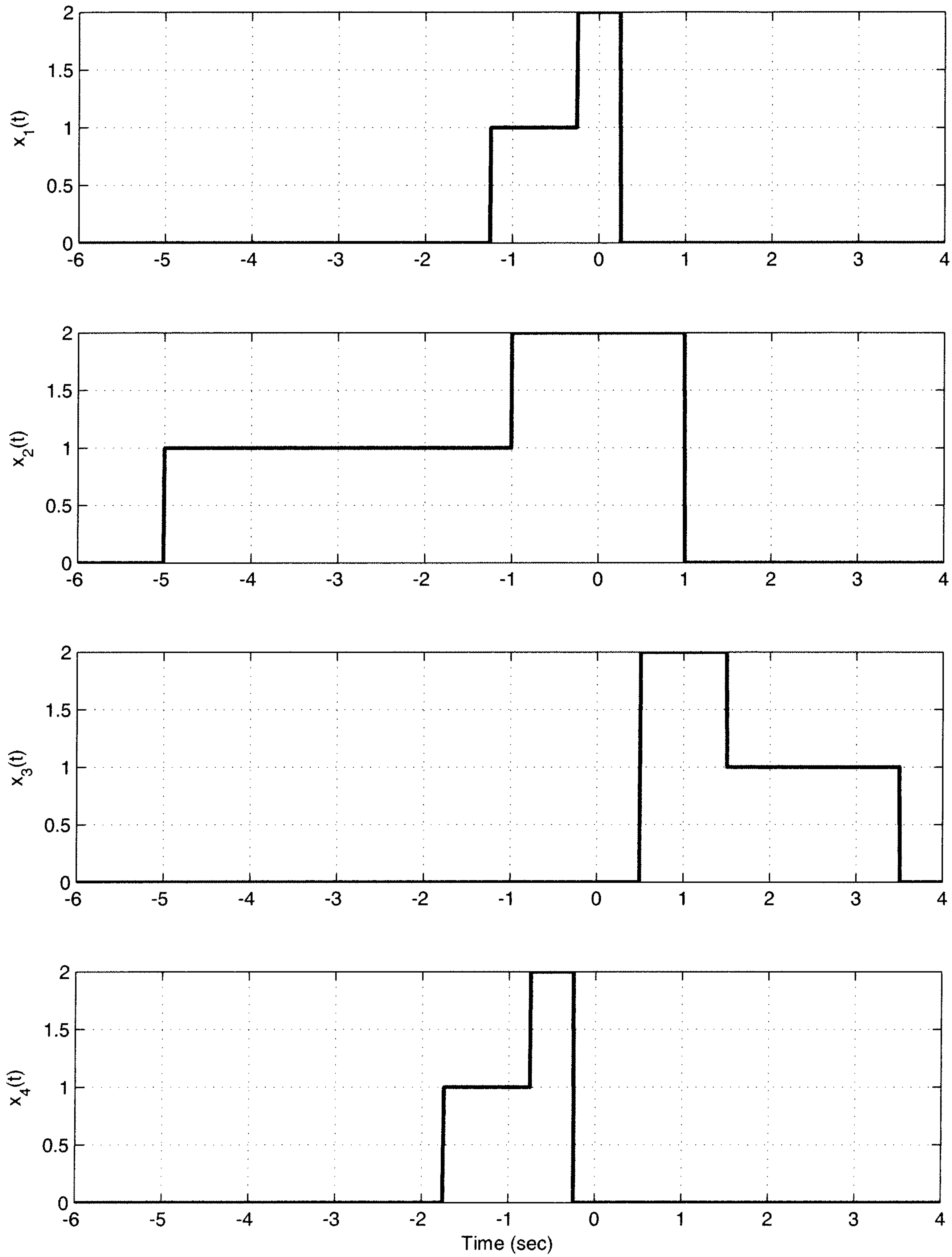
$1+2t = -2.5 \quad t = -1.75 \quad x_4(-1.75) = x(-2.5)$

$1+2t = -0.5 \quad t = -0.75 \quad x_4(-0.75) = x(-0.5)$



$x_4(t)$

#2



#3

(a) $\int_{-\infty}^{\infty} e^{-t} u(t-5)\,dt = \int_5^{\infty} e^{-t}\,dt = -e^{-t}\Big|_5^{\infty} = \boxed{e^{-5} = 0.00674}$

