**ECE 300**
**Signals and Systems**
Homework 4

**Due Date:** Tuesday October 2. 2007 *at the beginning of class*

**Problems**

**1.** For the following system models, determine if the model represents a BIBO stable system. If the system is not BIBO stable, give an input $x(t)$ that demonstrates this.

a) $y(t) = \dot{x}(t) + x(t)$  b) $y(t) = \sin\left(\frac{1}{x(t)}\right)$

c) $y(t) = x^2(t)$  d) $y(t) + y(t)x(t) = x(t)$

**2.** ZTF Problem 3-19. For this problem, first expand the first equation out using trigonometric identities. Secondly, expand out the second equation given in the problem using trigonometric identities, and then determine expressions for $A(t)$ and $\theta(t)$ using the two expansions. Plot the resulting waveform assuming $f_0 = 220$ Hz and $\Delta f_0 = 3$ Hz (ignore the parameters given in the book). Plot the waveform from 0 to 300 $T_0$ (i.e., plot over 300 periods) using 10,000 points (this sounds like a job for **linspace**) Finally, listen to the waveform using **soundsc(x,1/dt)**, where dt is the interval between samples (**dt = t(2) – t(1)**). Turn in your code and you plot.

**3.** (**Matlab Problem**) Read the **Appendix**, then

Donwload **Fourier_Sine_Series.m** from the class website.

**a)** If you type (in Matlab's command line) Fourier_Sine_Series(5) you should get a plot like that shown in Figure 1. As you increase the number of terms in the Fourier series, you should get a better match to the function. Run the code for N=100 and turn in your plot.

**b)** Read through the code and answer the following questions:

- What variable in the code represents the Fourier series approximation to the input?
- In extending this program to implement a full trigonometric Fourier series, does the computation for $a_0$ need to be inside a for loop?

**c)** Copy **Fourier_Sine_Series.m** to a file named **Trig_Fourier_Series.m** and implement a full trigonometric Fourier series representation. This means you will have to compute the average value $a_o$ and the $a_k$, and then use these values in the final estimate.

**d)** Using the code you wrote in part **c**, find the trigonometric Fourier series representation for the following functions (defined over a single period)

$$f_1(t) = e^{-t}u(t) \quad 0 \le t < 3$$

$$f_2(t) = \begin{cases} t & 0 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \le t < -1 \\ 1 & -1 \le t < 2 \\ 3 & 2 \le t < 3 \\ 0 & 3 \le t < 4 \end{cases}$$

Use N = 10 and turn in your plots for each of these functions. Also, turn in your Matlab program for one of these. *Notes: (1) the values of **low** and **high** will be different for each of these functions! (2) we are going to use these functions again, so once your code is working it is best to comment out the function description.*

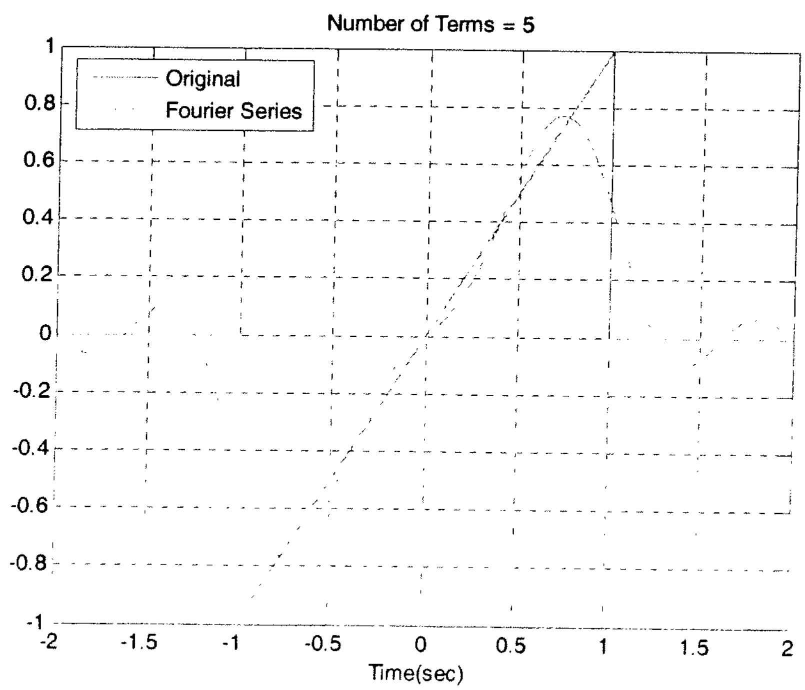



Figure 1. Trigonometric Fourier series for problem 2b.

#1

a) $y(t) = \dot{x}(t) + x(t)$

not BIBO stable for $x(t) = u(t)$

at $t=0$ $y(0) = \dot{u}(0) + u(0) = \delta(0) + u(0) = \infty$

b) $y(t) = \sin\left(\frac{1}{x(t)}\right)$

no matter what $x(t)$ is, even $x(t) = 0$, $-1 \le y(t) \le 1$ BIBO stable

c) $y(t) = x^2(t)$

for $|x(t)| < M$ $|y(t)| \le M^2$ so BIBO stable

d) $\dot{y}(t) + y(t)x(t) = \dot{x}(t)$

$y(t)[1 + x(t)] = x(t)$

$$y(t) = \frac{x(t)}{1 + x(t)}$$

for $x(t) = -1$ $y(t) = \infty$ not BIBO stable

(#2) ZTF 3-19

$x(t) = \cos(\omega_0 t) + \cos([\omega_0 + \Delta\omega] t)$

$\cos(\omega_0 t + \Delta\omega t) = \cos(\omega_0 t)\cos(\Delta\omega t) - \sin(\omega_0 t)\sin(\Delta\omega t)$

so $x(t) = \cos(\omega_0 t) + [\cos(\omega_0 t)\cos(\Delta\omega t) - \sin(\omega_0 t)\sin(\Delta\omega t)]$

$= [1 + \cos(\Delta\omega t)]\cos(\omega_0 t) - [\sin(\Delta\omega t)]\sin(\omega_0 t)$

The assumed form of the solution is

$x(t) = C(t)\cos(\omega_0 t + \theta(t))$

$= C(t)[\cos(\omega_0 t)\cos(\theta(t)) - \sin(\omega_0 t)\sin(\theta(t))]$

equating terms we get

$C(t)\cos(\theta(t)) = 1 + \cos(\Delta\omega t)$

$C(t)\sin(\theta(t)) = \sin(\Delta\omega t)$

Squaring, we get $C(t)^2\cos^2(\theta(t)) + C(t)^2\sin^2(\theta(t)) = [1+\cos(\Delta\omega t)]^2 + [\sin(\Delta\omega t)]^2$

$C(t)^2[\sin^2(\theta(t)) + \cos^2(\theta(t))] = C(\cdot)^2$

so $C(t)^2 = [1 + \cos^2(\Delta\omega t) + 2\cos(\Delta\omega t)] + \sin^2(\Delta\omega t)$

$= 1 + \underbrace{\cos^2(\Delta\omega t) + \sin^2(\Delta\omega t)}_{=1} + 2\cos(\Delta\omega t)$

$= 2 + 2\cos(\Delta\omega t) = 2(1 + \cos(\Delta\omega t))$

$= 2\left[2\cos^2\left(\frac{\Delta\omega t}{2}\right)\right] = 4\cos^2\left(\frac{\Delta\omega t}{2}\right)$

$C(t) = 2\left|\cos\left(\frac{\Delta\omega t}{2}\right)\right|$

$\tan(\theta(t)) = \frac{C(t)\sin(\theta(t))}{C(t)\cos(\theta(t))} = \frac{\sin(\Delta\omega t)}{1 + \cos(\Delta\omega t)}$

$\theta(t) = \tan^{-1}\left(\frac{\sin(\Delta\omega t)}{1 + \cos(\Delta\omega t)}\right)$

(#2) ZTF 3-19 

Alternate form

$$x(t) = \cos(\omega_0 t) + \cos([\omega_0 + \Delta\omega]t)$$

use $\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$x(t) = 2\cos\left(\frac{2\omega_0 t + \Delta\omega t}{2}\right)\cos\left(-\frac{\Delta\omega t}{2}\right)$$

$$= 2\cos\left(\frac{\Delta\omega t}{2}\right)\cos\left(\omega_0 t + \frac{\Delta\omega t}{2}\right)$$