(b) $\int_{-\infty}^{\infty} t^2 [u(t-6) - u(t-5)]\,dt = -\int_{-\infty}^{\infty} t^2 [u(t-5) - u(t-6)]\,dt$

$= -\int_5^6 t^2\,dt = -\frac{t^3}{3}\Big|_5^6 = -\left[\frac{6^3 - 5^3}{3}\right] = \boxed{-30.33}$

(c) $\int_{-\infty}^{\infty} t^2 \delta(t-2) = \boxed{4}$

(d) $\int_{5}^{\infty} t^2 \delta(t-2) = \boxed{0}$

(e) $\int_{-\infty}^{\infty} \delta(t-3)\,\delta(t-4)\,dt = \boxed{0}$

(f) $\int_{-\infty}^{\infty} u(t-3)\,\delta(t-4)\,dt = u(4-3) = u(1) = \boxed{1}$

(g) $\int_{-\infty}^{t} e^{-(t-\lambda-1)} \delta(\lambda-2)\,d\lambda = \boxed{e^{-(t-3)} u(t-2)}$

(h) $\int_{-\infty}^{t} e^{-2(t-\lambda)} \delta(\lambda+1)\,d\lambda = \boxed{e^{-2(t+1)} u(t+1)}$

(i) $\int_{-\infty}^{t-1} e^{-3(t-\lambda)} \delta(\lambda-1)\,d\lambda = \boxed{e^{-3(t-1)} u(t-2)}$

(j) $\int_{-t}^{\infty} e^{-(t-\lambda)} \delta(\lambda+2)\,d\lambda = \boxed{e^{-(t+2)} u(t-2)}$

need $-t < -2$ or $t > 2$

(#4)

(a) $x(t) = \sin(2t) + \cos(3t+30°)$

$x(t+T_0) = \sin(2t+2T_0) + \cos(3t+3T_0+30°)$

$= x(t)$ if $2T_0 = q(2\pi)$ $3T_0 = r(2\pi)$ q, r integers

$T_0 = q\pi = r\frac{2}{3}\pi$ $q = 2$ $r = 3$ works

$\boxed{\text{periodic, } T_0 = 2\pi}$

(b) $x(t) = \cos(2t) + \cos(\pi t)$

$x(t+T_0) = \cos(2t+2T_0) + \cos(\pi t + \pi T_0)$

$= x(t)$ if $2T_0 = q(2\pi)$ $\pi T_0 = r(2\pi)$ q, r integers

$T_0 = q\pi = 2r$ no q, r integers will solve

$\boxed{\text{not periodic}}$

(c) $x(t) = e^{-t}\cos(t)$

$x(t+T_0) = e^{-t}e^{-T_0}\cos(t+T_0)$

$= x(t)$ if $e^{-T_0} = 1$ $T_0 = (2\pi)q$ q integer

The only solution is $T_0 = 0$ $\boxed{\text{not periodic}}$

(d) $x(t) = 2e^{j2t} + 3e^{j(3t+2)}$

$x(t+T_0) = 2e^{j(2t+2T_0)} + 3e^{j(3t+3T_0+2)}$

$= 2e^{j2t}e^{j2T_0} + 3e^{j(3t+2)}e^{j3T_0}$

$= x(t)$ if $2T_0 = q(2\pi)$ $3T_0 = r(2\pi)$ q, r integers

$T_0 = q\pi = \frac{2}{3}r\pi$ $q = 2$ $r = 3$ works

$\boxed{\text{periodic } T_0 = 2\pi}$

#4 (continued)

(e) $x(t) = e^{j\pi t} - e^{-j(t+3)}$

$x(t+T_0) = e^{j\pi(t+T_0)} - e^{-j(t+T_0+3)}$

$= e^{j\pi t} e^{j\pi T_0} - e^{-j(t+3)} e^{-jT_0}$

need $\pi T_0 = q(2\pi)$ $\quad T_0 = r(2\pi)$ $\quad$ no integers q, r

so ~~not~~ periodic

(f) $x(t) = \sin(2t) + e^{j(\frac{t}{2}+1)}$

$x(t+T_0) = \sin(2t+2T_0) + e^{j(\frac{t}{2}+\frac{T_0}{2}+1)}$

$= \sin(2t+2T_0) + e^{j(\frac{t}{2}+1)} e^{j\frac{T_0}{2}}$

need $2T_0 = q(2\pi)$ $\quad \frac{T_0}{2} = r(2\pi)$

$T_0 = q\pi$ $\quad T_0 = r4\pi$ $\quad q=4 \;\; r=1$ works

$\boxed{\text{periodic with period } 4\pi = T_0}$

#5 $\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ $\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