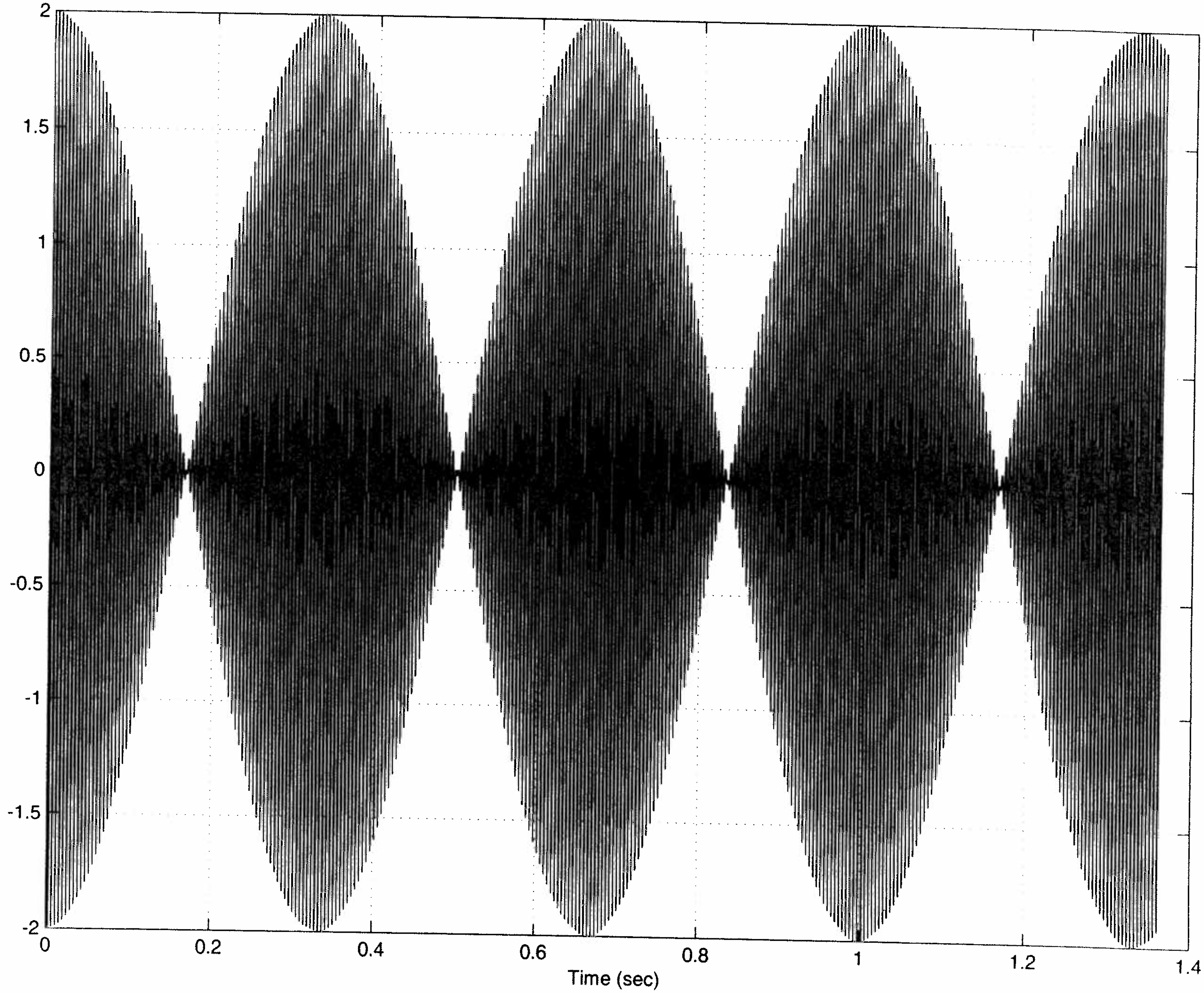
Make this fit

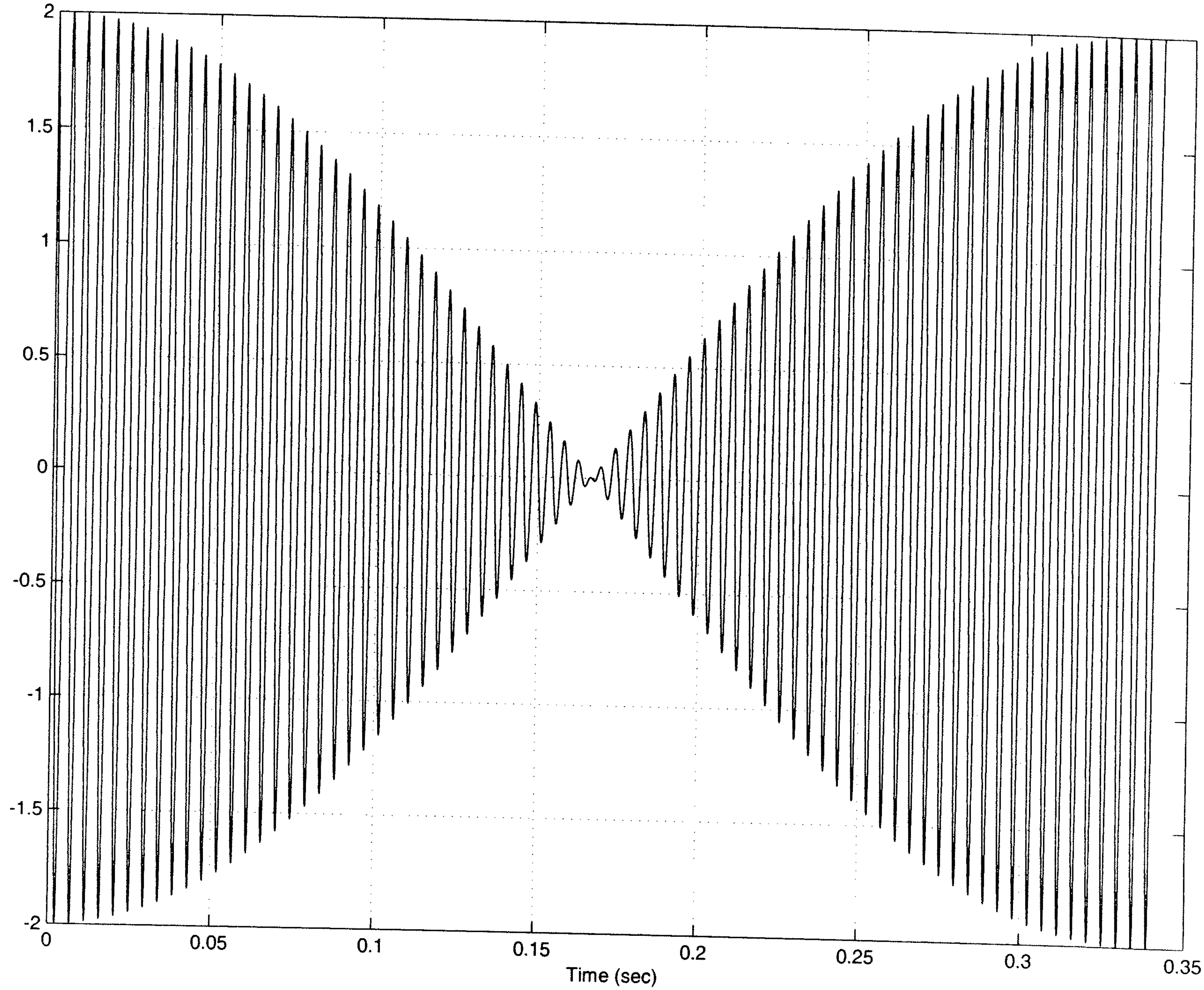
$$x(t) = A(t)\cos(\omega_0 t + \theta(t))$$