(a) $2\cos(\theta)\sin(\theta) = 2\left(\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right)\left(\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right) = 2\left(\dfrac{e^{j2\theta} - e^{-j2\theta}}{4j}\right)$

$= \dfrac{e^{j2\theta} - e^{-j2\theta}}{2j} = \boxed{\sin(2\theta)}$

(b) $\cos^2(\theta) = \left(\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right)\left(\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right) = \dfrac{e^{j2\theta} + e^{-j2\theta} + 2}{4}$

$= \left(\dfrac{e^{j2\theta} + e^{-j2\theta}}{2}\right)\dfrac{1}{2} + \dfrac{1}{2} = \boxed{\dfrac{1}{2} + \dfrac{1}{2}\cos(2\theta)}$

(c) $\sin^2(\theta) = \left(\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right)\left(\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right) = \dfrac{e^{j2\theta} + e^{-j2\theta} - 2}{-4}$

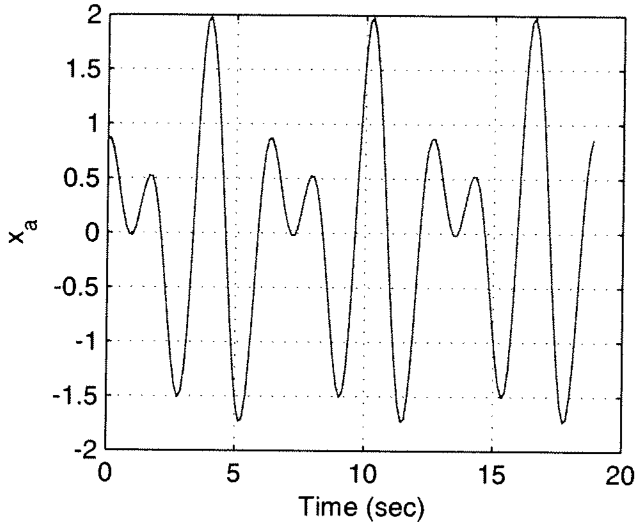
$= \dfrac{1}{2} - \dfrac{1}{2}\left(\dfrac{e^{j2\theta} + e^{-j2\theta}}{2}\right) = \boxed{\dfrac{1}{2} - \dfrac{1}{2}\cos(2\theta)}$

(d) $\dfrac{d}{d\theta}\left[\dfrac{e^{j\theta} + e^{-j\theta}}{2}\right] = \dfrac{je^{j\theta} - je^{-j\theta}}{2} = -\dfrac{e^{j\theta} + e^{-j\theta}}{2j} = -\left[\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right]$

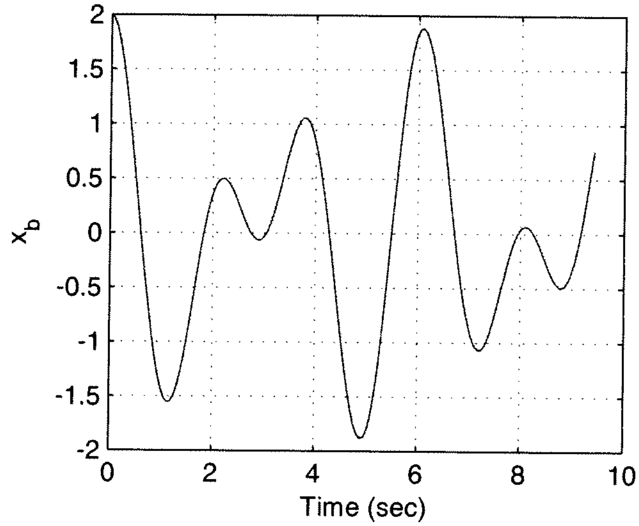
$= \boxed{-\sin(\theta)}$

(e) $\dfrac{d}{d\theta}\left[\dfrac{e^{j\theta} - e^{-j\theta}}{2j}\right] = \dfrac{je^{j\theta} + je^{-j\theta}}{2j} = \dfrac{e^{j\theta} + e^{-j\theta}}{2} = \boxed{\cos(\theta)}$