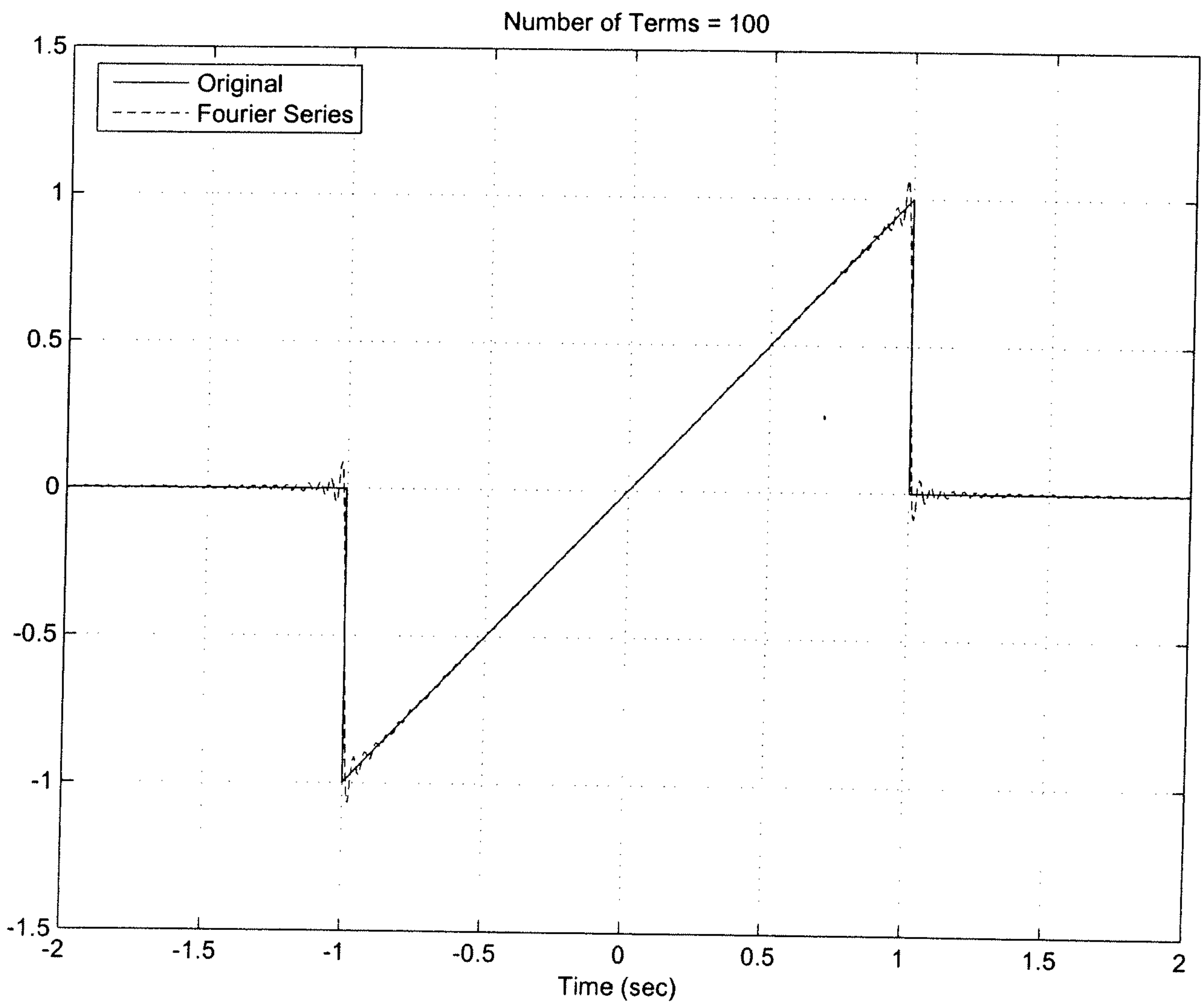
$$A(t) = 2\cos\left(\frac{\Delta\omega t}{2}\right)$$

$$\theta(t) = \frac{\Delta\omega t}{2}$$

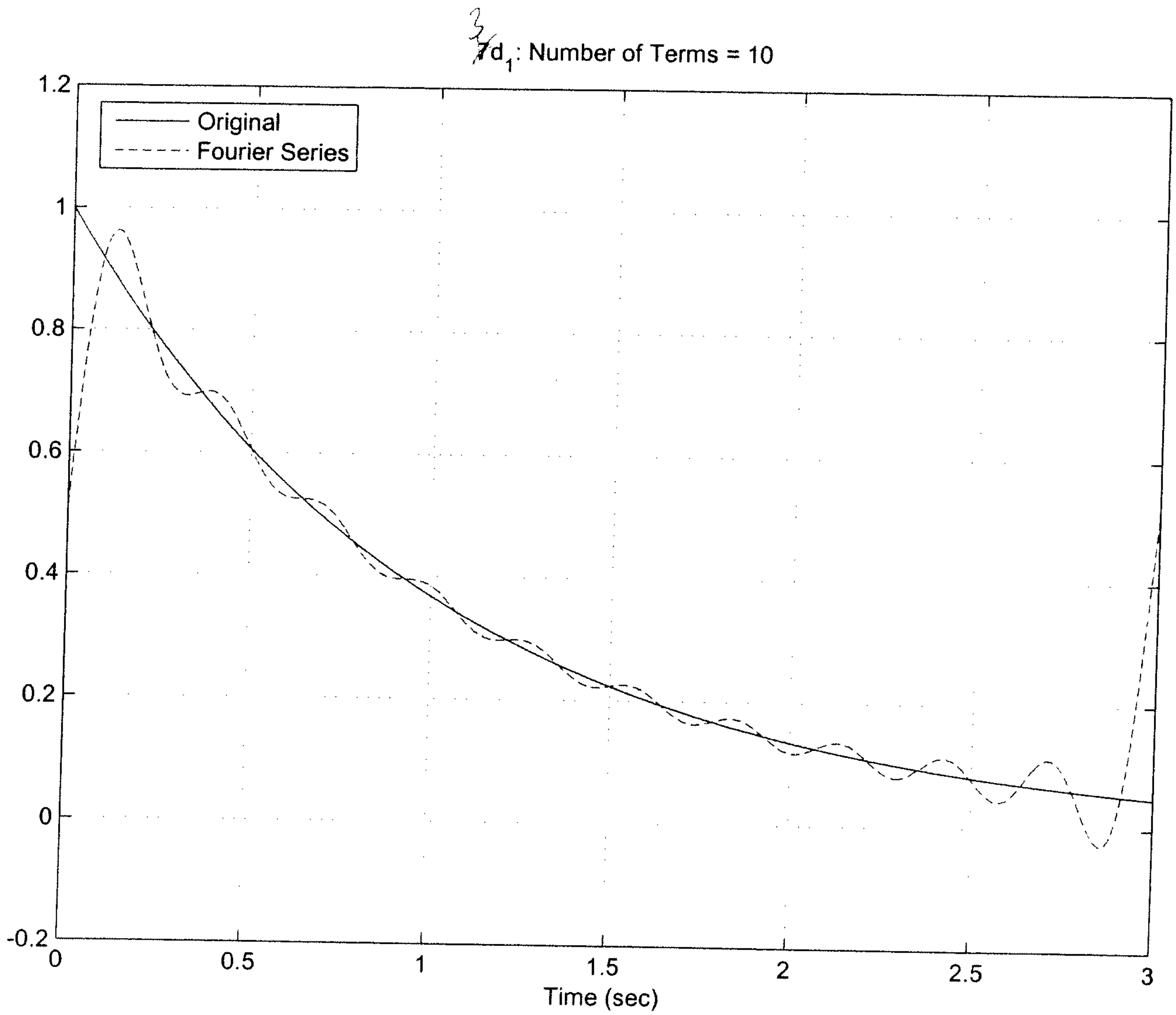
```
%
%  Problem 3-19 from the text
%
 f0 = 220;
 df = 3;
 T0 = 1/f0;
 t = linspace(0,300*T0,10000);
 dt = t(2)-t(1);
 A = 2*abs(cos(df*2*pi*t/2));
 theta = atan2( sin(df*2*pi*t),1+cos(df*2*pi*t));
 x = A.*cos(f0*2*pi*t+theta);
 orient landscape
 plot(t,x); grid; xlabel('Time (sec)');

 pause
 soundsc(x,1/dt);
```


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Time (sec)