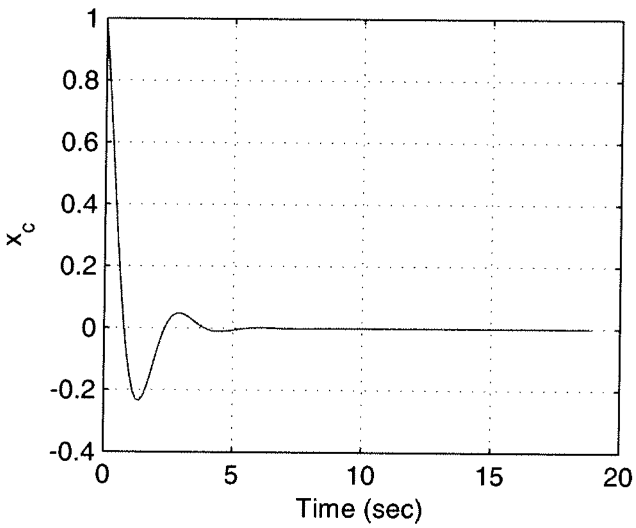
#6



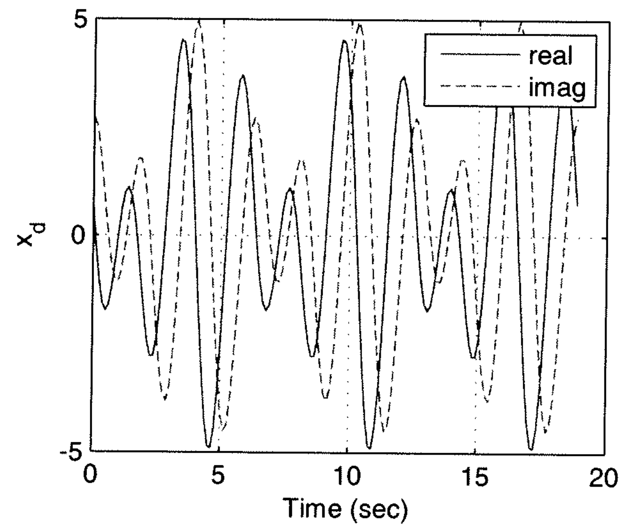
x_a
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
Time (sec)



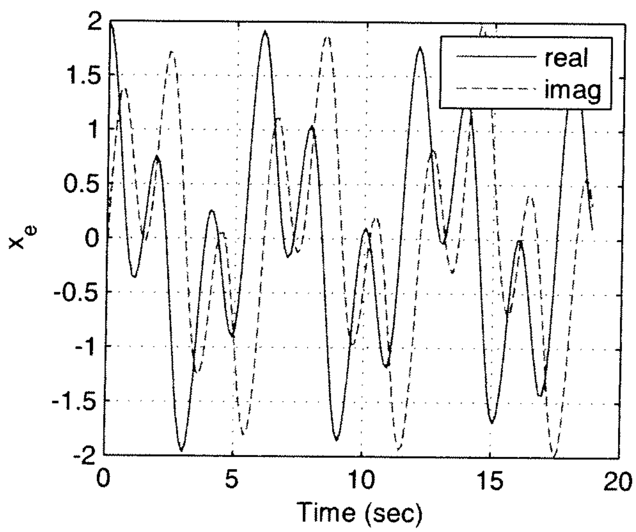
x_b
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10
Time (sec)