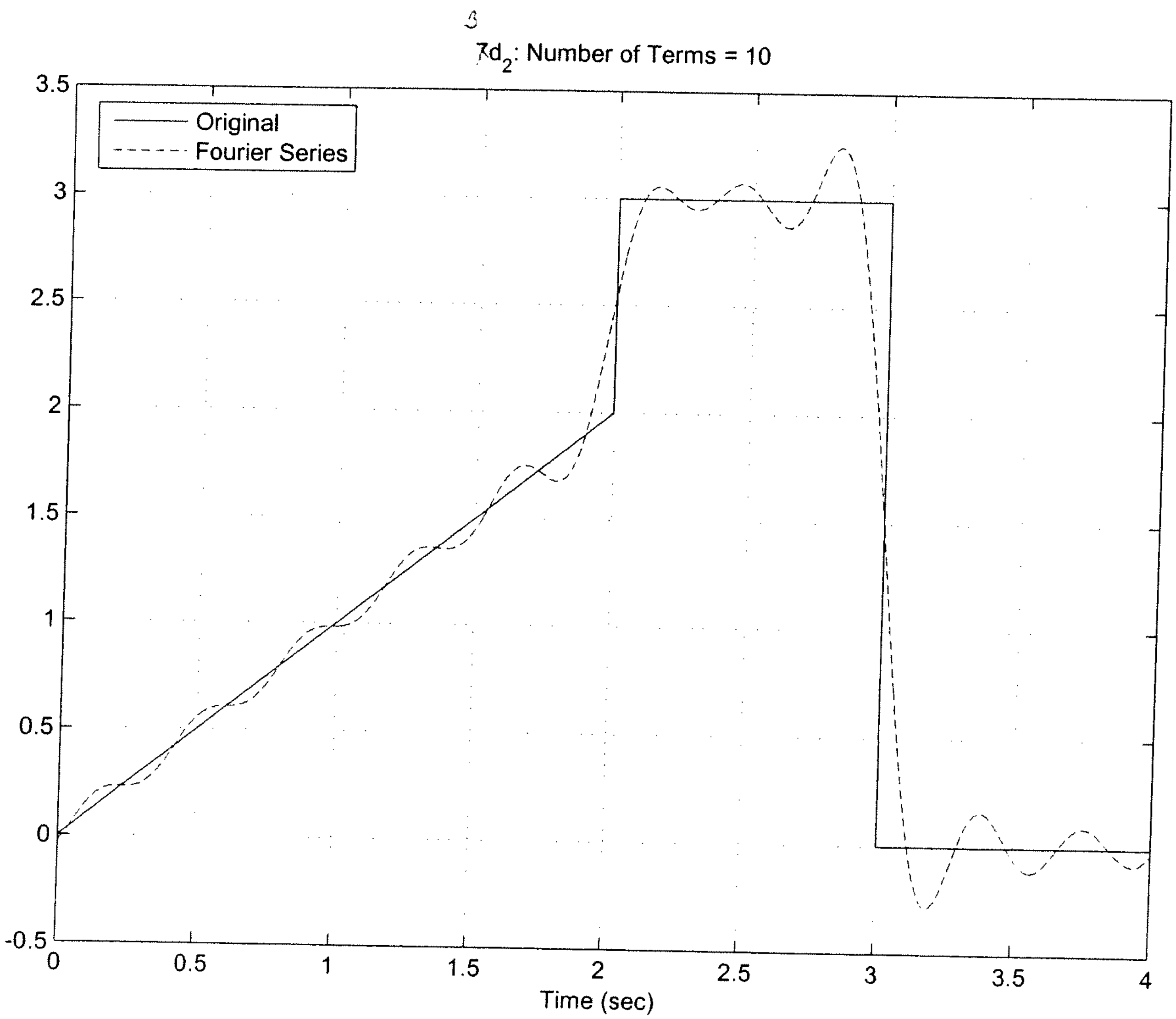

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Time (sec)