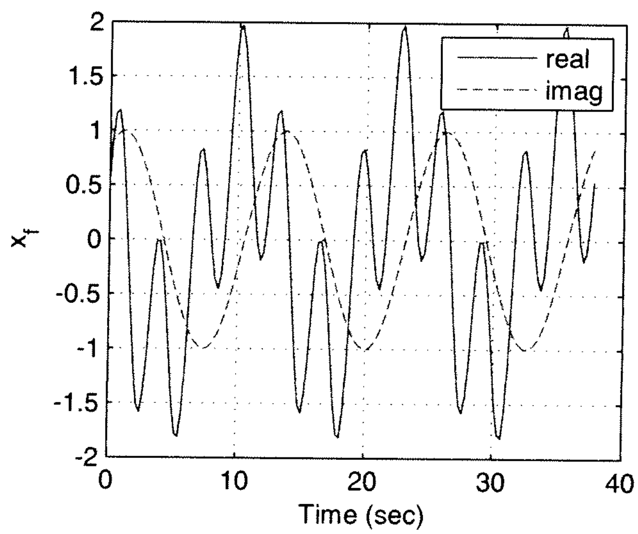
x_c
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
20
Time (sec)



x_d
real
imag
5
0
-5
0
5
10
15
20
Time (sec)



x_e
real
imag
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
Time (sec)



x_f
real
imag
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
Time (sec)

```
%
%  problem 1 (or 7)
%
t = linspace(-2,6,1000);
x1 = 0.5*unit_rect((t-2),4) + 0.5*unit_rect((t-3),5);
x2 = unit_rect(t,1)+ unit_triangle(t,1);
x3 = unit_step(t+1)-2*unit_step(t-1)+3*unit_step(t-4)-2*unit_step(t-5);
x4 = unit_rect(t-2,4)-unit_triangle(t-2,2);
figure;
orient tall
subplot(4,1,1); plot(t,x1,'LineWidth',2); grid; ylabel('x_1(t)');
subplot(4,1,2); plot(t,x2,'LineWidth',2); grid; ylabel('x_2(t)');
subplot(4,1,3); plot(t,x3,'LineWidth',2); grid; ylabel('x_3(t)');
subplot(4,1,4); plot(t,x4,'LineWidth',2); grid; ylabel('x_4(t)'); xlabel('Time (sec)');

%
%  problem 2
%
g = @(t) unit_rect((t+1),3)+unit_rect(t,1);

t = linspace(-6,4,1000);
g1 = g(2*t);
g2 = g(t/2);
g3 = g(1-t);
g4 = g(1+2*t);
figure;
orient tall
subplot(4,1,1); plot(t,g1,'LineWidth',2); grid; ylabel('x_1(t)');
subplot(4,1,2); plot(t,g2,'LineWidth',2); grid; ylabel('x_2(t)');
subplot(4,1,3); plot(t,g3,'LineWidth',2); grid; ylabel('x_3(t)');
subplot(4,1,4); plot(t,g4,'LineWidth',2); grid; ylabel('x_4(t)'); xlabel('Time (sec)');
%
%  problem 4
%
figure;
orient tall
T0 = 2*pi;
t = [0:T0/50:3*T0];
x = sin(2*t)+cos(3*t+30*pi/180);
subplot(3,2,1); plot(t,x); grid; xlabel('Time (sec)'); ylabel('x_a');
%
T0 = pi;
t = [0:T0/50:3*T0];
x = cos(2*t)+cos(pi*t);
subplot(3,2,2); plot(t,x); grid; xlabel('Time (sec)'); ylabel('x_b');
%
T0 = 2*pi;
t = [0:T0/50:3*T0];
x = exp(-t).*cos(2*t);
subplot(3,2,3); plot(t,x); grid; xlabel('Time (sec)'); ylabel('x_c');
%
T0 = 2*pi;
```

```
t = [0:T0/50:3*T0];
x = 2*exp(j*2*t)+3*exp(j*(3*t+2));
subplot(3,2,4); plot(t,real(x),'-',t,imag(x),'--'); legend('real','imag');
grid; xlabel('Time (sec)'); ylabel('x_d');

T0 = 2*pi;
t = [0:T0/50:3*T0];
x = exp(j*pi*t)-exp(j*(t+3));
subplot(3,2,5); plot(t,real(x),'-',t,imag(x),'--'); legend('real','imag');
grid; xlabel('Time (sec)'); ylabel('x_e');

T0 = 4*pi;
t = [0:T0/50:3*T0];
x = sin(2*t)+exp(j*(0.5*t+1));
subplot(3,2,6); plot(t,real(x),'-',t,imag(x),'--'); legend('real','imag');
grid; xlabel('Time (sec)'); ylabel('x_f');
```