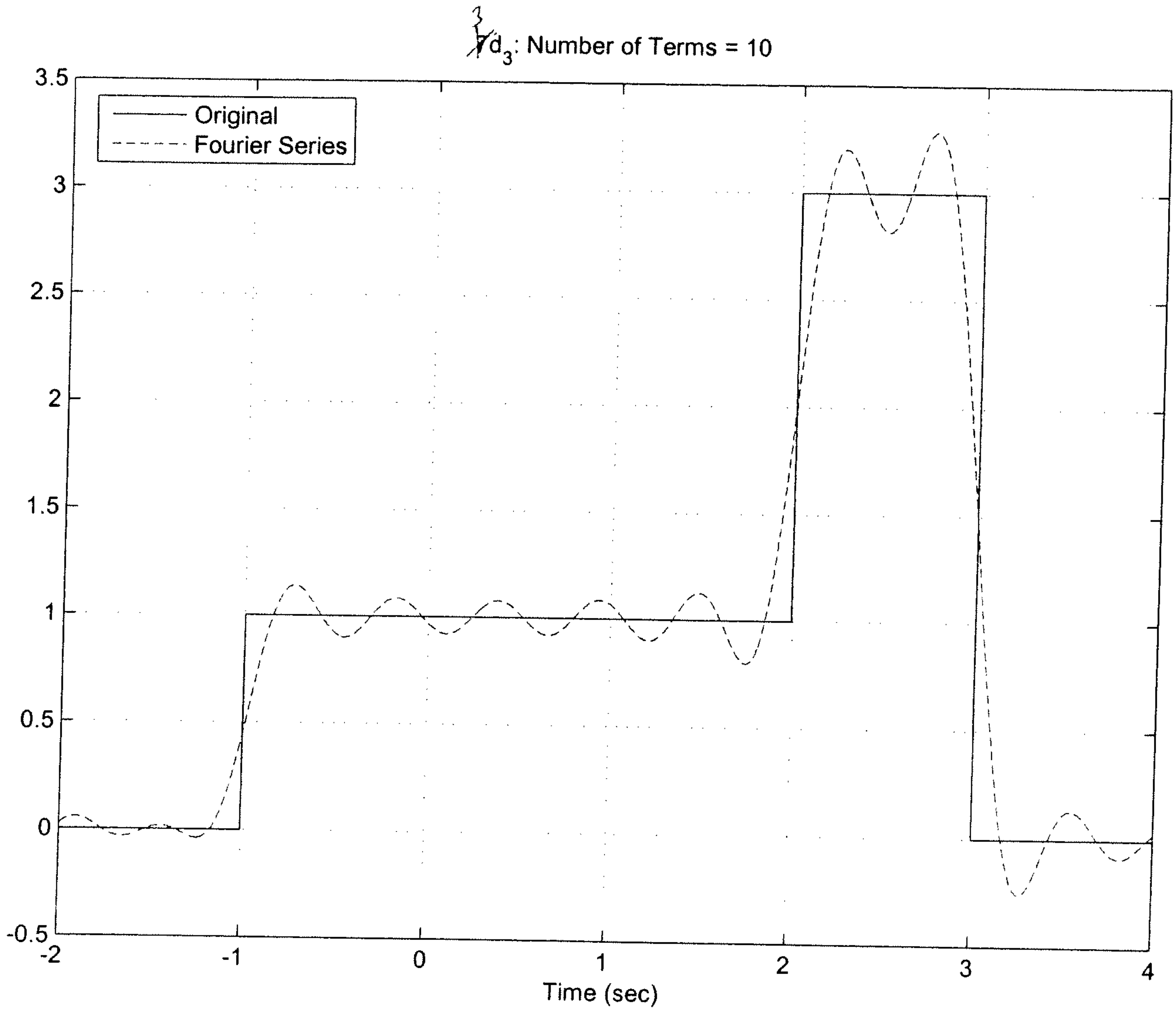


Number of Terms = 100
Original
Fourier Series
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Time (sec)



3/7d$_1$: Number of Terms = 10
Original
Fourier Series
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3
Time (sec)


7d2: Number of Terms = 10
Original
Fourier Series
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (sec)


3
7d3: Number of Terms = 10
Original
Fourier Series
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-2
-1
0
1
2
3
4
Time (sec)

```
%
%  Thsir routine implements a trigonometric Fourie Series
%
% Inputs: N is the number of terms to be used in the series
%
function Trig_Fourier_Series(N)
%
%  one period of the function goes from low to high
%
%  low = -2;
%  high = 2;
%
%  function f1
%
%   low = 0;
%   high = 3;
%
%  function f2
%
%  low = 0;
%  high = 4;
%
%  function f3
%
low = -2;
high = 4;
%
%  the difference between low and high is one period
%
 T = high-low;
 w0 = 2*pi/T;
%
%  the periodic function
%
  x  = @(t) 0.*(t<-1)+t.*((-1<=t)&(t<1))+0.*(t>=1);
  x = @(t) exp(-t);
  x = @(t) t.*(t<2)+3*((2<=t)&(t<3))+0.*(t>=3);
   x = @(t) 0.*(t<-1)+1*((-1<=t)&(t<2))+3*((2<=t)&(t<3))+0.*(t>=3);
%
%  get the average value
%
 a0 = (1/T)*quadl(x,low,high);
%
%   find b(1) to b(N), a(1) to a(N)
%
 for k = 1:N
     arg = @(t) x(t).*sin(k*w0*t);
     b(k) = (2/T)*quadl(arg,low,high);
     arg = @(t) x(t).*cos(k*w0*t);
     a(k) = (2/T)*quadl(arg,low,high);
 end;
%
```

```
% determine a time vector over one period

 t = linspace(low,high,1000);
%
% Find the Fourier series representation
%
 est = a0;
 for k = 1:N
     est = est + b(k)*sin(k*w0*t)+a(k)*cos(k*w0*t);
 end;
%
% plot the results
%
 plot(t,x(t),'-',t,est,'--'); grid; xlabel('Time (sec)');
 legend('Original','Fourier Series','Location','NorthWest');
 title(['7d_3: Number of Terms = ', num2str(N)]);
%
```